Arithmetick:

VULGAR,
DECIMAL,
INSTRUMENTAL,
ALGEBRAICAL,

Adams 7.67.15

In Four Parts.

Containing

I. VULGAR ARITHMETICK, In *whole Numbers* and *Fractions*, in a plain and easie method.

II. DECIMAL ARITHMETICK, The Ground and Reason thereof, and its Use illustrated by divers *Examples*.

III. INSTRUMENTAL ARITHMETICK, Performing, by *Decimal Scales*, all kind of *Reductions* (with more expedition than by *Decimal Tables*.) Also new *Scales*, whereby the *Square* and *Cube Roots* may be extracted by Inspection only: Both of them new *Artifices*; nothing of the like kind having been before published in any Language — With the Description of *Nepair's Bones* (according to their best Contrivance) and the use of them in *Multiplication*, *Division*, and *Extracting* of *Roots*.

IV. ALGEBRAICAL ARITHMETICK, Containing an Abridgment of the Precepts of that Art, and its Use, illustrated by Questions of divers kinds.

Whereunto is added, the Construction and Use of several Tables of *Interest* and *Annuities*, *Weights* and *Measures*, both of our own and other Countries.

The Fourth Edition, corrected and enlarged.

By WILLIAM LEYBOURN.

LONDON, Printed by *Tho. James* for *George Sawbridge*, at the *Bible* on *Ludgate-Hill*. 1678.

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~~96,781~~

TO THE READER.

HEre is presented unto thee a short *Treatise* of *Arithmetick*, I confess there are enough (if not too many) already extant, notwithstanding I have *adventured* to publish this, more out of *variety* than for any *necessity*, knowing that in the perusal thereof thou wilt find something worthy thy *Labour*, and what in other *Books* of this kind is wanting.

The whole *Treatise* is divided into four Parts. The first contains *VULGAR ARITHMETICK* in *whole Numbers* and *Fractions*: And in every *Rule* there are *Examples* for Practice added, and *Questions* also wrought by those single *Rules*. In *Multiplication* I have added divers *Compendiums*, or brief ways of *Multiplying*, whereby Sums (having 2 or 3 Figures in the Multiplier) may be performed, without any burthen or charge to the Memory, more than in ordinary *Multiplication*, and yet no other (or at most very few) Figures set down but the Product it self. And in *Division* (which is the most difficult of the four *Species*)

there are Five varieties, ſo that every man may make uſe of that which he beſt underſtands or fancies: And in the working of the *Golden Rule*, &c. I have (to expreſs variety) made uſe ſometimes of one kind of *Diviſion*, and ſometimes of another.

The Second Part contains *DECIMAL ARITHMETICK*, with the Ground and Reaſon thereof. Alſo *Tables* of the *Moneys*, *Weights*, and *Meaſures*, uſed in *England*, with Directions for the making of thoſe *Tables*, and of any other. And laſtly, there are *Examples* wrought in *Decimal Numbers*, in all the moſt uſual *Rules* of *Arithmetick*, and thoſe *Examples* are incumbered with as many *Fractions* as can poſſibly happen in any *Queſtion* concerning *Buying* or *Selling*.

Unto this Second Part there is added an *Appendix* containing certain *Rules* of *Exchanges*, with *Tables* of the *Weights* and *Meaſures* of *Forreign Countries*, compared with the *Weights* and *Meaſures* uſed in *London*, with an Example to illuſtrate the uſe of each *Table*: Alſo, There are ſeveral Tables calculated at *6 per Cent. Compound Intereſt*, by which the true valuation of any *Leaſe* or *Annuity*, or *Money forborn* or *rebated*, may be eaſily known, with an Example ſhewing the uſe of each Table.

The third Part is of *INSTRUMENTAL ARITHMETICK*, which performeth by a new Artifice, by me contrived, any *Queſtion Arithmetical*

Arithmetical in a *Decimal* way, without the help of *Decimal Tables*, by which the whole work of *Reduction* is avoided, there being certain *Scales* of *Engliſh Money*, *Weights*, and *Meaſures* divided, and ſo diſpoſed, that by them (by inſpection only) the *Decimal Fraction* of either *Money*, *Weight*, or *Meaſure*, may be ſet down as exactly, and in leſs time, than they could have been taken out of the *Decimal Tables* in the Second Part of this *Treatiſe*. And on the contrary, any *Decimal Fraction* may be reduced into its proper parts of the integer with the ſame facility, ſpeed and exactneſs. I have alſo in this Third Part (the better to illuſtrate the uſe, and commodiouſneſs of theſe *Scales*, a Figure whereof is inſerted at the beginning of the Third part) gone through all the moſt uſual *Rules* of *Arithmetick*, giving *Examples* in each *Rule*, by which the *Reader* may plainly perceive what labour there is ſaved by uſing the *Scales*, the whole work of *Reduction* being taken away, and the *Fraction* immediately ſet down at firſt view, without *Addition*; or being already ſet down, *reduced* to the known part of the *Integer*, without *Subſtraction*.

Unto theſe *Decimal Scales*, I have now added Two other *Scales*, for the *Extraction* of *Roots*; By the one you may find the *Root* of any *Number* the *Square* thereof being given; Or, if the *Root* be given, you may find the *Square Number* anſwering thereunto, and

that by inſpection only, without the help of either *Pen* or *Compaſs* — And as this Line doth for extracting the *Square Root*, the other doth the like for the *Cube Root*.

And to make this Third part of *Inſtrumental Arithmetick*, yet the more compleat, I have more largely (than in the former *Editions*) inſiſted upon the *Deſcription* and *Uſe* of *Nepairs Bones*; largely treating of their *Uſe* in *Multiplication*, *Diviſion*, and *Extraction* of *Square* and *Cube Roots*. And laſtly,

The Fourth Part containeth an *ABRIDGMENT of the PRECEPTS of ALGEBRA*, firſt written in French by *James de Billy*, a Tranſlation whereof came to my hands ſome years ſince: In the peruſal whereof, finding the *Precepts* very plain and eaſie, and conſidering that We have but very little of this kind of *Arithmetick* in our Engliſh Tongue; I have adventured to inſert it here as a Fourth Part, thereby to make this *Work* the more compleat; unto which Tranſlation there is further added divers *Queſtions* of good conſequence, which were not in the *Original*, as by comparing them together may appear.

This Treatiſe thus finiſhed, I freely offer unto thee, deſiring thy friendly acceptance, and pardon, for ſuch faults as may poſſibly have eſcaped the Preſs, or my ſelf, and in ſo doing thou wilt encourage him, who is

A Friend to all that are Mathematically affected,

William Leybourn.

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ADVER-

ADVERTISEMENT.

IF any Gentleman, or other Person, desire to be instructed in any of the Sciences Mathematical, as *Arithmetick*, *Geometry*, *Astronomy*, the use of the *Globes*, *Trignometry*, *Navigation*, *Surveying of Land*, *Dialling*, or the like; Either at their own Houses, his Habitation, or such other convenient Place as the Party shall direct, the *Author* hereof will be ready to attend them at times appointed.

Also, If any Persons would have their Land, or any Ground for Building *Surveyed*, or any Edifice or Building *Measured*, either for the *Carpenters*, *Bricklayers*, *Plaisterers*, *Glasiers*, *Joyners*, or *Masons* Work, he is ready to perform the same either for *Master Builder* or *Workman*.

Likewise, If any desire to have about his House or Garden, any kind of Sun-Dial, or Dials, of what kind soever, either fixed or movable, he will prepare or make for them such as they shall desire.

You may hear of him at the House of Mr. *Walter Hayes*, at the Cross-Daggers in *More-fields*, next door to the *Popes-Head* Tavern, who makes and selleth all sorts of Mathematical Instruments: Also, At Mr. *Robert Mordens* at the sign of the *Atlas* in *Corn-hill*, near the *Royal Exchange London*, where you may have the best and newest Globes of all Sizes, Maps, Sea Plates, and other Mathematical Instruments.

FINIS.

VULGAR Arithmetick.

The First Part.

Numeration.

NUMERATION, is accounted the First Part of *Arithmetick*, and it is to know how to read a Sum of figures expressed in writing; or to write down any Sum to be expressed.

To the doing of which there are four things necessary.

First, To know their *Number*, which is *Nine*.

Secondly, Their *shapes*, which are 1. 2. 3. 4. 5. 6. 7. 8. 9. Of which the first toward the left hand ever signifieth *One*, the second *Two*; &c.

Thirdly, To know the *value* of their *places*.

Laſtly, How their proper *ſignification* is altered thereby.

The *value* of their places is thus, When two, three, or more figures ſtand in one Sum, that is, without any *Point*, *Line* or *Comma* betwixt them, as 321., that place next the right hand where the figure 1 ſtandeth, is called the place of *Unity*, or *Unities*; and the figure 1 ſtandeth in that place only for *one*, and the figure 2 when it is found in that firſt place, ſtands only for *two*; and the like of the reſt.

But in the Sum 321, above expreſſed, the figure 2 is in the ſecond place, and every place contains the value of that place before towards the right hand ten times; and therefore the figure 2 doth not ſignifie *Two*, but (in this ſecond place) ten times two, that is *Twenty*. And ſo the figure 3, if it had been in that place had ſignified ten times *Three*, that is *Thirty*; but being here in the third place, it ſignifies ten times *Thirty*, that is *Three hundred*. And ſo the whole Sum 321, is to be read: *Three hundred twenty and one.*

It is hereby ſeen, how their proper ſignifications, which were *Three*, *Two*, and *One*, are altered by being thus placed, and the Sum, which otherwiſe had been but *Six*, is *Three hundred twenty one*, as before.

In like ſort if there had been more places, as Seven, the value is quite through increaſed ten times, by being a place bound towards the left hand; as in the Sum 1111111, The figure 1 in the ſecond place ſtands for ten times one (that is *Ten*,) in the third, for ten times ten (which is one *Hundred*,) in the fourth, for *Ten hundred*, (which is called One Thouſand,) in the fifth, for ten Thouſand, in the ſixth, for ten times *Ten thouſand*, (which is *One hundred thouſand*) in the laſt,

laſt (here the ſeventh) place, for *Ten hundred thouſand*, which is called a *Million*: and ſo on, if there were more places, obſerving the ſame order.

Now to read this readily, make a prick over the place of *Unity*, another the third from it, and over every third ſtill towards the left hand, for ſo thoſe points will be over the places of *Unities*, *Thouſands*, and *Millions*; and ſo beginning at the laſt, that is, at the left hand; read *One Million*, and becauſe the three following towards the right, ſignifie properly *One hundred* and *eleven*, but the prick belonging to them is in the place of thouſands, call them *One hundred and eleven thouſand*, and the three remaining being under the point over *Unity*, ſignifie only *One hundred* and *eleven*; and all three points read together in one ſum, is, *One million, one hundred* and *eleven Thouſand, one hundred and eleven.*

In like manner, if this number 73598624, were given to be read (according to former directions) make a prick over every third figure, beginning with the firſt figure towards the right hand, (which is the place of *Unity*) and then will your number ſtand thus,

7 3 5 9 8 6 2 4

Then for the ready reading thereof (becauſe the third prick ſignifieth *Millions*) call all the figures towards the left hand, ſtanding from that prick, Millions, which in this example are 7 and 3, ſo then this number contains 73 *Millions*, 598 *Thouſand*, 624 *Six hundred twenty four*, Which in words at length we read, *Seventy three Millions, five hundred ninety eight thouſand, ſix hundred twenty four.*

Let thus much ſuffice concerning the placing of large numbers, for the ready reading of them, only take theſe four *Tables* following, for illuſtration of

what hath been hitherto delivered in words, the very ſight whereof is better than a whole Chapter of information.

The firſt Table is thus to be read] *One* in the firſt place, ſignifies *One*. *One* in the ſecond place, ſignifies Ten. *One* in the third place, ſignifies a *hundred*, &c. as in the Table.

The ſecond Table is thus to be read] In this Table you ſhall find the laſt number thereof to conſiſt of theſe figures, 357. 846. 903. with a point or comma betwixt every third figure, for diſtinction ſake, and alſo every three figures in their order are connected together with this ⏞ brace, which denominates the places of *Millions*, *Thouſands*, *Hundreds*; ſo that the laſt number of this Table will evidently appear to be 357 *Millions*, 846 *Thouſands*, 903 *Nine hundred and three.*

The third Table] Is only certain rows of figures, ſet together, and orderly diſpoſed, having the ſignification or reading of the ſame Numbers in words at length to them annexed, and is only inſerted for the better ſatisfaction of ſuch as ſhall doubt whether they perfectly underſtand what hath been before taught.

The fourth Table] Is much like the ſecond, only it conſiſteth but of one number and extends three places farther then the greateſt number in the ſecond Table doth: *viz.* to twelve places; which figures are thus to be read, 736 *Millions of Millions*, 842 *Millions*, 708 *Thouſand*, 645 *Six hundred forty five.*

(I. *Table.*)

One in the	place signifies
first	1 one
second	10 ten
third	100 a hundred
fourth	1000 a thousand
fifth	10000 ten thousand
sixth	100000 a hundred thousand
seventh	1000000 a Million
eigth	10000000 ten Millions
ninth	100000000 a hundred Millions

(II. TABLE.)

(III. Table.)

Number of places.		
1	8	(eight.
2	54	(fifty four.
3	762	(seven hundred sixty two.
4	3483	(three thousand four hundred eighty three.
5	97621	(ninety seven thousand, six hundred twenty one.
6	243794	(two hundred 43 thousand 7 hundred 94.
7	8749207	(eight millions, 749 thousand 8 hundred and 7.
8	73162481	(fifty seven mil. 3 hund. and 16 thous. 248
9	357846983	(three hundred fifty seven millions, eight hundred 46 thous. 9 hund. 83

(IIII. Table.)

7	hundred	millions of millions
3	ten	
6	one	
8	hundred	millions
4	ten	
2	one	
7	hundred	thousands
0	ten	
8	one	
6	hundred	hundreds
4	ten	
5	one	

Addition.

Addition.

ADDITION, is the collecting or gathering together of two or more ſums, either of one, or of divers denominations; into one ſum, which is called the [*Aggregate*] [*Total*] or [*Groſs ſum.*]

In Addition of Numbers of one denomination, the order is, to ſet the numbers to be added one directly under the other; that is to ſay, *Unites* under *Unites*, *Tens*, under *Tens Hundreds* under *Hundreds*, &c.

The Rule.

Having placed your numbers to be added in due order, one under another; draw a line under them, and begin at the lowermoſt figure towards your right hand, and add that to the next figure above, and the ſum of them to the next figure above that; proceeding in this order, till you have added the whole line together; which when you have done, conſider how many tens are contained in that line; and for every ten, keep one Unite *in your minde, to be added to the next row; but if there be any odd* Digits, *you muſt ſet them beneath the ſtroak, juſt under the line you added together; Having thus finiſhed the addition of one line, proceed to the next; and from thence to the third; and ſo forward, be there never ſo many.* The examples following will make this plain.

Example 1. Let the numbers given to be added together be 7832, 5609, 376, 8547, having thus placed

ced them in order one under another, as in the Margine is done; draw a line under them, then begin your Addition, at the lowermoſt figure towards your right hand; Saying 7 and 6 is 13, and 9 is 22, and 2 is 24: Now (becauſe in 24 there is two tens, and 4 remaining) I place the 4 under the line, and carry the two tens to the next row of *Tens*; Saying, 2 which I carried and 4 makes 6, and 7 makes 13, and 3 makes 16; in which row there is but one ten contained, and 6 remaining, which 6 I ſet under the line, and carry the ten to the next row of *Hundreds*, ſaying, 1 that I carried and 5 makes 6 and 3 makes 9, and 6 makes 15, and 8 makes 23, in which 23, ten is contained two times, and 3 remaining; the 3 I place under the line, and carry the two tens to the next row of *Thouſands*, ſaying, 2 which I carried and 8 makes 10, and 5 makes 15, and 7 makes 22, in which, ten is contained two times, and 2 remaining; which 2 I ſet under the line, and becauſe there is never another row to be added (to which I ſhould carry the two tens) I therefore ſet 2 down alſo under the line towards the left hand, as you ſee done in the margine: So the *total* or *croſs ſum* of theſe numbers, being added together, is 22364.

Thouſands	Hundreds	Tens	Unites
7	8	3	2
5	6	0	9
	3	7	6
8	5	4	7
22	3	6	4

Example 2. *A Man hath in his Orchard* 136 *Apple-Trees*, 76 *Pear-trees*, 107 *Cherry-Trees*, *and* 36 *Plum-Trees*, *and he deſires readily to know how many Trees he hath in all.*

Place your numbers one under another, as in the margine, and then begin to add them together, at

your right hand, Saying, 6 and 7 make 13, and 6 make 19, and 6 make 25; place 5 under the line, and carry 2 to the next row; Saying, 2 and 3 is 5, and 7 is 12, and 3 is 15, place 5 under the line, and carry 1 to the next row; Saying, 1 and 1 is 2, and 1 is 3; which 3 I set under the line, and (becauſe there was no tens contained in that line, therefore) the total is 355, and ſo many *Trees* are in the *Orchard*.

Apple-trees	136
Pear-trees	76
Cherry-trees	107
Plum-trees	36
Trees in all	355

Other Examples for Practice.

95432	321	9161
76100	1986	235
2570	23	72
832	1107	9
Total 174934	Total 3437	Total 9477

Addition of Numbers of divers Denominations.

I. Addition *of Engliſh Money.*

The moſt uſual Coyns uſed in *England* are *Pounds*, *Shillings*, *Pence*, and *Farthings*, of which

4 Farthings, 12 Pence, 20 Shillngs } make { 1 Penny, 1 Shilling, 1 Pound } thus Charactred. { d. s. li.

For a Farthing we uſe *q*.

THE

ced them in order one under another, as in the Margine is done; draw a line under them, then begin your Addition, at the lowermoſt figure towards your right hand; Saying 7 and 6 is 13, and 9 is 22, and 2 is 24: Now (becauſe in 24 there is two tens, and 4 remaining) I place the 4 under the line, and carry the two tens to the next row of *Tens*; Saying, 2 which I carried and 4 makes 6, and 7 makes 13, and 3 makes 16; in which row there is but one ten contained, and 6 remaining, which 6 I ſet under the line, and carry the ten to the next row of *Hundreds*, ſaying, 1 that I carried and 5 makes 6 and 3 makes 9, and 6 makes 15, and 8 makes 23, in which 23, ten is contained two times, and 3 remaining; the 3 I place under the line, and carry the two tens to the next row of *Thouſands*, ſaying, 2 which I carried and 8 makes 10, and 5 makes 15, and 7 makes 22, in which, ten is contained two times, and 2 remaining; which 2 I ſet under the line, and becauſe there is never another row to be added (to which I ſhould carry the two tens) I therefore ſet 2 down alſo under the line towards the left hand, as you ſee done in the margine: So the *total* or *groſs ſum* of theſe numbers, being added together, is 22364.

Thouſands	Hundreds	Tens	Unites
7	8	3	2
5	6	0	9
	3	7	6
8	5	4	7
22	3	6	4

Example 2. *A Man hath in his Orchard* 136 *Apple-Trees*, 76 *Pear-trees*, 107 *Cherry-Trees*, *and* 36 *Plum-Trees*, *and he deſires readily to know how many Trees he hath in all.*

Place your numbers one under another, as in the margine, and then begin to add them together, at

your right hand, Saying, 6 and 7 make 13, and 6 make 19, and 6 make 25; place 5 under the line, and carry 2 to the next row; Saying, 2 and 3 is 5, and 7 is 12, and 3 is 15, place 5 under the line, and carry 1 to the next row; Saying, 1 and 1 is 2, and 1 is 3; which 31 ſet under the line, and (becauſe there was no tens contained in that line, therefore) the total is 355, and ſo many *Trees* are in the *Orchard*.

Apple-trees	136
Pear-trees	76
Cherry-trees	107
Plum-trees	36
Trees in all	355

Other Examples for Practice.

95432	321	9161
76100	1986	235
2570	23	72
832	1107	9
Total 174934	Total 3437	Total 9477

Addition of Numbers of divers Denominations.

I. Addition *of Engliſh Money.*

The moſt uſual Coyns uſed in *England* are *Pounds*, *Shillings*, *Pence*, and *Farthings*, of which

4 Farthings, 12 Pence, 20 Shillings } make { 1 Penny, 1 Shilling, 1 Pound } thus Charactred. { d. s. li.

For a Farthing we uſe *q*.

THE

THE RULE.

In the Addition of Numbers of divers denominations this order is to be observed, *viz. Place all numbers of the same denomination one directly under another*, as Pounds *under* Pounds, Shillings *under* Shillings, Pence *under* Pence, *and* Farthings *under* Farthings. *Then draw a line under them, and begin your Addition with the least denomination first; Observing how many times the next greater denomination is contained in that least: and for every time carry one unite to the next denomination, as before you did the tens, setting down the remainder, if any be; Then adding the next denomination together, take notice how many times the next greater denomination is contained in the lesser; carrying for every time one to the next greater denomination.* Thus proceeding till you have gone over all the *denominations*, be they never so many.

Example 1. Let the numbers to be added together be 37 *l.* 16 *s.* 9 *d.* 3 *q.* 21 *l.* 9 *s.* 8 *d.* 1 *q.* 13 *l.* 12 *s.* 9 *d.* 2 *q.* Place the numbers as in the margine, draw a line under them, and begin with the least denomination (which in this example is *farthings*,) first, Saying, 2 *q.* and 1 *q.* is 3 *q.* and 3 *q.* is 6 *q.* which is one peny and 2 *q.* remaining; which 2 *q.* I place under the line, and carry the one peny to the next row, which is the place of pence; Saying, one peny and 9 *d.* is 10 *d.* and 8 *d.* is 18 *d.* which is 1 *s.* and 6. *d.* (*Now against the* 8, *I make a prick with my pen, for my better remembrance; to signifie, that there is one shilling to be carried to the place of shillings,*) then go on and say, 6, *d.* and 9. *d.* is 15 *d.* which is 1 *s.* and 3 *d.* therefore

li.	*s.*	*d.*	*q.*
37	16	9.	3.
21	09.	8.	1
13	12	9	2
72.	19	3	2

fore againſt 9, I make a prick with my pen, and (becauſe that is the laſt number) I ſet down the odd 3 *d.* under the place of pence, and (being I find two pricks in the line of pence, therefore) I carry 2 *s.* to the place of ſhillings, ſaying, 2 *s.* which I carried, and 12 *s.* is 14 *s.* and 9 *s.* is 23 *s.* which is one pound and 3 *s.* remaining, I make a prick againſt 9, and going on, ſay, 3 *s.* and 16 *s.* is 19 *s.* which (being there is no more numbers to be added, and being alſo leſs than 20 *s.*) I ſet under the line, and finding one prick in the line of ſhillings, I therefore carry one to the place of pounds, ſaying, one which I carried and 3 is 4, and 1 is 5, and 7 is 12. Set down the 2 under the line (as in addition of numbers of one denomination) and carry one to the next row; Saying, one that I carried and 1 is 2, and 2 is 4, and 3 is 7, which being the laſt I ſet down, and ſo the total or groſs ſum is 72 *l.* 19 *s.* 3 *d.* 2 *q.*

Example 2. Let the numbers to be added be 29 *l.* 16 *s.* 8 *d.* 32 *l.* 17 *s.* 9 *d.* 81 *l.* 13 *s.* 11 *d.* and let it be required to find the total or groſs ſum. Here in this Example the leaſt denomination is *pence*, therefore I begin with them, and ſay, 11 *d.* and 9 *d.* is 20 *d.* which is 1 *s.* and 8 *d.* make a prick againſt the 9, and ſay 8 *d.* and 8 *d.* is 16 *d.* that is 1 *s.* and 4 *d.* make a prick againſt the 8, and ſet down the odd 4 *d.* Then (becauſe there are two pricks in the line of pence) you muſt carry 2 *s.* to the place of ſhillings, Saying, 2 *s.* which I carry, and 13 *s.* is 15 *s.* and 17 *s.* is 32 *s.* which is 1 *li.* 12 *s.* make a prick againſt 17, and ſay 12 *s.* and 16 *s.* is 28 *s.* make a prick againſt 16, and (becauſe there is no more numbers to be added) ſet down the odd 8 *s.* under ſhillings, and (being there is two pricks in the line of ſhillings)

li.	*s.*	*d.*
29	16	8.
32	17.	9.
81	13	11
144	08	4

shillings) carry 2 to the place of pounds, saying, 2 and 1 is 3, and 2 is 5, and 9 is 14, set down 4 and carry 1 to the next line; and say 1 and 8 is 9, and 3 is 12, and 2 is 14, which (becaus it is the last) you set down, so is the total or gross sum 144 *li.* 8 *s.* 4 *d.*

Other Examples for Practice.

li.	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*
29	18	7	3	36	2	8.
63	11	2.	1.	29	0	2
129	4	0	2	31	16.	9.
3	7	10	1	6	2	5
226	1	8	3	103	2	0

II. *Addition of* Troy-weight.

Troy-weight is a weight used in *England*, by the which is weighed, *Bread*, *Gold*, *Silver*, *Pearl*, &c. the most usual denominations of which weight are *Pounds*, *Ounces*, *Peny-weights*, and *Grains*; of which

24 Grains
20 Peny-weight } make
12 Ounces

1 Peny-weight
1 Ounce
1 Pound

thus charact. *pw.* *ou.* *lib.*

for a grain we write *gr.*

The Addition of *Troy-weight* (and consequently of any other weight or measure whatsoever, either Domestique, or Forreign) differeth nothing at all from the Addition of English Coin last taught, if the affinity of one denomination to another be first known; for

for whereas in Money, becauſe 12 *d.* make 1 *s.* you therefore obſerve how many twelves there are in the addition of your pence, and for every 12 you add one ſhilling to the place of ſhillings, ſo in the addition of *Troy-weight*, knowing that 24 *gr.* make one *peny-weight*, you muſt therefore in the addition of *Grains* of *Troy-weight*, obſerve how many times 24 you find in your line of *Grains*, and for every 24, carry one to the place of *peny-weights*; likewiſe, in the addition of *peny-weights*, you muſt conſider how many times 20 is contained in your line, and for every 20 carry one to the place of *ounces*, (becauſe 20 peny-weights make an ounce.) Alſo in the addition of *Ounces* Troy, you muſt obſerve how many times 12 you find in your line of *ounces*, and for every 12 carry one to the place of Pounds; Then laſtly, Add your Pounds together, as numbers of one denomination.

Example. Let the numbers to be added together be 7 *lb.* 11 *ou.* 13 *pw.* 19 *gr.* 6 *lb.* 7 *ou.* 16 *pw.* 19 *gr.* 3 *lb.* 7 *ou.* 9 *pw.* 6 *gr.* Place your numbers as in Addition of Money, each under other, according to their reſpective denominations, as in the margine: then draw a line under them, and begin your Addition with the leaſt denomination firſt, *viz.* Grains, Saying 6 *gr.* and 19 *gr.* is 25 *gr.* which is one peny-weight and one grain, make a prick againſt 19, and carry the odd grain to the number above, ſaying, 1 *gr.* and 19 *gr.* is 20 *gr.* which (becauſe it is leſs than one peny weight) I let under the line, then finding one prick in the line of grains, (I therefore) carry one to the place of peny-weights, ſaying 1 and 9 is 10, and 16 is 26, which is one ounce, and 6 *pw.* make a

lb.	*ou.*	*pw.*	*gr*
7	11	3.	19.
6	07	16	19.
3	07	09	06
18	02	19	20

a prick againſt 16, and ſay 6 and 13 is 19, which (being leſs than an ounce) ſet under the line, then for the one prick, carry 1 to the place of ounces, ſaying 1 and 7 is 8, and 7 is 15, which is one pound and 3 *ounces*, make a prick at 7, and ſay 3 and 11 is 14, which is one pound and 2 ounces, make a prick againſt 11, and ſet down the 2 ounces, and for the 2 pricks carry 2 pounds to the place of pounds, ſaying 2 and 3 is 5, and 6 is 11, and 7 is 18, which ſet under the place of pounds, ſo is your Addition ended, and the ſum is 18 *lb.* 2 *ou.* 19 *pw.* 20 *gr.*

Other Examples for Practice,

lb.	*ou.*	*pw.*	*gr.*	*lb.*	*ou.*	*pw.*	*gr.*
32	9.	12	16	0	10	17.	11
17	11.	6	9.	0	6.	0	5
34	8.	15.	10	0	0	19	8.
8	10	4	7	0	5	2	19
94	3	18	18	1	10	16	19

III. *Addition of* Avoirdupois little weight.

There is another kind of weight moſt commonly uſed in *England*, called *Avoirdupois little weight*, by which is weighed all ſorts of Wares or Merchandize Garblable, as *Sugar*, *Pepper*, *Cloves*, &c. This weight is commonly divided into theſe denominations, *Pounds*, *Ounces*, *and Drams*, of which

16 Drams, 16 Ounces } make { 1 Ounce, 1 Pound } thus characted { *ou.*, *lb.*

For a Dram we write *dr.*

I

In the Addition of *Avoirdupois-weight*, you must observe the very same method and order as in *Money* and *Troy-weight*, having due respect to the quantity of the denominations; as in the Addition of *drams* to make a prick at every 16, setting down the remainder, and for every prick carrying a unite to the next place. The preceding Rules being so copious in this particular, I shall forbear to make any verbal illustration; but only give you some *Examples* ready wrought, together with the most usual parts into which the *Weights* and *Measures* now used in *England* are divided; which to the ingenious will be sufficient.

Examples *of Addition of* Avoirdupois little weight.

lb.	*oz.*	*dr.*	*lb.*	*oz.*	*dr.*
12	11.	09	06	13.	07.
76	05	12.	05	09.	12
32	10.	00	06	03	09.
91	07.	13.	10	00	00
32	13.	07	05	07	09
246	00	09	34	02	05

IV. *Addition of* Avoirdupois great Weight.

There is a weight commonly used in *England*, by which is weighed all commodities that are sold by the hundred, as *Corants*, *Wooll*, *Flesh*, *Butter*, *Cheese*, and the like, the which hundred weight containeth 112 pounds, and the hundred weight is divided into *Quarters*, *Pounds*, and *Ounces*, so that

16 Ounces, 28 Pounds, 4 Quarters } make { 1 Pound, 1 quarter of a C., 1 Hund. weig. } thus charact. { *lb.*, *qr.*, *C.*

For an Ounce we write *on.*

Exam-

a prick againſt 16, and ſay 6 and 13 is 19, which (being leſs than an ounce) ſet under the line, then for the one prick, carry 1 to the place of ounces, ſaying 1 and 7 is 8, and 7 is 15, which is one pound and 3 *ounces*, make a prick at 7, and ſay 3 and 11 is 14, which is one pound and 2 ounces, make a prick againſt 11, and ſet down the 2 ounces, and for the 2 pricks carry 2 pounds to the place of pounds, ſaying 2 and 3 is 5, and 6 is 11, and 7 is 18, which ſet under the place of pounds, ſo is your Addition ended, and the ſum is 18 *lb.* 2 *ou.* 19 *pw.* 20 *gr.*

Other Examples for Practice,

lb.	*ou.*	*pw.*	*gr.*	*lb.*	*ou.*	*pw.*	*gr.*
32	9.	12	16	0	10	17.	11
17	11.	6	9.	0	6.	0	5
34	8.	15.	10	0	0	19	8.
8	10	4	7	0	5	2	19
94	3	18	18	1	10	16	19

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There is another kind of weight moſt commonly uſed in *England*, called *Avoirdupois little weight*, by which is weighed all ſorts of Wares or Merchandize Garblable, as *Sugar*, *Pepper*, *Cloves*, &c. This weight is commonly divided into theſe denominations, *Pounds*, *Ounces*, *and Drams*, of which

16 Drams, 16 Ounces } make { 1 Ounce, 1 Pound } thus characted { *ou.* *lb.*

For a Dram we write *dr.*

I

In the Addition of *Avoirdupois-weight*, you muſt obſerve the very ſame method and order as in *Money* and *Troy-weight*, having due reſpect to the quantity of the denominations; as in the Addition of *drams* to make a prick at every 16, ſetting down the remainder, and for every prick carrying a unite to the next place. The preceding Rules being ſo copious in this particular, I ſhall forbear to make any verbal illuſtration; but only give you ſome *Examples* ready wrought, together with the moſt uſual parts into which the *Weights* and *Meaſures* now uſed in *England* are divided; which to the ingenious will be ſufficient.

Examples *of Addition of* Avoirdupois little weight.

lb.	*on.*	*dr.*	*lb.*	*on.*	*dr.*
12	11.	09	06	13.	07.
76	05	12.	05	09.	12
32	10.	00	06	03	09.
91	07.	13.	10	00	00
32	13.	07	05	07	09
246	00	09	34	02	05

IV. *Addition of* Avoirdupois great Weight.

There is a weight commonly uſed in *England*, by which is weighed all commodities that are ſold by the hundred, as *Corants*, *Wooll*, *Fleſh*, *Butter*, *Cheeſe*, and the like, the which hundred weight containeth 112 pounds, and the hundred weight is divided into *Quarters*, *Pounds*, and *Ounces*, ſo that

16 Ounces, 28 Pounds, 4 Quarters } make { 1 Pound, 1 quarter of a C., 1 Hund. weig. } thus charact. { *lb.*, *qr.*, *C.*

For an Ounce we write *on.*

Exam-

a prick againſt 16, and ſay 6 and 13 is 19, which (being leſs than an ounce) ſet under the line, then for the one prick, carry 1 to the place of ounces, ſaying 1 and 7 is 8, and 7 is 15, which is one pound and 3 *ounces*, make a prick at 7, and ſay 3 and 11 is 14, which is one pound and 2 ounces, make a prick againſt 11, and ſet down the 2 ounces, and for the 2 pricks carry 2 pounds to the place of pounds, ſaying 2 and 3 is 5, and 6 is 11, and 7 is 18, which ſet under the place of pounds, ſo is your Addition ended, and the ſum is 18 *lb.* 2 *on.* 19 *pw.* 20 *gr.*

Other Examples for Practice,

lb.	*on.*	*pw.*	*gr.*	*lb.*	*on.*	*pw.*	*gr.*
32	9.	12	16	0	10	17.	11
17	11.	6	9.	0	6.	0	5
34	8.	15.	10	0	0	19	8.
8	10	4	7	0	5	2	19
94	3	18	18	1	10	16	19

III. *Addition of* Avoirdupois little weight.

There is another kind of weight moſt commonly uſed in *England*, called *Avoirdupois little weight*, by which is weighed all ſorts of Wares or Merchandize Garbleable, as *Sugar*, *Pepper*, *Cloves*, &c. This weight is commonly divided into theſe denominations, *Pounds*, *Ounces*, *and Drams*, of which

16 Drams, 16 Ounces } make { 1 Ounce, 1 Pound } thus characted { *on.* *lb.*

For a Dram we write *dr.*

1

In the Addition of *Avoirdupois-weight*, you must observe the very same method and order as in *Money* and *Troy-weight*, having due respect to the quantity of the denominations; as in the Addition of *drams* to make a prick at every 16, setting down the remainder, and for every prick carrying a unite to the next place. The preceding Rules being so copious in this particular, I shall forbear to make any verbal illustration; but only give you some *Examples* ready wrought, together with the most usual parts into which the *Weights* and *Measures* now used in *England* are divided; which to the ingenious will be sufficient.

Examples *of Addition of* Avoirdupois little weight.

lb.	*on.*	*dr.*	*lb.*	*on.*	*dr.*
12	11.	09	06	13.	07.
76	05	12.	05	09.	12
32	10.	00	06	03	09.
91	07.	13.	10	00	00
32	13.	07	05	07	09
246	00	09	34	02	05

IV. *Addition of* Avoirdupois great Weight.

There is a weight commonly used in *England*, by which is weighed all commodities that are sold by the hundred, as *Corants*, *Wooll*, *Flesh*, *Butter*, *Cheese*, and the like, the which hundred weight containeth 112 pounds, and the hundred weight is divided into *Quarters*, *Pounds*, and *Ounces*, so that

16 Ounces } make { 1 Pound
28 Pounds } make { 1 quarter of a C.
4 Quarters } make { 1 Hund. weig.

thus charact. *lb.* *qr.* *C.*

For an Ounce we write *on.*

Exam-

Examples *of Addition of* Avoirdupois great Weight.

C.	*qr.*	*li.*	*ou.*	*C.*	*qr.*	*li.*	*ou.*
37	03.	21	12	05	01	07	07
09	01	06	03	03.	02	18.	06.
33.	02	20.	00	00	01	06	08
10	00	00	00	11	03.	04	00
12	03.	07	03	06	01	10	05
103	02	27	02	17	01	11	10

I might farther proceed to give you Examples of Addition of common *Engliſh Meaſures*, *viz.* of *Long meaſures*, *Liquid meaſures*, and *Dry meaſures*, as alſo of *Time*, *Motion*, &c. but the preceding Examples being of ſufficient extent, I ſhall forbear to trouble either my ſelf or the Reader with that which I conceive ſuperfluous: Only, before I leave *Addition*, I will give you a brief view of the moſt uſual *Meaſures* uſed in *England*, which take as followeth. And

V. *Of* Liquid Meaſures.

Liquid Meaſures are thoſe by which all ſorts of Liquid ſubſtances are meaſured, of which (according to the Statute of 12 *Hen.* 7 *chap.* 5.) a *Pint* is the leaſt, from which the greater *Liquid meaſures* are deduced, according as is expreſſed in the Table following.

	make	
2 Pints		1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon
8 Gallons		1 Firkin of Ale, Sope, or
9 Gallons		1 Firkin of Beer (Herrings.
10 ½ Gallons		1 Firkin of Salmon or Eels
2 Firkins		1 Kilderkin
2 Kilderkins		1 Barrel
42 Gallons		1 Tierce of Wine
63 Gallons		1 Hogſhead
2 Hogſheads		1 Pipe or Butt
2 Pipes or Butts		1 Tun of Wine

VI. *Of*

VI. *Of* Dry Measures.

Dry Measures are these by which all kind of dry substances are measured, as *Corn*, *Salt*, *Coles*, *Sand*, *&c.* of which a pint is the least.

2 Pints	make	1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon
2 Gallons		1 Peck
4 Pecks		1 Bushel Land Measure
5 Pecks		1 Bushel Water measure.
8 Bushels		1 Quarter
4 Quarters		1 Chaldron
5 Quarters		1 Wey.

VII. *Of* Long Measures.

Long Measure is that by which is measured *Cloth*, *Land*, *Board*, *Glass*, *Pavement*, *Tapestry*, &c. of which measures (according to the Statute of 33 *Ed.* 1. and 25 *El.*) a *Barley Corn* is the least. So that

3 Barley Corns	make	1 Inch
2 Inches		1 Foot
3 Foot		1 Yard
3 Foot 9 Inches		1 Ell
6 Foot		1 Fadome
5½ Yards, or 16½ Foot		1 Pole or Perch
40 Perches		1 Furlong
8 Furlons		1 English mile.

VIII. *Of* Time.

Time consisteth of *Years*, *Months*, *Weeks*, *Days*, *Hours* and *Minutes*. So that

Examples *of Addition of* Avoirdupois great Weight.

C.	qr.	li.	ou.	C.	qr.	li.	ou.
37	03.	21	12	05	01	07	07
09	01	06	03	03.	02	18.	06.
33.	02	20.	00	00	01	06	08
10	00	00	00	11	03.	04	00
12	03.	07	03	06	01	10	05
103	02	27	02	17	01	11	10

I might farther proceed to give you Examples of Addition of common *Engliſh Meaſures*, *viz.* of *Long meaſures*, *Liquid meaſures*, and *Dry meaſures*, as alſo of *Time*, *Motion*, &c. but the preceding Examples being of ſufficient extent, I ſhall forbear to trouble either my ſelf or the Reader with that which I conceive ſuperfluous: Only, before I leave *Addition*, I will give you a brief view of the moſt uſual *Meaſures* uſed in *England*, which take as followeth. And

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Liquid Meaſures are thoſe by which all ſorts of Liquid ſubſtances are meaſured, of which (according to the Statute of 12 *Hen.* 7 *chap.* 5.) a *Pint* is the leaſt, from which the greater *Liquid meaſures* are deduced, according as is expreſſed in the Table following.

	make	
2 Pints		1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon
8 Gallons		1 Firkin of Ale, Sope, or
9 Gallons		1 Firkin of Beer (Herrings.
10 ½ Gallons		1 Firkin of Salmon or Eels
2 Firkins		1 Kilderkin
2 Kilderkins		1 Barrel
42 Gallons		1 Tierce of Wine
63 Gallons		1 Hogſhead
2 Hogſheads		1 Pipe or Butt
2 Pipes or Butts		1 Tun of Wine

VI. *Of*

VI. *Of* Dry Meaſures.

Dry Meaſures are theſe by which all kind of dry ſubſtances are meaſured, as *Corn*, *Salt*, *Coles*, *Sand*, &c. of which a pint is the leaſt.

2 Pints	make	1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon
2 Gallons		1 Peck
4 Pecks		1 Buſhel Land Meaſure
5 Pecks		1 Buſhel Water meaſure.
8 Buſhels		1 Quarter
4 Quarters		1 Chaldron
5 Quarters		1 Wey.

VII. *Of* Long Meaſures.

Long Meaſure is that by which is meaſured *Cloth*, *Land*, *Board*, *Glaſs*, *Pavement*, *Tapeſtry*, &c. of which meaſures (according to the Statute of 33 *Ed.* 1. and 35 *El.*) a *Barley Corn* is the leaſt. So that

3 Barley Corns	make	1 Inch
2 Inches		1 Foot
3 Foot		1 Yard
3 Foot 9 Inches		1 Ell
6 Foot		1 Fadome
5½ Yards, or 16½ Foot		1 Pole or Perch
40 Perches		1 Furlong
8 Furlons		1 Engliſh mile.

VIII. *Of* Time.

Time conſiſteth of *Years*, *Months*, *Weeks*, *Days*, *Hours* and *Minutes*. So that

60 Minutes		1 Hour
24 Hours		1 Day Natural
7 Dayes		1 Week
4 Weeks	make	1 Moneth of 28 days
13 Moneths, one day 6 hours.		1 Year

IX. *Of* Apothecaries Weight.

The Weights uſed by *Apothecaries* are *Grains, Scruples, Drams*, and *Ounces*, of which

20 Grains		1 Scruple	℈
3 Scruples	make	1 Dram	ʒ
8 Drams		1 Ounce	℥
12 Ounces		1 Pound	lb

thus charact.

By help of theſe Tables, and the rules and cautions before expreſſed, any Man may make Addition of any of the above-ſaid meaſures one with another, and therefore I ſhall forbear to illuſtrate them by Examples, but leave them to every Mans own practice, and thus I conclude *Addition*.

The Proof of Addition.

Having placed your numbers in order, and added them together, and ſet the Total under the line, *cut off the upper number by drawing a line with your Pen betwixt that and the others, then add all the numbers together except the uppermoſt, and ſet the Total of them under the Total before found; then add this laſt Total, and the firſt number which you cut off with your Pen together, and if the ſum of thoſe two numbers be equal with your Total ſum firſt found, then is your work right, otherwiſe not.*

Example. In the firſt Example of whole numbers, the ſums to be added were 7833, 5609, 376, and 8547, theſe numbers placed in due order, and added together, the total or groſs ſum of them is 22364, now to prove whether this Total be true or not,

not, I cut off the uppermoſt number, (to wit 7832,) with a daſh of the Pen, and I add the other three numbers together, namely, 5609, 376, and 8547, and the Total of them is 14532; which number being added to 7832 (the number cut off) the ſum of them is 22364, exactly agreeing with the Total firſt found; clearly evidencing that the Addition was truly performed: but if they had diſagreed, then the work had been erroneous. The like courſe muſt be taken for the proof of thoſe ſums which have different denominations, as in *Money* and *Weight*, as by the examples following will appear.

	7832
	5609
	376
	8547
firſt Total	223664
laſt Total	14532
Proof	22364

Other Examples proved.

1 *Example of* Money.

	li.	*s.*	*d.*	*q.*
	37	16	9	3
	21	9	8.	1
	13	12	9	2
1 Total	72	19	3	2
2 Total	35	2	5	3
Proof	72	19	3	2

2 *Example of* Troy-Weight.

	lb.	*oũ.*	*pw.*	*g.*
	32	9	12	16
	17	11	6	9
	34	8	15	10
	8	10	4	7
	94	3	18	18
	61	6	6	2
	94	3	18	18

There are other ways to prove *Addition*, by caſting away all the *nines* in numbers of one denomination, and of all the *twelves*, *twenties*, and *nines*, in *pounds*, *ſhillings* and *pence*, &c. but this as the moſt certain and eaſie I imbrace; and thus much for Addition, and the proof thereof.

Subſtraction.

SUBSTRACTION is the taking of one or more ſmall ſums out of a greater, as 7 *s.* out of 12 *s.* or 37 *li.* out of 100 *li.* or 137 foot out of 983 foot, and the like.

As in *Addition*, the ſums to be added may be either of one, or of divers denominations; ſo likewiſe they may be in *Subſtraction*, and the manner of placing them is the ſame; for you muſt ſet *Unites* under *Unites*, *Tens* under *Tens*, *Hundreds* under *Hundreds*, &c.

Example 1. Of *Subſtraction* of numbers of one denomination. *Let it be required to Subſtract* 234 *out of* 986. Place the numbers one under the other as you ſee done in the Margine, draw a line under them, and begin with the firſt figure towards your right hand, which is 4; ſaying, take 4 from 6, and there remains 2, place 2 under the line, and go to the next figure which is 3; ſayig, take 3 from 8, and there remains 5, place 5 under the line, and go to the next figure which is 2; ſaying, take 2 out of 9 and there remains 7, place 7 under the line, and your Subſtraction is ended; and it is evident by the work, that if you take 234 out of 986, there will remain 752, which you may thus prove. For if you add the 234, to 752, you ſhall find the ſum of that addition to be 986, which is equal to the whole ſum from which 234 was ſubſtracted.

Number given	986
Number to be ſubſtracted	234
Remainder.	752

Example

Example 2. *Let it be required to ſubſtract* 2976 *out of* 96527. Place the numbers one under another, as in the Margine you ſee done; then draw a line under them, and beginning with the firſt figure towards your left hand; ſay, take 6 out of 7, and there remains 1, place 1 under the line, and proceed to the next figure; ſaying, 7 out of 2 I cannot (wherefore you muſt always add [10] to the number above, which in this Example is 2, and it makes it 12, (therefore take 7 out of 12, and there remains 5, place 5 under the line, and (becauſe you added 10 to the 2 to make it 12, you muſt) carry a Unite to the next figure; ſaying, one which I carried and 9 is 10, take 10 out of 5, which I cannot; therefore I muſt add 10 to 5, and it makes 15, and ſay 10 out of 15, and there remains 5; place 5 under the line, and (becauſe you added 10 to 5 to make it 15, you muſt therefore) carry a unite to the next figure: ſaying, one which I carried, and 2 is 3, take 3 out of 6, and there remains 3, place 3 under the line, and becauſe there is no more figures to be ſubſtracted from the number above, you muſt ſay, nothing from 9, and there remains 9, ſet the 9 under the line, and your Subſtraction is ended.

96527	
2976	
93551	

Other Examples for Practice.

	li.	*Reams of Paper*		*Sheep.*
Lent	5762	Bought	9765	From 1000
Paid	378	Sold	6529	Take 394
Reſts to pay	384	Unſold	3236	Remains 606

Subſtraction of Numbers of divers denominations.

Of Engliſh Money.

In Subſtraction *of Numbers of divers denominations, you muſt obſerve the ſame order as in* Addition, *namely, to place every number in due order, with reſpect to the denomination, as* pounds *under* pounds, ſhillings *under* ſhillings, *&c. the greater number always uppermoſt; and drawing a line under them, begin with the leaſt denomination firſt, ſubſtracting it from the line above, and ſetting the* remainder *under the line, as in whole numbers; but if the* pence *or* ſhillings *in the upper row, be ſmaller than thoſe in the neather row, you muſt add* 12 d. *or* 20 s. *to the ſmaller number, that ſo Subſtraction may be made, as by the* Examples *following will appear.*

Example 1. *Let it be required to ſubſtract* 38 *li.* 12 *s.* 8 *d. out of* 269 *li.* 18 *s.* 10 *d.* Place your numbers as in the Margine; then beginning with the leaſt denomination firſt, (which in this Example is pence) ſay, 8 *d.* from 10 *d.* and there remains 2 *d.* ſet the 2 *d.* under the line, and proceed to the next denomination, which is ſhillings; ſaying, take 12 *s.* out of 18 *s.* and there remains 6 *s.* place 6 *s.* under the line, and go to the pounds; ſaying, 8 out of 9, and there remains 1, place 1 under the line, and ſay 3 out of 6, and there remains 3; then (becauſe there is no more figures to be ſubſtracted) ſay, nothing out of 2, and there remains 2, which ſet under the line, ſo is your ſubſtraction ended, and the remainder is 231 *li.* 6 *s.* 2 *d.*

	li.	*s.*	*d.*
Lent	269	18	10
Paid	38	12	8
Reſt	231	6	2

Example

Example 2. *Let it be required to ſubſtract* 2628 *li.* 16 *s.* 10 *d. out of* 9320 *li.* 10 *s.* 7 *d.* Place the numbers in order, and beginning with the pence, ſay, 10 *d.* out of 7 *d.* I cannot (therefore I muſt add 12 *d.* (which is one ſhilling) to 7 *d.* and it makes 19 *d.*) but 10 *d.* out of 19 *d.* and there remains 9 *d.* ſet the 9 *d.* under the line, and (becauſe I added 12 *d.* to 7 *d.*) I muſt therefore carry one to the place of ſhillings, ſaying, 1 *s.* which I carried, and 16 *s.* is 17 *s.* then 17 *s.* from 10 *s.* I cannot take, therefore I muſt add 20 *s.* (which is one pound) to 10 *s.* and it makes 30 *s.* then 17 *s.* out of 30 *s.* and there remains 13, ſet 13 under the line, and carry one to the place of pounds; ſaying, one which I carried and 8 is 9, take 9 out of 0 I cannot, but 9 out of 10, and there remains 1; ſet 1 under the line, and carry a unite to the next place, ſaying, 1 which I carried and 2 is 3, take 3 out of 2 I cannot, but three out of 12, and there remains 9; place 9 under the ſame line, and carry 1 to the next place, ſaying, 1 which I carry and 6 is 7, take 7 out of 3 I cannot, but 7 out of 13, and there remains 6, place 6 under the line, and carry one to the next row; ſaying, 1 and 2 is 3, take 3 from 9, and there remains 6, place 6 under the line, ſo is your ſubſtraction ended, and the remainder is 6691 *li.* 13 *s.* 9 *d.*

	li.	*s.*	*p.*
Lent	9320	10	07
Paid	2628	16	10
Reſts	6691	13	9

Example 3. *Suppoſe a Man had let to another Man* 1000 *pound, and that the borrower had paid thereof at one time* 127 li. *at another time* 430 l. 10 s. *and at a third payment* 50 li. *and the Creditor would know how much he hath received, and how much is owing of his Debtor.*

Place the numbers as here you see, first the sum of money lent, and draw a line under it; then set the sums paid at several times one under another, and draw a line under them: Then add all the sums which have been paid at several times together, which make 607 *li.* 10 *s.* which is the sum which the Debtor hath paid in all; then substract this 607 *li.* 10 *s.* from 1000 *li.* and there will remain 392 *li.* 10 *s.* and so much is still owing to the Creditor.

	li.	*s.*	*d.*
Money lent	1000	00	00
Paid at several times	127	00	00
	430	10	00
	50	00	00
Paid in all	607	10	00
Rests to pay	392	10	00

Other Examples for Practice.

	li.	*s.*	*d.*
Lent	2601	13	6
Paid	98	7	9
Rests	2503	5	9

	li.	*s.*	*d.*
Owing in all	100	00	00
Paid in all	36	10	06
Rests to pay	63	09	06

	li.	*s.*	*d.*	*q.*
Lent	3625	16	08	03
Paid at several times	100	00	00	00
	336	10	06	02
	039	12	09	02
	100	00	00	00
Paid in all	576	03	04	00
Rests to pay	3049	03	04	03

The Proof *of* Substraction.

The *Proof* of *Substraction* is performed by *Addition*, for adding the number to be substracted to the remainder, the sum of them must be equal to the number given, if you have truly wrought: As in the first example of numbers of one denomination,

The

The number given is —— 986
The number to be ſubſtracted is —— 234
The remaind.r is —— 752
Proof —— 986

Add the number to ſubſtracted 234, to the remainder 752, the ſum of them is 986, equal to the number given.

Examples for Practice proved.

	li.	s.	d.		li.	s.	d.
Lent	62	18	09	Borrowed	100	00	00
Paid	37	19	06	Received	36	13	04
Reſts	24	19	03	Due	63	06	08
Proof	62	18	09	Proof	100	00	00

Other Examples in Weight *and* Meaſure.

1 Example in *Troy-weight.*

Bought	li.	ou.	pw.	gr.
Of Silver	07	11	13	19
Sold	05	07	03	05
Unſold	02	04	10	14
Proof	07	11	13	19

2 Example in *Avoirdupois great weight.*

Bought	C.	q.	li.	ou.
Sold	37	03	22	11
	13	01	23	06
Reſts	24	02	27	05
Proof	37	03	22	11

3 Example in *Avoirdupois little weight.*

Bought	li.	ou.	dr.
Sold	84	12	13
	26	08	11
Reſts	58	04	02
Proof	45	12	13

4 Example in Time.

From	days	ho.	m.
Take	364	23	50
	76	09	22
Reſts	288	14	28
Proof	364	23	50

Queſtions

Queſtions performed by Addition *and* Subſtraction.

Queſtion 1. *What number is that which being added to* 376 *ſhall make* 1000? Subſtract 376 from 1000, the remainder is 624, the number ſought.

Queſtion 2. *What number of Pounds, Shillings and Pence muſt be added to* 36 *li.* 17 *s.* 3 *d. to make that ſum up* 100 *l.* Subſtract 36 *li.* 17 *s.* 3 *d.* from 100 *li.* the remainder is 63 *li.* 2 *s.* 9 *d.* which added to 36 *li.* 17 *s.* makes 100 *li.*

Queſt. 3. *In the year of our Lord* 1440, *the famous Art or Myſtery of Printing was invented, I would know how long it is ſince that time to this year of our Lord,* 1655. From 1655, ſubſtract 1440, the remainder is 215, and ſo many years are expired ſince *Printing* was invented.

Queſtion 4. *An Army conſiſting of* 13721 *Horſe, and* 26850 *Foot, in an Ingagement there were ſlain* 3760 *Horſe, and* 7523 *Foot; the Queſtion is, how many were ſlain in all, and how many Horſe, and how many Foot eſcaped.* From the 13721 Horſe that went out, ſubſtract the 3760 that were ſlain, there remains 9961, and ſo many Horſe eſcaped: Alſo from the 26850 Foot which went out, ſubſtract the 7523 which were ſlain, and there remains 19327, the number of Foot which eſcaped; and by adding the 3760 Horſe which were ſlain, to the 7523 Foot that were ſlain, their Total is 11283, and ſo many were ſlain in all.

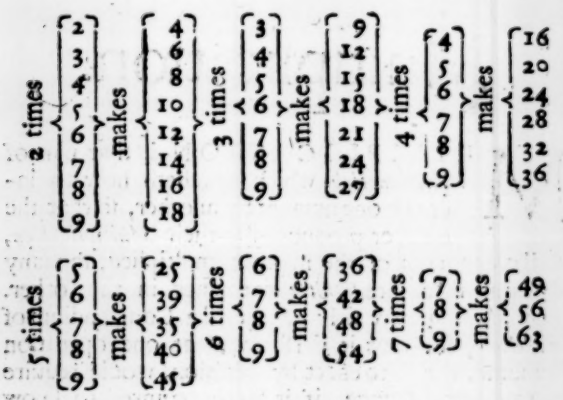
Multiplication.

MULTIPLICATION is that part of *Arithmetick* which teacheth how to increase one number by another, so that the number produced by their *Multiplication*, shall contain one of the numbers multiplied, so many times as there are *Unites* contained in the other. *Multiplication* may fitly be termed a Compendium of *Addition*, for that it performeth at one operation the same, which to effect by Addition, would require many. For instance, if it were required to know how much 7 times 5 is, to perform this by Addition, I must set seven fives, or five sevens, one under another, and adding them together, I shall find that either of their Totals shall contain 35; but this by *Multiplication* is performed with far more brevity, as by *Examples* hereafter shall appear.

Before you enter upon the practice of *Multiplication*, it is necessary to remember the Product produced by the multiplication of any one of the nine Digits, by any other of the same; as readily to know, that 4 times 5 is 20, 6 times 7 is 42, 2 times 9 is 18, 7 times 9 is 63, 8 times 9 is 72, *&c.* which this Table following will plainly declare, and must be perfectly learned by heart, before you attempt to multiply great numbers.

Multipli-

Multiplication Table.



8 times 8 9 makes 64 72 9 times 9 makes 81.

The use of the TABLE *of* MULTIPLICATION, *and the manner how it is to be read.*

This Table sheweth what the sum of any two Digits multiplied one by another doth amount unto, and is thus to be read, 2 times 2 makes 4, 2 times 3 makes 6, 2 times 4 makes 8: Also 6 times 4 makes 24, 7 times 8 makes 56, 8 times 8 makes (or is) 64, 9 times 9 is 81, *&c.*

Another Table of Multiplicatiou.

1	9	8	7	6	5	4	3	2	1
2	18	16	14	12	10	8	6	4	
3	27	24	21	18	15	12	9		
4	36	32	28	24	20	16			
5	45	40	35	30	25				
6	54	48	42	06					
7	63	56	49						
8	72	64							
9	81								

THis Table is thus to be read; In the first Row, or Column, towards the left hand, and also at the top of the Table, you have the nine *Digits* in bigger Figures than the rest; the Figures in the first Column beginning with 1, and so proceeding by 2, 3, 4, *&c.* to 9. Those at the top of the Table beginning with 9 towards the left hand, and so backwards by 8, 7, 6, *&c.* to 1 at the right hand.

Now if by this Table you would know how much 8 times 7 is, find 8 among the great Figures at the head of the Table, and look down that *Row* or *Column*, till you come against 7 of the great Figures in the first *Column*, against which you shall find 56, and so much is 8 times 7, or eight multiplied by 7.

In the same manner may you find that 7 times 9 is 63, 5 times 6 is 30, 3 times 4 is 12, and so of any two of the nine Digits.

In Multiplication there are three terms commonly used, that is to say;

The

The *Multiplicand*,
The *Multiplier*, and
The *Product*.

The *Multiplicand*, is the number to be multiplied.
The *Multiplier*, is the number by which the Multiplicand is Multiplied: and
The *Product*, is the number which is produced by the multiplication of the Multiplicand and the Multiplier together.

Thus, if it were required to multiply 8 by 7, here 8 is the *Multiplicand*, 7 the *Multiplier*, and 56 is the *Product*, for 8 times 7, or 7 times 8 is 56.

In *Multiplication* it mattereth not which of the two numbers is made the *Multiplicand*, or which the *Multiplier*, for the *Product* produced by either, will be the same; but the usual way is to make the *greater* number the *Multiplicand*, and the *lesser* number the *Multiplier*.

THE RULE.

The numbers to be multiplied must be set one under another, viz. *the* Multiplicand (*or great number*) *above*, *and the* Multiplier (*or lesser number*) *below*, *the last figure of the* Multiplier *under the last figure of the* Multiplicand, *then draw a line under them*; *and* (*having learned the preceding Tables perfectly by heart*) *multiply every Digit of the* Multiplier, *into every Digit of the* Multiplicand, *setting the several Products under the line*; *then having finished your* Multiplication, *draw a line*, *and add all the Products together*, *and the sum of those Products is the general* Product *of the whole multiplication*, *as by the following* Examples *will appear*.

Example 1. *Let it be required to multiply* 736 *by* 7

First,

First, I write down 736 the Multiplicand, and under it 7 the Multiplier, and under them I draw a line; then I multiply 7 into every Digit of the Multiplicand; saying, 7 times 6 is 42, place 2 under the line, under 7, and for the four tens keep 4 in mind; then say again, 7 times 3 is 21, and 4 which I kept in mind is 25; place 5 under the line, and keep the two tens in mind; then say again, 7 times 7 is 49, and 2 which I kept in mind is 51; place 1 under the line, and the 5 tens kept in mind (because there is no more figures to be multiplied) I set down under the line also, so is the work ended, and the Product of this multiplication is 5152.

736	*Multiplicand*
7	*Multiplier*
5152	*Product.*

Example 2. *Let it be required to multiply* 3417 *by* 5. Place the numbers one under another, and draw a line under them, as in the Margine; then begin your Multiplication, saying, 5 times 7 is 35, place 5 under the line, and keep the three tens in mind; then say again, 5 times 1 is 5, and 3 which I kept in mind is 8, place 8 under the line, and (because it is less than 10, I keep nothing in mind) then say again, 5 times 4 is 20, place a cypher under the line, and keep the two tens in mind: Lastly, say 5 times 3 is 15, and 2 which I kept in mind is 17, which 17 (being the last number) I place under the line, and so is my Multiplication ended, and the *Product* is 17085.

3417
5
17085

¶ You may be satisfied of the truth of this work, if you will take the pains to set down the Multiplicand 3417, five times one under another, and

and add them together, as ſo many ſeveral ſums, ſo ſhall you find the *Total* of that Addition, to be 17085, exactly the ſame with the Product of this Multiplication.

Example 3. In the two fore-going examples, the *Multiplier* conſiſted but of one *Digit*, we are now to ſhew how *Multiplication* is performed, when the Multiplier conſiſts of more than one figure, therefore in this Example, *Let it be required to multiply* 5704 *by* 37. Place your numbers, and draw a line under them as you ſee in the Margine: Then begin your Multiplication in in this manner: Saying, 7 times 4 is 28, ſet 8 under the line, and keep the two tens in mind, then ſay 7 times nothing is nothing, but the two tens in mind is 2, ſet 2 under the line, then ſay 7 times 7 is 49, ſet 9 under the line, and keep 4 in mind, then laſtly, ſay 7 times 5 is 35, and 4 in mind is 39, which being the laſt number to be multiplied, I ſet down under the line; ſo is the multiplication of one of the Digits (namely 7) finiſhed.

```
  5 7 0 4 Multiplicand
    3 7 Multiplier
  -------
  3 9 9 2 8
  1 7 1 1 2
  -------
  2 1 1 0 4 8 Product
```

Then begin to multiply the ſecond digit, ſaying 3 times 4 is 12, place 2 in the ſecond line, one place towards the left hand, and keep 1 in mind, then ſay 3 times nothing is nothing, but 1 in mind is 1, ſet down 1 by the 2 in the ſecond line; thirdly, ſay 3 times 7 is 21, place 1 in the ſecond line, and keep the two tens in mind: Laſtly, ſay 3 times 5 is 15, and 2 is 17, which 17 (becauſe there is no more figures to be multiplied) I place in the ſecond line alſo.

Having thus done, I draw a line under them, and add theſe two lines together, as in common Addition

of

of numbers of one denomination; Saying 8 is 8, place 8 under the line; then say 2 and 2 is 4, place 4 under the line; then say 1 and 9 is 10, place a cipher under the line; and carry one to the next place: saying 1 and 1 is 2, and 9 is a 11, place 1 under the line, and carry 1 to the next row, saying 1 and 7 is 8, and 3 is 11, place 1 under the line, and carry 1 to the next place; saying, 1 which I carry and 1 is 2, place 2 under the line, and so is your Multiplication ended, and the product is 211048.

Example 4. *Let it be required to multiply* 57325 *by* 4032. Place the multiplicand and the multiplier one under another, and draw a line as before, then proceed to the multiplication as formerly; saying First, 2 times 5 is 10, set down a Cypher, and keep 1 in mind; then 2 times 2 is 4, and 1 in mind is 5, place 5 under the line: then 2 times 3 is 6, set 6 under the line; then 2 times 7 is 14, set down 4 and keep one in mind; then 2 times 5 is 10, and 1 in mind is 11, which 11 (being the last) I set down.

	57325	Multiplicand
	4032	Multiplier
	114650	
	171975	
229300		
231134400		Product

The Multiplication of one of the digits being finished, proceed to the Multiplication of the next; Saying, 3 times 5 is 15, set down 5 in the second line, a place more towards the left hand, and keep 1; then 3 times 2 is 6, and 1 kept is 7, set down 7; then 3 times 3 is 9, set down 9; then 3 times 7 is 21, set down 1 and keep 2 in mind; then 3 times 5 is 15, and 2 in mind is 17; which being the last set down also.

Two of the figures of the multiplier being finished,

proceed to the third, which (in this example) being a cypher, you may wholly neglect, and proceed to the multiplication of the fourth figure; only remember to remove the Product of the fourth figure one place more to the left hand, as in the example you may see, for the cypher though it be not written down, yet it must keep its place, and the figures following must be removed a place farther.

Then for the Multiplication of the fourth and last digit, say 4 times 5 20, set down a cypher, (under 9) and keep 2 in mind: then 4 times 2 is 8, and 2 in mind is 10, set down a cypher; and keep 1 in mind: then 4 times 3 is 12, and 1 is 13, set down 3 and keep 1: then 4 times 7 is 28, and 1 kept is 29, set down 9, and keep 2: then 4 times 5 is 20, and 2 is 22; which 22 (beaause the multiplication is ended) set down also.

Having thus multiplied all the digits severally, draw a line under their Products, and add them altogether as in the former Example, so shall you find their general Product to be 231134400.

Other Examples for Practice.

73260	50762
45003	4567
219780	355334
366300	304572
293040	253810
	205048
3296919780	233830054

The Proof of Multiplication.

The most certain proof of *Multiplication* is by *Division*,

vision, but because *Division* is not yet known, I will here shew a near way by which *Multiplication* may be proved. Which is thus.

Make a Cross *as in the Margine, then, any sums being multiplied, you may prove the truth of your work in this manner, (1) Cast away all the nines which you can find in the* Multiplicand, *what remaineth set on the right side of the* Cross. *(2) Cast away also the nines in the* Multiplier, *and what remains set on the left side of the* Cross. *(3) Multiply the figure on the right side of the* Cross, *and out of that* Product *cast away the nines also, setting the figure remaining over the* Cross, *then (4) Cast away all the nines in the* Product, *and if the figure remaining be the same with that which standeth over the* Cross, *then is your Multiplication truly performed, otherwise not.*

Example 1. Let it be required to prove the Sum in the margine.

```
    3 2 4
      2 3
  -------
  1 2 9 7 2
  8 6 4 8
  -------
  9 9 4 5 2
```

1. Cast away all the nines in the *Multiplicand*; Saying, 4 and 3 is 7, and 2 is 9, which being rejected, there remains 4, which I set on the right side of the Cross, Then

2. Cast away all the nines in the *Multplier*: Saying, 2 and 3 is 5, (which being less than 9) I set on the left side of the Cross. Then

3. Multiply 4 by 5, saying 4 times 5 is 20, from which cast all the nines which are two, and there remains 2, place 2 over the Cross. And

4. Cast away all the nines in the *Product*; Saying, 2 and 5 is 7, and 4 is 11, cast away 9 and there remains 2: which exactly agrees with the figure over the *Cross*, and demonstrates that the Multiplication is truly performed,

Compendiums in Multiplication.

1. If the *Multiplier* consist of Cyphers in the last place or places, you may omit the Multiplication of them, and place the former figures of the Multiplier under the Multiplicand: Thus, if it were required to multiply 3257 by 2600; place the numbers as you see in the margent, then multiplying 3257 by 26, the Product will be 84682, to which if you add two Cyphers, (because there were two Cyphers in the Multiplier) it will be 8468200, which is the true product of the multiplication.

```
 3257
 2600
------
19542
6514
------
8468200
```

2. If it be required to multiply any number by 10, 100, 1000, 10000, *&c.* You have no more to do, but to add so many Cyphers to the *Multiplicand*, as there are Cyphers in the *Multiplier*: Thus if you were to multiply 365 by 10, the product will be 3650, or by 100, it would be 36500, or by 1000, it would be 365000, or by 10000, it would be 3650000.

3. If any number given were to be multiplied by 5, you may abbreviate your work thus. Add a Cypher to the *Multiplicand*, take half that number, and it shall be the product required. Thus if it were required to multiply 8727 by 5, add a Cypher to the *Multiplicand*, then it is 86270 the half whereof is 43135, which is the product required.

To multiply by any of the nine Digits without charging the memory.

To *multiply any number by* 2, Either double the number in your mind, or add it, by setting it down twice, so 57325 produceth 114650.

To

To multiply any number by 3, To the number given add the double thereof, the ſum is the product, ſo 57325 produceth 114650.

To Multiply any number by 4, Double the duplication in your mind, ſo 57325 produceth 229300.

To multiply any number by 5, Conceive a cypher added to the given number, and in your mind half thereof is the product, thus a cypher added to 57325, maketh it 573250, the half whereof is 286625.

To multiply any number by 6. Take half adding a cypher, and add to the half, the figure ſtanding next before: Thus 573250 produceth 343950.

To multiply any number by 7. Take half and add it to the double of the former figure, ſuppoſing a cypher added as before; So 57325 thus ordered, produceth 401275.

To Multiply any number by 8. Double each former figure and ſubſtract it from the following, ſo 57325 produceth 458600.

To multiply any number by 9. Suppoſe the number multiplyed by 10, then ſubſtract each former figure from the following, beginning with that next before the cypher, the remainder is the product, ſo 57325 produceth 515925.

Other Brief Rules of Multiplication.

Having ſhewed you ſome *Compendious* wayes of *Multiplying* by *Article Numbers*, as by 10, 100, 1000, *&c.* and alſo by the *nine Digits*, without any trouble or charge to the *memory*: I will now ſhew how you may expeditiouſly and certainly, *multiply* any ſum by divers other Numbers conſiſtidg of *two* or *three* places, without ſetting down many figures but the Product it ſelf,

To multiply any number by 11.

THE RULE.

Set the Multiplicand down twice, removing it one place to the left hand, add them together, the ſum is the Product of the Number multiplied by 11.

Example. *Let it be required to multiply* 97 *by* 11.

Set down 97 twice, removing one of them a place more to the right hand as you ſee here in the margine, then add them together, the ſum is 1067, which is equal to the Product of 97 multiplied by 11.

```
 97
 97
----
1067
```

To multiply any Number by 12, 13, 14, 15, 16, 17, 18, or 19.

THE RULE

To effect this, You have no more to do, but to *multiply the given number by* 1, 2, 3, 4, 5, 6, 7, 8, 9, *and in your Multiplication, continually to add that figure of the Multiplicand which ſtandeth on the right hand of the figure you are multiplying by, ſetting down the ſum for the figure of the Product.*

Example. *Let it be required to multiply* 3624 *by* 17.

Multiply in this manner by 7, Saying, 7 times 4 is 28, ſet down 8 and carry 2 —— then 7 times 2 is 14, and 2 which I carryed is 16, and 4 (*the figure of the Multiplicand which ſtands on the right hand of* 2) is 20: Set down 0, and carry 2 —— then 7 times 6 is 42, and 2 which I carried is 44, and 2 (*which ſtands on the right hand of* 6) is 46; ſet down 6 and carry 4 —— then 7 times 3 is 21, and 4 which I carried is 25, and 6 (*which ſtands on the right hand of* 3) is 31, ſet down 1 and carry 3, which 3 add to 3 (*the left hand figure of your Multiplicand,*)

it is 6, which ſet down, ſo the Product of 3624 multiplied by 17 is 61608.

In the ſame manner, may you multiply by 12, 13, 14, &c. as in theſe Examples

3764	625	4793
13	16	19
48932	10000	91067

To multiply any Number by 102, 103, 104, 105, 106, 107, 108, *or* 109.

THE RULE.

Multiply the Number given by 2, 3, 4, 5, 6, 7, 8, *or* 9, *ſetting the Product two places towards the right hand of the Multiplicand, the Product and the Multiplicand added together in the ſame order that they ſtand, ſhall be the Product of the whole Multiplication.*

Example. *Let it be required to multiply* 3924 *by* 106.

Set them down as in the Margine, then multiply 3624 by 6, it produceth 21744, which added to 3624, in the ſame order as they there ſtand, the ſum of that Addition will be 384144, which is equal to the Product of 3624 multiplied by 106.

3624	
106	
	21744
	384144

Other Examples.

765	6374
103	107
2295	44618
78795	682018

To multiply any number by 112, 113, 114, 115, 116, 117, 118, *or* 119

THE RULE.

Multiply the given Sum by 12, 13, 14, 15, 16, 17, 18, *or* 19, *as hath been ſhowen already, ſetting the Product two places to the right hand of the Multiplicand, then add this Product and the Multiplicand together in the ſame order as they ſtand, ſo ſhall the ſum of that Addition be equal to the Product of the Multiplication.* As by the Examples following is evident.

Multiply	4065
By	113
The Product multiply by	1352845
Real Product	459345

Multiply	7632
By	119
The Product Mul. by 19	145008
Real Product	908208

Queſtions performed by Multiplication only.

Queſtion 1. *If a piece of Land be* 236 *Perches long, and* 182 *Perches broad, how many ſquare Perches are contained therein?* Multiply 236 the length, by 182 the breadth, the product is 42952, and ſo many ſquare Perches are contained in ſuch a ſquare piece of Land.

Queſtion 2. *In a year there are* 365 *days natural, and in every day* 24 *hours, how many hours be there in a year?* Multiply 365 the number of days by 24, the number of hours, the product is 8760, and ſo many hours be there in a year.

Queſtion 3. *From* London *to* Coventry *it is accounted* 76 *miles; how many yards therefore is it from* London *to* Coventry? Multiply 1760 (which are the number of yards contained in one mile) by 76, the product is 133760, and ſo many yards are between *London* and *Coventry*.

DIVISION.

Diviſion.

DIVISION is the juſt contrary to *Multiplication*, for *that* turns *Small* denominations to *Greater*, as *Multiplication* turns *Greater* to *Smaller*: Or (in whole Numbers, of which only we yet ſpeak) *Diviſion* is the asking, *how many times one ſum is contained in another?* and the number which anſwereth to that queſtion is called the *Quotient*.

And the Number containing, which is to be divided, is called the *Dividend*.

And the Number contained, or by which the Dividend is to be divided, is called the *Diviſor*.

And as often as the *Dividend* contains the *Diviſor*, ſo often doth the *Quotient* contain *Unity*.

The ways of performing Diviſion are divers, I will begin with that which is moſt uſed and taught, which is as followeth.

THE RULE.

Place the Diviſor *under the* Dividend, *ſo that the figures next to the left hand ſtand directly one under the other, if the reſt of the* Diviſor *be not the greater; or if all the* Diviſor *be greater than that above it, then the ſaid* Diviſor *muſt be devolved one place further toward the right hand; having ſo placed them, try how many times the lower figures are contained in the upper figures, and*

and write that figure which answereth that question within a crooked line in the margine of the work, which is called the Quotient, *and by it multiply the first figure of the* Divisor, *and take the Product out of the figures directly over it, beginning the* Substraction *towards the left hand; then cancel that figure of the* Divisor, *and also that of the* Dividend *which hath been already used, with a light dash of a Pen, and write the* remain (*when the Product of the first figure multiplied by the* Quotient *is substracted, as before*) *just over the figure used and cancelled; Then proceed to do the like with the second, third, and fourth figure of the* Divisor, *if there be so many; till having cancelled it all, and set the remain orderly above the* Dividend, *you have finished one work.*

Now if the Dividend *have still some figures untouched towards the right hand, then remove the* Divisor *still towards the right hand, but one place at a time, and then again ask or try how many times the lower may be had in the upper, and write the answer in the* Quotient, *whether it be* 1 *or more, (only it cannot be above* 9) *or nothing, then put* 0 *in the* Quotient, *and multiply the* Divisor *by this new figure, and substarct the Product, setting the remainder orderly above, as before; this work must be repeated by removing the* Divisor *still one place towards the right hand, until* Unity *in the* Divisor *stand under* Unity *in the* Dividend, *and then the work is done.*

Example 1.

Let it be required to divide 4096 *by* 3.

Place them thus

4096 (1
3

Ask how many times 3 in 4, the Answer is 1, which is put within a crooked line by it self.

Then in your mind multiply the Divisor 3 by the Quotient

Quotient 1. And having ſaid
theſe words, *Once* 3 *is* 3, 1
preſently cancel the 3. And 4096 (1
having added theſe words, 3
Out of 4, cancel the 4. And
after theſe words, *and there remains* 1, write 1 juſt over the 4, as you ſee here done.

Then remove the Diviſor one place toward the right hand, ſaying, *how many times* 3 *in* 10, the anſwer is 3, 1
which write in the quotient; 4096 (13
and in your mind multiply the 3
Diviſor 3 by the quotient 3,
the product is 9, wherefore ſay, three times three is nine, out of ten, and there remains one, then (having cancelled the 10 and the 3,) write over them 1.

Again, remove the Diviſor 3
one place more, asking *how many times* 3 *in* 19? the anſwer 4096 (136
being 6, write 6 in the quotient, 3 3
and ſay 6 times 3 is 18 out of
19, and there remains 1, wherefore having cancelled the 3, and the 19, write 1 over 3, and remove the Diviſor once more, and ask *how many times* 3 *in* 16? anſwer is 5, which write in
the quotient, then in mind 1
ſay 5 times 3 is 15 out of 16 4096 (1365
and there remains 1, and 3 3
cancelling the 16 and the 3,
write over 3, 1: Now becauſe the *Diviſor* 3, is advanced ſo far till it is come to ſtand under 6 in the *Dividend*, which 6 is the place of *Unity* there, the ſaid *Diviſor* cannot be removed any more, and therefore the Diviſion is ended, and the *Quotient* being 1365, ſhews that the *Diviſor* 3 is contained in the *Dividend* 4096, 1365 times, and 1 remaining, which being

ing leſs then the Diviſor 3, doth not contain it once, but one third part of once, which (after the Reader hath skill in broken numbers) muſt be joyned to the quotient thus 1365 $\frac{1}{3}$

The beſt proof of this *Diviſion* is by *Multiplying* the quotient into the Diviſor, and to the Product add the remain; then if the work be well done, the ſum ſhall be equal to the Dividend.

So 1365 multiplied by 3 produceth 4095, to which adding the remain 1, the ſum 4096 is equal to the Dividend.

Here we divided only by one figure 3, becauſe the firſt example being eaſie and clear, ſhould be a fair Introduction to the ſecond.

Note, that if the Diviſor had been greater than 4, as 5, the work muſt have begun thus 4096 (

5

So the quotient would have been 819 $\frac{1}{3}$ which is one place leſs.

Example 2.

Let it be required to divide 1 3 1 0 7 2 0 by 4096.

Place them thus

1 3 1 0 7 2 0 (
4 0 9 6

Now the queſtion to be asked is, how many times 4096 is there in 13107? to find an anſwer to this queſtion, the Reader which hath but an indifferent faculty of judging, may do it, (for the moſt part) by conſidering the firſt figure in the Diviſor, as here 4, for preſently he knows that 4 times 4 is 16, which cannot be had in 13, therefore the firſt figure in the quotient muſt be leſs than 4.

Again, it cannot be much leſs, becauſe the ſecond place in the Diviſor is 0. He

He may therefore venture on 3, And having put 3 in the quotient, ſay 3 times 4 is 12, which ſubſtract out of 13 over it thus, ſay 2 out of 3, and there remains 1, and 1 out of 1 and there remains 0, cancelling the 0 and the 13, and ſet the remain 1 over the 4, as you ſee.

```
      1
  1 8 3 9
1 3 1 0 7 20 (3
  4 0 9 6
```

Then go on, ſaying 3 times 0 is 0, out of 11, and remains ſtill 11, again 3 times 9 is 27, take 7 out of 10 there remains 3, to be ſet over the 0, which is over 9, and 2 with 1 borrowed (to make the 0, 10, from the which the 7 was taken) is 3, and 3 out of 11, remains 8, which write over the place of 0 in the Diviſor, cancelling the 9 in it, and alſo thoſe figures of the Dividend 110, out of which you have taken any thing: Then laſtly, ſay 3 times 6 is 18, take 8 out of 7 I cannot, therefore (borrowing 10,) ſay 8 out of 17 remains 9, which write over the 7, then in the next place, the 10 borrowed is 1, and the 10 in the 18 is 1: Say therefore 1 and 1 is 2, out of 3, and the remain is 1, which write over the 3, having ſtill cancelled the figures which you have uſed orderly as you go.

Now the whole Diviſor being cancelled, it muſt be removed on one place further, and placed as here.

Now ask again, how many times 4096 in 8192? it will be found (becauſe of the 0 after the 4) as often as 4 in 8, that is two times, put therefore in the quotient 2, and work as before, ſaying, 2 times 4 is 8, out of 8 remains 0, then 2 times 0 is 0 out of 1 remains 1, then

```
      0 0
    0 1 1
  0 1 8 3 9 0
1 3 1 0 7 2 0 (320
  4 0 9 6 6 6
    4 0 9 9
      4 0
```

then 2 times 9 is 18, take 8 out of 9 remains 1, (which put over 9,) and 1 out of 1 in the next place remains 0, then lastly, 2 times 6 is 12, that is, 2 out of 2 there remains 0, and 1 out of 1, there also remains 0, cancel and put the remainder over as formerly.

Now again, the Divisor being all cancelled, should be removed; and ask how many times 4096 in 0000, the answer is nothing 0, which heing put in the quotient, the work is all done.

And the quotient 320, shews that the *Divisor* 4096 is contained in the *Dividend*, 1310720, three hundred and twenty times.

And whether the Divisor have 2, 3, 5, 7, or more places, the working is still like this, not differing from it at all.

Compendiums in *Division*. For brevity in some cases, where the *Divisor* towards the right hand hath one *Cypher* or more; those Cyphers may be placed orderly under the *Dividend* at first, and remain there till the work be done, with the rest of the *Divisor*, which must needs shorten the *Division*.

As if 2587645 *were to be divided by* 15000, *place them thus:*

```
2 5 8 7 6 4 5
1 5     0 0 0
```

And say how many times 15 in 25? answer is 1, which write in the quotient, and multiply in mind the Divisor by the quotient; saying once one is one, out of 2 there remains 1, then cancel the 2, and the 1 uuder it, and write the remaining 1 over it, as here is done, then say once 5 is 5 out of 5, and there remains 0, therefore cancel the two 5, and over the uppermost write 0.

```
  0 0
  3 1
1 0 3(7
2 5 8 7(6 4 5 (1 7 2
1 5 5 5 0 0 0
  1 1
```

Now

Now remove the Diviſor one place, and ask how many times 15, in 108? anſwer is 17, which write in the quotient, and multiply, ſaying 7 times 1 is 7, out of 10 remains 3, which write over the 0, then ſay 7 times 5 is 35, take 5 out of 8 remains 3, which write over 8, and 3 out of 3 remains 0.

Now again, remove the Diviſor, and ask how many 15 in 37? anſwer is 2, which write in the quotient, and ſay two times 15 is 30, out of 37 remains 7, and the Diviſion is ended, the remain being 7645.

Proof of this.

Multiply the quotient	172
by the Diviſor	15
	860
	172
The Product is	2580
Before which put the three Cyphers, And then it is	2580000
To which add the remain	7645
The Total is	2587645

Which is equal to the Dividend, and therefore the work is right.

So if one would divide any ſum by 10, 100, 1000, 10000, *&c.* he need but cut off the firſt, two firſt, three firſt, or four firſt figures towards the right hand, the other figures ſhall be the quotient, and thoſe cut off, the remain.

As if 2587645 be divided by 1000, the quotient is 2587, and the remain 645.

If

If you would know how many pounds Sterling are in 95670 shillings: having placed them thus, that is, 2 under 9, and 0 under 0, then divide by 2, which is very eaſie. So the quotient 4783, ſhews that in the number of ſhillings given, is contained 4783 pounds; and 10 ſhillings remaining.

```
x x    (1
x x x (0 (4783
x x x x
```

The reaſon why the Diviſor was 2, is becauſe there are 20 Shillings in one Pound, and therefore any number of Shillings is turned into Pounds, by dividing by 20, that is by 2, putting the Cypher under Unity, onely to fill a place at laſt. And this way of turning any number of Shillings into Pounds, may be eaſily effected by memory, if you ſuppoſe the laſt figure of your given number to be cut off with a line or comma, and taking the half of the other figures. Thus, let the given number of Shillings be 5739; Imagine the laſt figure 9 to be ſeparated from the reſt by a line thus 573|9; now by memory take take the half of 573, by ſaying in your mind the half of 5 is 2, (and one remaining which makes the 7 following 17) the of 17 is 8 (and one remaining, which 1 is 10 *s.* to be added to the 9. and the whole is 286 *li.* 19 *s.*

In this way of Diviſion (as in all others) if the *Remain* at laſt be greater than the *Diviſor*, the *Quotient* is not juſt, but too little, which may be remedied (without beginning the work again) by dividing the remainder onely by the ſame Diviſor, for thereof will ariſe a new quotient, which added to the former quotient the ſum will be the juſt quotient.

So if 7290 be divided by 27, the numbers being placed thus,

Becauſe 27 can be had in 72 but twice, put 2 in the

the quotient, saying 2 times 27 is 54, out of 72, and there remains 18, which write over 72, cancelling as before is shewed; then removing the Divisor, say how many times 27 in 189? answer is 7, but if one should mistake, and write 5 in the quotient, and say 5 times 27 is 137, out of 189, remains 54, and write it over as before; and remove the Divisor, and say, how many times 27 in 540? answer is 20, but should not be above 9; say therefore, 9 times 27 is 243, out of 540, and the remain is 297, which being the last remain, and greater than the Divisor, shews the quotient 259 is too little.

Wherefore divide the last remain 297, by the Divisor 27, saying, twenty seven is in 29 once, and write 1 in the new quotient, and say, once 27 is 27, out of 29 remains 2, which write over 9, and remove the Divisor, and say, 27 in 27, justly once, so write 1 in the quotient, so the quotient is 11; which added to the former quotient 259, gives 270, which is the true and whole quotient.

A Second way of Division.

Although (as I have said) the former way is more used, yet this may seem plainer and more natural to some, I will therefore give one example of it.

Example.

6477734 (19494		*Let it be required to divide* 6477734 *by* 334, where the first figure in the quotient is easily seen 1; substract once 334 out of 647, and write the remain under a line, then see how many times 334 in 3137, answer 9, by which multiply the Divisor 334 in another paper, the Product is 3006, which substract out of the remain, then the new remain is 131734.
3137734	*Rem.*	
3006	*Subst.*	
0131734	*Rem.*	
01002	*Subst.*	
0031534	*Rem.*	
003006	*Subst.*	
0001474	*Rem.*	Again, how many times 334 in 1317, answer is 4, for the third figure in the quotient; by which multiply the Divisor, the Product is 1002, which take out of the later remain, as in the Margine, then will remain 31534.
0001336		
....138	*Rem.*	

Now try how many times 334 in 3153, answer is 9; for the fourth figure in the quotient, by which multiply the Divisor (in a by-paper) the Product is 3006 (as might have been seen above in the second substraction) which substracted out of the later remain, there remains now 1474.

Lastly, ask how many times 334 in 1474, answer is 4, for the last figure in the quotient, by which multiply the Divisor 334, the Product is 1336, which substracted from 1474, there finally remains 138; which being less than the Divisor, shews the division is done.

Proof

Proof of this.

	334....
	3006...
The ſeveral ſubſtractions, and the final remain, added together.	.1002..
	..3006.
	...1336
	138
The Total equal to the Dividend	6477734

If the former (as I have ſaid more uſual) way ſeem difficult to Beginners, becauſe the Products of the Diviſor into the ſeveral figures in the quotient are not ſet down, but mentally made; and alſo becauſe the ſubſtraction of them begins towards the left hand: and laſtly, becauſe the remain is ſtill ſet above; yet this later way which agrees altogether with plain ſubſtraction before taught, I hope is ſo plain, that any diligent Reader may acquire it without a Tutor. And yet for the better ſatisfaction and help of the young Learner, I will add another way or two more of Diviſion.

A Third way of Diviſion.

There is another kind of Diviſion which is very much uſed, and is in great requeſt with thoſe who have moſt occaſion to divide great numbers, the manner of working is not much unlike the way before taught, one or two *Examples* will make it plain.

Example 1. *Let it be required to divide* 162483 by 1321. Set down your numbers as you ſee them placed in the Margine, *viz.* Firſt, ſet down 162483

the Dividend, then on the left hand thereof set the Divisor 1321 with a crooked line between them, then on the right hand thereof make another crooked line, which must serve to set the figures of the quotient in, so are your numbers placed in due order; then draw a line under the Dividend, and make a prick under the figure 4, (because so far the figures of the Divisor would extend if they had been placed underneath the Dividend, according as in the other examples) this prick serves only to shew how far you have proceeded in your work, and must at every division be removed a place further, till at length you come to the last figure of the Dividend: your numbers being thus placed with a line under them, you are ready for the work, which must be performed according to the directions of the following Rule.

1321) 162483 (

THE RULE.

Demand how often the Divisor *may be had in the* Dividend, *and place that number in the* quotient; *then multiply the* divisor *by the* quotient, *and place the* product *under the line: then subtract this* product *from the* dividend, *and set the* remainder *under the* product, *then make a prick under the next figure of the* dividend, *and bring that figure down to the* remainder, *and then proceed as before.*

Example. Your numbers being placed, as is before directed, you may begin your work in this manner. First, say how many times 1321 can I have in 1624? say once, place 1 in the quotient, by which 1 multiply the Divisor 1321, beginning at the left hand, saying, once one is 1, place 1 under the line, then once 2

is 2, ſet 2 under the line, then once 3 is 3, place 3 under the line: laſtly, once 1 is 1, place 1 under the line; then ſubſtract this 1321 from 1624, and there will remain 303. To this 303 bring down the next figure in the dividend, namely 8; (firſt, making a prick under the 8) ſo will that number be 3038, under which draw a line, and repeat the ſame work again, ſaying, how many times 1321 can I have in 3038, which may be had two times, place 2 in the quotient, by which 2 multiply the diviſor 1321; ſaying, 2 times 1 is 2, place 2 under the line: then 2 times 2 is 4, place 4 under the line: then 2 times 3 is 6, place 6 under the line: laſtly, 2 times 1 is 2, place 2 under the line, and ſubſtract this 2642 from 3038, and there will remain 396, to this 396 bring down the next figure of the dividend, which is 3, ſo is this number made 3963, under which draw a line, and repeat the work once again; ſaying, how many times 1321 can I have in 3963, which may be had 3 times, by which 3 multiply the diviſor 1321; ſaying, 3 times 1 is 3; then 3 times 2 is 6; then 3 times 3 is 9; and laſtly, 3 times 1 is 3, which place under the line, and ſubtract it from the line above, which in this example is the ſame number; therefore there remains nothing, and the work is ended; but if any remainder had been, that ſhould have been ſet under the line, as by the examples following will appear.

```
1321) 162483 (123
      1321
      -----
       3038
       2642
       ----
        3963
        3963
        ----
           0
```

Other Examples for Practice.

```
5624) 793058 (141
      5624
      23065
      22496
        5698
        5624
          74 remain-
             der
```

In this Example where 793058, is divided by 5624, you may perceive that the quotient is 141, and 74 remaining, so that the real quotieut is 141 $\frac{74}{5624}$.

The Proof of this Division.

This kind of Division is proved by Addition, for *if you add the several* products *arising from the multiplication of the several* quotients *into the divisor, and also add thereunto the* remainder *(if any be) the total of this Addition shall be equal to the dividend, if there be no error in the work.*

So in the example following, if you add 4325 the first product, and 30275 the second product, and 2796 the remainder together, in the same order as they now stand in the example, you shall find the total of this Addition to be 76321, equal to the dividend, which demonstrates the Work to be true.

```
4325) 76321 (17
      43·25
      33071
      30275
        2796 remaind.
      76321 Proof.
```

A

A Fourth way of Diviſion.

There is a fourth way of Diviſion uſed by ſome, not inferior to any of the preceding, for that it is no burthen to the memory, and it is alſo proved by Addition.

The manner of placing the figures is the ſame with the third kind of Diviſion laſt taught; And for the performance of this Work, this is

THE RULE.

Firſt write down the dividend, *and on the left hand thereof the* diviſor, *with a crooked line betwixt them, and on the right hand of the* dividend *make another crooked line wherein to place the figures of the* quotient, *then draw a line under the* dividend, *and alſo make a prick under that figure of the* dividend *under which the laſt figure of the* diviſor *would fall, were it to be placed as in the firſt kind of* diviſion. *This done, demand how often the* diviſor *may be found in thoſe figures of the* dividend, *and place that digit in the* quotient, *then by this digit multiply the* diviſor, *and ſet the* product *of this Multiplication directly under the* dividend, *beginning at the place where you made the prick, then ſubſtract this* product *from the figures of the dividend, and place the* remainder *over the* dividend, *cancelling the figures of the* dividend *as you proceed, ſo is the firſt figures of the* quotient *finiſhed, then make a prick under the next figure of the* dividend, *and demand how often the* diviſor *may be found in the laſt* remainder, *and the other figure being added thereto, which place in the* quotient, *and proceed in all reſpects as before, till you have pointed all the figures of the* dividend.

Example, *Let it be required to divide* 763258 *by* 2345) 763258 (

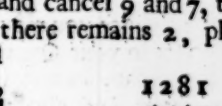
2345. Place your numbers as you see in the margine, and because there are 4 figures in the divisor, therefore make a prick under the fourth figure of the dividend, which is 2, and draw a line, then begin your Division in this manner: saying.

First, how many times 2245 can I have in 7632, (or how many times 2 can I have in 7) say 3 times, place 3 in the quotient, by which 3 multiply the divisor, saying 3 times 5 is 15, place 5 under the prick, then 3 times 4 is 12, and 1 is 13, place 3 under the line; then 3 times 3 is 9, and 1 is 10, place a cypher under the line; then 3 times 2 is 6, and 1 is 7, place 7 under the line; then substract 7035 from from 7632; saying, 5 from 12, and there remains 7, place 7 over 2, (cancelling the 5 and the 2) and bear one in mind, then 1 and 3 is 4, out of 13, there remains 9, place 9 over 3, and cancel 3 and 3, then 1 which I carried from 6, and there remains 5, place 5 over 6, and cancel 0 and 6; lastly, 7 from 7 there remains 0, which you need not set down, but cancel the two sevens, then will the work stand as above, and the remainder will be 597.

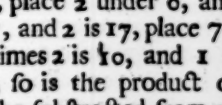
```
      597
2345) 7632 (3
      7035
```

Secondly, make a prick under the next figure of the dividend, namely 5, and say how many times 2245 can I have in 5975, answer two times, place 2 in the quotient, by which multiply the divisor, saying 2 times 5 is 10, place 0 under 5, and carry 1, then 2 times 4 is 8, and 1 is 9, place 9 under 7; then 2 times 3 is 6, place 6 under 9; lastly, 2 times 2 is 4, place 4 under 5; so is the product of this multiplication 4690, which you must substract from 5975, saying 0 from 5 and there remains 5, place 5 over 5 and cancel 0 and 5; then 9 out of 17, there remains

mains 8, place 8 over 7, and cancel 9 and 7, then 1 carried and 6 is 7, from 9 there remains 2, place 2 over 9, and cancel 6 and 9; laſtly, 4 from 5 reſts 1, place 1 over 5, and cancel 4 and 5, ſo have you finiſhed your ſecond figure, and your work will ſtand thus, and your remainder will be 128.



Thirdly, make a prick under the next figure of your dividend (namely under 8) and ask how many times 2345 can I have in 12858 (or how many times 2 can I have in 12) ſay 5 times, place 5 in the quotient, by which multiply the diviſor, ſaying 5 times 5 is 25, place 5 under 8, and carry 2; then 5 times 4 is 20, and 2 is 22, place 2 under 0, and carry 2; then 5 times 3 is 15, and 2 is 17, place 7 under 9, and carry 1; then 5 times 2 is 10, and 1 is 11, place 11 under 4 and 6; ſo is the product of this multiplication 11725 to be ſubſtracted from 12858, ſaying 5 from 8 reſts 3, place 3 over 8, and cancel 5 and 8; then 2 from 5 reſts 3, place 3 over 5, and cancel 2 and 5; then 7 from 8 reſts 1, place 1 over 8, and cancel 7 and 8; then 1 from 2 reſts 1, place 1 over 2, and cancel 1 and 2; laſtly, 1 from 1 reſts nothing, ſo is your work ended, which you ſhall find to ſtand as in the Margine, the remainder being 1133.



I had an intent here to have put an end to *Diviſion*, but there came into my mind

A Rule, by which you may certainly know what figure to ſet in your Quotient, and never to take one too great, or too little, but that which will juſtly ſerve: And alſo to perform (with eaſe and certainty) the hardeſt, and moſt difficult Sum that can be propoſed in *Diviſion*, without the affiſtance of *Multiplication*, only by *Addition* and *Subſtraction*, not burthening the memory at all.

In the practice of *Diviſion*, there is nothing more difficult than in large Sums (eſpecially if the firſt figures of the Diviſor be either 1, 2, 3, or cyphers, and the laſt figures 7, 8 or 9) to know certainly what figure to put in the Quotient, when you demand how often the diviſor may be had in the dividend; for the certain finding whereof (a little pains being taken before you begin your work) do thus.

Suppoſe you were to divide any Sum, as 1979909, by 309: Firſt, ſet down the nine Digits 1, 2, 3, *&c.* one under another, and againſt the figure 1, ſet 309 your diviſor, which doubled is 618, which ſet againſt 2, theſe added together make 927, which ſtands againſt 3: Add the diviſor 309 to 927, it makes 1236, which is againſt 4, to this add the diviſor, and it makes 1545, which ſtands againſt 5. And thus to every laſt number, ſtill add the *Diviſor* till you have gone through all the nine Digits, then will they be as in the Margine.

1	309
2	618
3	927
4	1236
5	1545
6	1854
7	2163
8	2472
9	2781

Having prepared this little Table, ſet your dividend and diviſor down, as in the third way of Diviſion, pricking the dividend, and drawing a line under it, as is there directed, and as you ſee here done.

309)

309) 1097909 (3553

927

1709

1545

959

927

32

Then laying your little Table before you, look in it for 1097, the four first figures of the dividend, which you cannot exactly find there, but the neareſt number leſs which you moſt always take when you cannot find the juſt number you look for) is 927, againſt which ſtands 3, ſet 3 in the Quotient, and ſubſtract 927, out of 1097, and there will remain 170, to which bring down 9, the next figure of your dividend, and it is 1709, Look this number in your Table, which you cannot find, but the next leſs is 1545, againſt which ſtands 5, ſet 5 in the Quotient, and ſubſtract 1545 from 1709, there will remain 164, to which bring down the next figure of your dividend (which here is a cypher) making it 1640. Look this 1640 in the Table, which you cannot find, but the next leſs is 1545, againſt which ſtands 5, ſet 5 in the Quotient, and ſubſtract 1545 out of 1640 and there will remain 95; to which bring down the laſt figure of your dividend 9, making it 959. Look this number in the Table, or the next leſs, which is 927, againſt which ſtands 3, ſet 3 in the Quotient, and ſubſtract 927 from 959, the remainder is 32. So is your diviſion ended, and the Quotient is 3553 $\frac{32}{309}$ And with what eaſe and certainty this is effected; no Multiplication being uſed, I leave to the Reader to judge.

The

The Proof of this Division.

This kind of division is also proved by Addiion; for, *If you draw a line under the work, and add all the figures between the two lines together, (in order as they then stand) taking in the remainder (if any be) the Total of this addition will be equal to the Dividend, if the work be true.*

Other Examples for Practice proved.

```
                (4
            2 2 2(7 3
        542) 7 6 3 5 3 (140
            ---------
            5 4 2 8 0
            2 1 6
            ---------
            7 5 8 8 0
To which add      4 7 3  the remainder.
            ---------
The sum is    7 6 3 5 3  equal to the dividend.
                6
            1 1 6 6
            7 4 4 9 4
    5678) 2 3 4 5 6 7 8 (413
            2 2 7 1 2 1 4
              5 6 7 3
              1 7 0
            ---------
            2 3 4 5 0 1 4
To which add      6 6 4  the remainder.
            ---------
The Total is  2 3 4 5 6 7 8  equal to the dividend.
```

Questions

Queſtions performed by Diviſion only.

Queſtion 1. *If a piece of Land lying in a long ſquare or Parallelogram, contain* 42952 *ſquare perches, and one of the ſides thereof be* 236 *perches long, how long muſt the other ſide be?* Divide 42952 by 236, the quotient will be 182, and ſo many perches long muſt the other ſide be.

Queſtion 2. *In a year there are* 8760 *hours, and in every natuarl day there are* 24 *hours, I demand how many dayes be there in a year?* Divide 8760 by 24, the Quotient will be 395, and ſo many dayes be there in a year.

Queſtion 3. *The diſtance from* London *to* Coventry *is* 133760 *yards, and in one mile there is contained* 176 *yards, now I would know how many miles it is from* London *to* Coventry; Divide 133760 by 1760, the quotient will be 76, and ſo many miles it is from *London* to *Coventry*.

Theſe Queſtions performed by *Diviſion* only, are the converſe of thoſe that were performed by *Multiplication*, which I the rather make choice of, that the Reader might ſee how *Multiplication* and *Diviſion* prove each other.

There are one or two more kinds of *Diviſion*, ſomething like theſe laſt, but I ſhall forbear exemplifying them; for much variety helps to make a book rather great than uſeful.

¶. Here is to be noted that in the following Rules, where there is continual uſe of *Diviſion*, I ſometimes uſe one kind of *Diviſion*, and ſometimes another, for variety ſake, but the Practitioner may uſe which he is beſt skill'd in, for they all produce the ſame effect.

Reduction.

Reduction.

IS Two-fold, First, That which turns *Great* denomination into *Smaller*; as Pounds into Shillings or Pence, this is done by *Multiplication*, as followeth.

Example. 1.

Let it be asked how many pence are contained in 719 li. 11 s. 7 d.?

First, a Shilling is contained in a pound 20 times, therefore multiply 719 by 20, or (which is the same, but shorter) by 2, and put 0 to the product, as in the margine, this shews, that in 729 *li.* there are 14580 Shillings. To which add 11 *s.* it makes 14591 Shillings.

li.	
729	
20	*multiply*
14580	
11	*add*
14591	
12	*multiply*
29182	
14591	
175092	
7	*add*
175099	

Again, because one penny is contained in one shilling 12 times, multiply 14591 by 12, it produceth 175092, to which add the 7 pence, so the sum will be 175099, and so many pence are contained in 729 *li.* 11 *s.* 7 *d.*

Example 2.

Let it be asked how many pints are contained in 4 Tuns, 3 Hogsheads, and 27 Gallons?

First,

First, 1. Tun is equal to 4 Hogsheads, therefore 4 Tun is equal to 16 Hogsheads, to which add the 3 Hogsheads, so there is 19 intire Hogsheads.

Again,, because one Hogshead contains 63 Gallons, multiply 19 by 63, it produceth 1197 Gallons, to which add 27, it gives 1224 Gallons.

Lastly, because every Gallon contains 8 pints, multiply 1224 by 8, it produceth 9792, and so many pints are contained in 4 Tuns, 3 Hogsheads, and 27 Gallons.

After the same sort might dry Measures be reduced, as Quarters to Bushels, Pecks, or Gallons, and likewise all weights and Outlandish Coins, of which the proportion of the greater to the lesser is (before) known or given.

```
   63
   19  multiply
  ----
  567
  63
 1197
   27  add
 ----
 1224
    8  multiply
 ----
 9792
```

Secondly, It is often requisite to turn *Smaller* denominations to *Greater*: this is done by *Division*, as followeth.

Example 1.

Let it be asked how many pounds are contained in 80976 shillings?

Divide 80976 by 20, the quotient is 4048 *l.* and 16 *s.* remaining, which is the true answer.

```
  ~~1~~(1
8 ~~0~~ ~~9~~ ~~7~~(6 (4048
~~2~~ ~~2~~ ~~2~~ ~~2~~ ~~0~~
```

Example 2.

Let it be asked how many pounds are in 109754 *d.*?

Because a pound contains a shilling 20 times, and a shilling contains a peny 12 times, therefore if 109754 be divided first by 12, the quotient shall be

9146

9146 shillings and 2 pence over; then if 9146 be divided by 20, the quotient is 457 pounds and six shillings remaining; so that 109754 pence is equal to 457 *l.* 6 *s.* 2 *d.*

Or if 109754 had been at first divided by 12 times 20, that is 240, (which is the number of pence contained in a pound) the quotient had been 457 pounds and 74 pence remaining, which is all one with the former; for 74 pence is equal to 6 shillings 2 pence. More instances shall not need herein, because the thing of it self is very clear.

Progression.

IS also of two sorts, the first is of certain numbers in *Arithmetical Proportion* from 1; that is, such as differ equally, as 1, 2, 3, 4, 5, 6, where the common difference is 1, (as is easily seen,) or 1, 3, 5, 7, 9, 11, where the common difference is 2, or any other, as 1, 8, 15, 22, 29, 36, where the common difference is 7, this is called *Arithmetical Progression.*

2. Secondly, of certain numbers in *Geometrical* Proportion from 1, that is such as increase by a common Multiplication, as 1, 3, 4, 8, 16, 32, where the common Multiplier is 2, that is the first by 2, produceth the second, and the second multiplied by 2, produceth the third, and so on.

Or as, 3, 9, 27, 81, 243, where the common Multiplier is 3, this is called *Geometrical Progression.*

Both the common difference (in the first) and the common Multiplication (in the latter) shall for shortness hereafter be called *the common excess.*

First,

First, now of the first sort, or *Arithmetical Progression*, the principal use of this is,

1. *If the number of place, and common excess be given, to find the last number.*

2. *When the number of places, and the last number is given, to find the aggregate, or total sum of all the numbers.*

3. *When the last number, and the total sum is given, to find the number of places.*

4. *The number of places, and total sum being given, to find the last number.*

5. *The last number, and number of places given, to find the common excess.*

6. *The last number and common excess being given, to find the number of places.*

I will instance in no more, few of these ever hap ning to be used.

For the first of these, let there be given the number of places 100
The common excess 1
To find the last number also; 100

THE RULE.

Multiply the number of places less by 1, *by the common excess, and to the product add the first number: the sum is equal to the last number.*

So here, multiply 99 by 1, the Product is 99, (for 1 neither multiplies nor divides) to this add the first number 1, it gives 100 for the last number.

Or let the numbers be 1, 7, 13, 19, 25, 31, where the common excess is 6, and the number of places also 6.

Now, if the number of places less by 1, that is 5, be multiplied by the common excess, which is 6, the product is 30, to which adding the first num-

ber which is 1, the laſt number 31, is thereby compoſed. This is ſo eaſie that it needs no proof.

2. For the ſecond, which is, The laſt number, and the number of places given, to find the total ſum of all the numbers.

THE RULE.

Add the firſt and laſt numbers together, and multiply the ſum by half the number of places, the product is equal to the aggregate or ſum of all the numbers added together.

So if to the firſt number 1 be added the laſt number 100, it gives 101, which multiplied by 50 (which is half the number of places) produceth 5050, which is equal to all the hundred numbers added together.

And hereby may that vulgar queſtion be anſwered, which is,

If a man take up 100 *ſtones placed a yard one from another, all in a right line by one at once, and bring them back one by one to his firſt ſtanding, how many yards doth he go backwards and forwards?*

It is ſhewed before that he goes forward 5050 yards, and he muſt needs come back juſt as much; that is, in all 10100 yards, which is 5 miles and 3 quarters; wanting 20 yards.

Or ſecondly, ſuppoſe the numbers were 1, 9, 17, 25, 33, 41. Whereof the common exceſs is 8, the firſt and laſt added gives 42, which multiplied by 3, (half the number of places) the Product is 126, which is the ſum of them all.

3. For the third thing, that is, by the laſt number and the total, to find the number of places.

THE

THE RULE.

Add the first and last numbers, and by the sum of them divide the total, the quotient will be equal to half the number of places.

```
5 2 8 (3
  4 2
```

This is so plain, it needs no clearing.

4. For the fourth, if the total, and number of places be given, to find the last number.

THE RULE.

Divide the total by half the number of places, the quotient is a number, from which if 1 be taken; the rest is the last number.

As let the numbers be 1, 3, 5, 7, 9, 11, 13, 15, or any other (in Arithmetical proportion) whatsoever. The sum of these is to be 64, and the number of places is 8; the half of it 4. Now if 64 be divided by 4, the quotient is 16, from which, if 1 be taken, there remains 15 for the last number.

```
2 0
6 4 (1 6
4 4
```

5. Now for the fifth variety, if the last number and number of places be given, to find the common excess.

THE RULE.

From the last number take 1, and the remain shall be the Dividend; then from the number of places also take 1, and make this later remain the Divisor; then the quotient of this Division shall be the common excess.

Example. Let the numbers be 1, 4, 7, 10, 13, 16, from 16 take 1, remains 15, for the dividend; then from 6 (which is the number of places) take also 1 remains 5 for the Divisor.

Now when 15 is divided by 5, the quotient is 3,

And 3 is also the common excess, or difference between 1 and 4, or 4 and 7, *&c.*

6. Lastly, let the last number, and the common excess be given, to find the number of places.

THE RULE.

From the last number take 1, *and divide the remain by the common excess; then to the quotient add* 1, *the sum is the number of places.*

As, let the numbers be 1, 5, 9, 13, 17, 21, 25, 29, from 29 take 1, remains 28, which divided by 4 (which is the excess) the quotient is 7, to which add 1, the sum is 8, which is the number of places, as the Reader may easily count.

Geometrical Progression.

I shall not be so large in this as in the former, because these things are of little use to the Arithmetician, except where a number is to be many times doubled, tripled, or the like, which cannot be so easily abridged here, as in the other, because there the last number arising of many *Additions* of the excess to 1, was easily found by one multiplication: but here the last number being made by many *Multiplications* of the excess, is therefore many times harder than the other.

The varieties here shall be but two.

1. *The common excess, and number of places being given, to find the last number.*

2. *The excess, and last number being given, to find the total sum.*

The first of these may thus be found. Let the numbers be 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, the excess is 2, the places 10, find out the fifth number (which

(which is easily done, for any one may reckon so far by heart, that is here 16, and multiply 16 by 16, it produceth 256, which is the ninth number: lastly, multiply 256 by the excess 2, thence ariseth 512, the number desired.

So if the places had been more, as 72, having found the 9 number 256, multiply it by 256, thence comes 65536 for the 17th number, which multiplied by the excess 2, gives 131072 for the eighteenth place, which multiplied by 131072, gives 17179869184, for the 35 place; and that multiplied again by the excess 2, gives 34359738368, for the 36 place, that multiplied by 34359738368, the product will be 1180591620717411327424, for the 71 place, which lastly, multiplied by the excess, gives 2361183241434822654848, for the 72 place, which is the last number of the Progression required to be found.

Perhaps this may seem somewhat tedious, but where things cannot be performed without labour, the Reader must content himself with such Rules as make it less; for it is certain, that this way is much shorter than to have multiplied still by the excess 71 times, which else he must have done.

All this notwithstanding, he is not bound to use the same numbers, much less in other questions where the number of places is not the same: but whereas I began from the place 9, he may begin at 8, 10 or 12, or where he pleases, so as he remembers still where he is; for this is general, *If the number belonging to any place whatsoever, be multiplied by it self, the product shall be the number belonging to twice so many places want one place.*

Now for the second thing, which is to find the sum of all the numbers.

THE RULE.

From the laſt number take the firſt, and divide the remain by the exceſs want 1, then multiply the quotient by the exceſs, and to the product add the firſt number, the ſum of them is equal to the ſum of all the numbers.

So if from the laſt number, or 72 place, be taken 1, remain is 2361183241434822654847 which ſhould be divided by 1, (that is the exceſs want 1, for the exceſs is but 2) but becauſe 1 neither multiplies nor divides, that labour is ſaved: Now multiply this remain by the exceſs, the Product is 4722366482869645309696, to which adding the firſt number 1, by making the figure 6 next the right hand to be 7, you have the total ſum of all the 72 numbers.

A Queſtion reſolved by Geometrical Progreſſion.

A Londoner *ſojourning in a Country Market Town, in Winter made himſelf a new Freez Suit and Coat, on which were ſet 6 dozen of Buttons of Silk and Silver; a Baker being in his company liked it ſo well, he would buy it of him; the Citizen conſented to let him have it, paying for the firſt Button a ſingle Barley-corn, for the ſecond 2, for the third 4, and ſo on, doubling to the laſt.*

The Bargain was liked on both parts for the preſent, but ſhortly after revoked, for it could not be performed, and no man can be holden to an impoſſibility.

But why this could not be performed, may be judged: Firſt, by inquiring the worth of ſo much Barley in Money: And ſecondly, the weight of it; and how it ſhould be removed.

1. For the firſt, allowing 10000 corns to a pint (which is more than enough) then 5120000 Corns make a quarter; and yet (for ſhortning the Diviſi-

on)

on) we will allow 10000000 Corns to a quarter; by which dividing the whole number of Corns (which is done by cutting off the first 7 figures towards the right hand) the quotient will be 472236-648286964, and so many whole quarters there are, omitting the remain, as in this case inconsiderable.

Now allowing Barley were to be sold at 15 *d.* the Bushel (which is cheap) it is so many Angels; and therefore dividing by 2, it is 2361183241434 82 pounds sterling; which is in words, *Two hundred thirty six millions of millions, one hundred and eighteen thousand, three hundred twenty four millions, one hundred forty three thousand, four hundred eighty two pounds,* which I take to be too much for any Trades-man to get or keep.

And reckoning Land for ever at twenty years purchase, if this sum of pounds be divided by 20, the quotient is the yearly rent of 118059162071 74 pounds.

And this divided again by 365 (the number of days in a year) the quotient is 323449759 18, that is above *thirty two thousands of millions a day for ever.* So great a vanity may be concluded on for want of a little premeditation.

2. Now secondly, for the weight of it, if we put 8 Bushels to weigh 2 hundred pound weight, (for sure it doth weigh more) then the whole number of quarters multiplied by 2, gives the weight of all to be 944473296573928 hundred weight, and if this be divided by 20, (which is but cutting off one figure toward the right hand, and dividing the rest by 2) or which is all one, cut off one figure from the number of quarters, the quotient 47223664828696 is so many Tuns. And therefore it will require 47223664828 ships of 1000 Tuns a piece to carry it: And consequently, if every Nation in the World

had above 10000 ſuch ſhips, yet there muſt be above *four millions* of ſuch Nations; which I ſuppoſe are not to be found in this World.

And here I will leave this, having uſed this long example, (which though it require more labour as all great examples do, yet the ſame skill will do it, as if the places had been fewer) that the Reader being throughly exerciſed thereby, may the eaſier leap over others which are ſhorter.

The GOLDEN RULE, or, Rule of Three Direct.

THis is the moſt *uſeful* and moſt *eaſie Rule* in *Arithmetique*, and deſerves a *Golden Name*. It is when there are three numbers given, or known, to find a fourth in proportion with them.

But 4 numbers are in proportion, and called *Proportional*, when as the firſt is to the third, ſo is the ſecond to the fourth.

As if there were given 3, 4, and 6, to find a fourth, which may be to 4, as 6 to 3, that is double, and that fourth number is 8; and this is called *Proportion Direct*; and the Rule whereby it is done, *The Direct Rule*.

There is alſo another proportion called *Reciprocal*; which is when as the firſt is to the third, ſo is the fourth to the ſecond: As 3, 4, 6, and 2, this is called *The Reverſe Rule*.

In *Direct proportion*, the product of the two middle numbers multiplied together, is ever equal to the product

product of the first and last multiplied together, which serves not only for a *Proof*, but a ground of the *Rule*, which *Rule* shall here follow: the *Reverse Rule* being deferred till we have done with this.

The Rule Direct.

Multiply the second term (or number) by the third, and divide the Product *by the first; the* Quotient *shall be the fourth number desired.*

Example. Let the three numbers given be 2, 6, 3, multiply 6 by 3, the Product is 18; then divide 18 by 2, the Quotient is 9, which is the fourth number in proportion with 2, 6, and 3.

For as 2 to 3, so 3 times 2, which is 6, is to 3 times 3, which is 9.

And so the Product 18 divided by 2, and the Quotient 9, causeth that the Product of 2 into 9 shall be also 18, and consequently if 2 be the first of the 4 proportional numbers, and 6 and 3 the two middlemost, then 9 is the last.

Otherwise.

Divide the second by the first, and multiply the third by the quotient, the product shall be the fourth.

So if one divide 6 by 2, the quotient is 3, by which multiply 3, the product is 9, for the fourth number, as before. Other ways this Rule might be expressed, but where the first way is so shott and clear, there many other ways would rather trouble than help the person that should use them.

In the first way, which here we mean to use, and no other) if the *first* number be 1; then the product of the *second* and *third* gives the *fourth*, without any *Division:* Or, if the *second* or *third* number be 1, then there needs no *Multiplication*, but dividing the greater of them by the first, the quotient (in whole

whole numbers, for yet we ſpeak of them) is the fourth number which was ſought.

Note 1.

To know when to uſe the Direct, or the Reverſe Rule. Conſider, if *more*, require *more*; or if *leſs*, require ſtill *leſs*; then uſe the *Direct Rule*. But if *more* require *leſs*, or *leſs more*, then uſe the *Reverſe Rule*, this will be eaſily underſtood when we come to Examples.

Note 2.

To know how to place the three numbers when they are confuſedly given. Remember that 2 of them are always of one denomination, as both *pounds*, or both *ſheep*, or both *yards*, or *acres*; and the other number hath another denomination: now know, that this ſingle number is ever the ſecond number in order.

And one of the other two, namely, that which hath ſome relation to this *ſecond*, is the *firſt*; and the other is the *third* number, whoſe relation is ſought for in the *fourth*, whence its plain that the *ſecond* and *fourth* are alſo of the ſame denomination.

And having premiſed theſe things, let us now exemplifie the Rule in ſome queſtions.

Queſtion 1.

If three yards of Cloth coſt 4 li. *what ſhall* 21 *yards coſt?*

Set the numbers in order, as in the example. If 3 yards coſt 4 *li.* what 21 yards? Here you ſee that the firſt number, and the third number, are both of one denomination, *viz.* both *yards*, and the ſecond number is of another denomination, namely *pounds*, wherefore the *fourth* number which is ſought for, muſt be alſo *pounds*; therefore multiplying (according

to

to the Rule before given) the *second* number by the *third*, and dividing the Product by the *first*, the quotient shall answer the question.

First, 21 multiplied by 4, (which is the third number multiplied by the second) produceth 84, which divided by 3 the first number, the quotient is 28 *li.* and so much shall 21 yards cost: for 28 is to 4, as 21 to 3, seeing each contains either 7 times.

And the work will stand thus.

Yards Pounds Yards
If 3 cost 4 what 21
4
x li. ——
84 (28 84
33

Question 2.

If 4 men eat 2 Pecks of Corn in a Week, how many Pecks shall serve 100 men?

Place your numbers as here you see, then multiply 100 by 2, (that is the third number by the second, and the Product is 200, which divided by 4, the quotient is 50, for the number of Pecks required.

Men Pecks Men
If 4 eat 2 what 100
2
——
200

x x x (50 *Pecks*
x x

Question 3.

If 20 sheep cost 13 pound 13 shillings 4 pence, what is that for every sheep?

Turn

Turn the Shillings and Pounds into Pence; thus:

s.	*li.*
13	13
12	240
26	520
13	26
156	3120
	3120
	156
	4
	3280

Multiply 12 *s.* by 12, the Product is	156
And 13 *li.* by 240 (because 240 pence make one pound) the Product is	3120
To which add the 4 *d.*	4
It makes in all	3280

Then the question will be, If 20 sheep cost 3280 pence, what shall one sheep cost?

sheep *pence*
If 20 cost 3280 what 1?

x *d.*	*x* *d.*
x x 80 (164	*x* 6 4 (13
x x x 0	*x* x x
	x

By the Rule before delivered, I should multiply the second number by the third, but in this example, the

the third numher being 1, it doth not multiply; I therefore divide 3280 the ſecond number by 20 the firſt number, and the quotient 164. is the price of one ſheep in pence, which divided by 12, the quotient is 13 *s.* and 8 *d.* remaining, the price of every ſheep therefore is 13 *s.* 8 *d.*

Queſtion 4.

How many 10 *inch Tiles will pave a Floor that contains* 16 *ſquare yards?*

Firſt remember there are 36 inches in one yard in length; which multiplied into 36, gives 1296, for the ſquare inches in one ſquare yard; multiply 1296 therefore by 16, thence comes 20736, the ſum of all the 16 yards in inches.

Secondly, ſeeing every Tile is 10 inches in length, and 10 in bredth, multiply 10 by 10, it produceth 100 for the ſquare inches in one tile, See the manner of work.

36	10
36	10
126	100
108	
1296	
16	
7776	
1296	
20736	

Then by the Golden Rule,

If

If 100 inches require 1 tile; what shall 20736 inches require?

inches *tile* *inches*
If 100 require 1 what 20736.

```
      0 |
  1 1 7 | 3 6 ( 207
  1 1 1 | 0 0
```

Here because 1 doth neither multiply nor divide (as hath been several times intimated) I therefore divide the third 20736, by the first 100, the quotient is 207, and 36 remaining.

So it appears, that 207 is too little, and 208 too much to do the Work: the just number being 207 $\frac{36}{100}$, we shall not trouble the Reader with this till he know something of Fractions.

Question 5.

If 100 li. give 6 li. interest for a year, how much shall 750 li. give?

Multiply 750 by 6, the product is 4500, which divided by 100, the quotient is 45 *l.* for the thing required.

li. *li.* *li.*
If 100 give 6, what 7 5 0?

```
      6
  -----
  4 5 0 0
```

```
          li.
  4 5 0 0 ( 45
  1 1 0 0
```

Question 6.

If 750 l. gives 45 l. interest for a year, what shall 100 l. give?

Mul-

Multiply 45 by 100, the product is 4500, which divided by 750, the quotient is 6 *l.* for the interest of 100 *l.* for a year.

li. *li.* *li.*
If 750 give 45, what 100
100

4500

Many other questions might be added, but the Rule is so plain, that it needs them not; and so general, that he which can resolve one, may as well resolve any other: And for that reason, and because in all the Rules which follow, this Rule will be constantly made use of, I will say no more of it here.

The Golden Rule *Reverse.*

IF 12 *Workmen do any piece of work in* 8 *months, how many Workmen shall do the same in* 2 *months?*

THE RULE.

Multiply the first *term by the second; and divide the Product by the* third, *the quotient is the number desired.*

Here 12 is not the first number, though it be first named; but the three numbers placed in order, stand thnd, 8, 12, 2, for the middle term must always be of the same denomination with that which is required.

Now multiply 12 by 8, the product is 96, which divided by 2, the quotient is 48, which answers the question, As in this example, *months*

months	*men*	*months*
8	12	2
	8	

2) 9 6 (48

8

16

16

For if 8 months require 12 men; then (a fourth part of 8) 2 months, shall require four times 12, that is 48 men.

For here *less* requires *more*; that is, *less time*, *more hands*; and therefore it is wrought by the *Reverse Rule*.

Question 2.

How many Ells of Tapestry will serve to hang a Room 3 yards high, 6 yards long; and 5 yards broad? not regarding Doors, Windows or Chimney, but as if there were no such.

First, multiply 6 by 3, the product is 18, which doubled (because there are 2 sides called *lengths*) is 36 yards for all the length.

Secondly (for the same reason) multiply 3 by twice 5, that is by 10, the product is 30 yards, for all the bredth; which added to 36, gives 66 yards, equal to all the length and bredth in yards.

But now because Ells, that is, *Flemish* Ells (for such measure are Hangings sold by) is equal to 3 quarters of a yard, that is, their Ell is to our Yard as 3 to 4. Say therefore, if 4 give 66, what 3? multiply 66 by 4, it produceth 264; then divide 264 by 3, the quotient is 88. Again, multiply 88 by 4, and divide the product (which is 352) by 3, the quotient is 117,

and

and 1 remaining: to which the diviſor 3 being applyed; the number juſtly anſwering the queſtion is 117 Ells, and one third part of an Ell.

Note 1.

Becauſe here we had to deal with things which had equal length and bredth, that is ſquare yards, and ſquare ells; therefore one multiplication and diviſion was not ſufficient to proportion this: but if inſtead of working by 4 and 3, we had done it by their ſquares which is 16 and 9, it might have been performed at once; thus multiply 66 by 16, the product is 1056, which divide by 9, the quotient is $117\frac{1}{3}$, as before, but I began not with this way, for I ſuppoſed my Reader ignorant of ſquares.

Note 2.

It might alſo have been done, by reducing all the terms into quarters of a yard at the firſt, and after the number is found, reducing them again to ells; but becauſe it is more proper to work thus, till fractions have been taught: I leave that, and proceed to another queſtion.

Queſtion 3.

If 1 *Cloſe would graze* 21 *Horſes for* 6 *weeks; then (ſuppoſing no waſte to be made) how many Horſes would it feed for* 7 *weeks?*

Multiply 21 by 6, it produceth 126, which divided by 7, the quotient is 18. At that rate therefore it would keep 18 Horſes for 7 weeks.

Queſtion 4.

If 1 *Cloſe will feed* 18 *Horſes for* 7 *weeks, how long ſhall it feed* 63 *Horſes?*

Multiply (according to the rule) 18 by 7, the

product is 126, which divide by 63, the quotient is 2, therefore 2 weeks it ſhall keep them.

The like way ſerves for Hay, Oats, or any other proviſion for Man or Beaſt; which may be of uſe in *Garriſons*, and ſuch like caſes where ſcarcity may be feared, to proportion either the *months* to the *meat*, or *meat* to the *mouths*.

Before I leave this *Rule*, (becauſe it comes not ſo much in uſe and Practice as the *direct Rule* doth, and therefore may be more apt to be forgotten) I will, to exerciſe the Reader therein, propoſe the following *Queſtions*, giving the Anſwers of them, and leave the Practice to the Reader to find out of himſelf, the better to fix it, the Rule in his memory.

Queſtion 1.

If 12 men would raiſe a Frame in 10 days; in how many days would 8 men raiſe the ſame?

Here, becauſe the fewer men would require the longer time, though the number be 12, 10; 8, yet you ſhall (by obſerving what hath been already delivered in this Rule) find the fourth proportional (which is the number anſwering the Queſtion) to be 15, and ſo many men will do the work in 8 days.

Queſtion 2.

If 60 yards of Hangings of three quarters broad would hang a Room; How many yards of half a yard in bredth would ſerve to hang the ſame Room?

Anſwer Ninety yards.

Queſtion 3.

If a yard being 12 inches in bredth do require 12 inches in length to make a foot ſquare; What number of inches in length will make a foot ſquare, when the bredth of the board is 16 inches?

Anſwer 9 inches.

Que-

Question 4.

If the base or end of any solid (as a piece of Timber or Stone) being 144 inches, do require 12 inches in length of that piece to make a solid foot; What number of inches in length will make a solid foot, when the square at the end is 216 inches?

Answer, 8 inches.

I will say no more of this Rule; Neither will I treat of the *Double Rule of Three*, as a rule by it self; but come to the *Rule of five Numbers*, which is an abridgment of the other.

The Golden Rule *Compound of five Numbers.*

Question 1.

IF *a hundred pound weight* that is 112 pound weight) *carried 126 miles cost* 14 s. *how much shall three quarters of a hundred* (that is 84 pound) *cost, being carried* 40 *miles?*

THE RULE.

Multiply the three last numbers one into another; (that is) the third *by the* fourth, *and that product by the* fifth; *the last product shall be the* dividend.

Again, *Multiply the* two first *numbers together; the product shall be the* Divisor. *This Division being made, the* Quotient *will be the number of shillings desired.*

Example of the former Question.

Firſt, Place your numbers according to the tenor of the queſtion thus:

li.	*miles.*	*s.*	*li.*	*miles.*
112	120	14	84	40
120		12		
2240		28		
112		14		
13440		168 *pence*		
		84		
		672	13440)564480(42	*d.*
		1344		
		14112	53760	
		40	2688	
		564480	26880	

Your numbers being placed in order, reduce the 14 *s.* into pence, and it is 168 *d.* then multiply 168 by 84, the product is 14112, which multiplied by 40 the later product, it produceth 564480 for the dividend.

Then multiply 112 by 120, it produceth 13440 for the Diviſor.

Divide 5644800 by 13440, the quotient will be 42 pence, which is 3 *s.* 6 *d.* and anſwers the queſtion.

In this Rule, the *firſt* number and *fourth*, alſo the *ſecond* and *fifth*; and alſo the *third* and *ſixth*, are of like denomination and nature.

Queſtion 2.

If 100 li. *for* 6 *monet!s yield* 3 li. *intereſt*, *what ſhall* 625 li. *yield for* 36 *moneths?* Place

Place them thus; 100, 6, 3, 625, 36.

Multiply the three laſt, as before is ſhewed, the later product is 67500 for the *dividend*; and the 2 firſt multiplied make 600 the *Diviſor*, then divide 67500 by 600 the (or 675 by 6, which is all one) the quotient will be 112 whole pounds; and 300 (or 3) remaining, which becauſe it is half the Diviſor, ſignifies the half of a pound; that is 10 ſhillings. So the anſwer to the queſtion is 112 *li.* 10 *s.*

li.	*m.*	*li.*	*li.*	*m.*
100	6	3	625	36
6			3	
600			1875	
			36	
6) 675 (112			11250	
6			5625	
			67500	

Which might have been given in one denomination, namely 2250 ſhillings, if before the work the pounds had been turned into ſhillings, by multiplying them by 20, as hath been ſhewed before.

But ſince moſt queſtions, except ſuch as are ſtudied for the purpoſe, are apt to end in ſome Fraction, I ſhall next treat of Fractions.

Only firſt, having ſpoken of the double Rule of Three, this may let you know, that all queſtions which are wrought at once by the compound Rule of Five, may be done at twice by the ſingle Rule of Three; and the doing of them ſo by two Operations, is called, *The Double Rule.*

As in our laſt queſtion, there are two things conſi-

derable, the difference of money; and the difference of time.

First, for the money.

Say, if 100 *li.* give 3 *li* what 625 *li?* answer $18\frac{12}{100}$ *li.*

Secondly, for the time.

Say, if 6 *mo.* give $\frac{12}{100}$ *li.* what 36 *mo.* answer $112\frac{12}{100}$ *li.*

But this will be better understood anon; and then the Reader may use that which he likes best.

Of Fractions.

THE word *Fraction* signifies a *breaking*, or *breach* of any intire thing into parts; and when a number is broken so, the parts (which must needs be every one less than the whole; and the whole is accounted but *One* or *Unity*) being less than Unity, are called Fractions (that is, fragments or pieces) of Unity. Now the Unite, or intire number which is to be broken, may be any thing, as one *pound*, in respect of which, *Shillings*, and *Pence*, and *Farthings* are *Fractions*; or, one *shilling*, in respect of which, *pence* and *farthings* are *Fractions*; or, one *peny*, in respect of which, *farthings* are *Fractions*; and the like of *Weights* and *Measures*, or any other thing to be broken into parts.

In *Fractions*, we shall treat first of *Numeration*, then of *Multiplication* and *Division*, then of *Reduction*; and lastly, of *Addition* and *Subtraction*.

The reason of this Order will soon be seen; for *Mul-*

Multiplication and *Diviſion* are here much eaſier than *Addition*, &c. and therefore ought to be learned before them.

NUMERATION.

NUmeration is nothing elſe but the way of writing Fractions; and that this may be done, we muſt conſider that any *Unite*, or *Number* repreſenting an *Unite*, may be broken into two parts equal; and then each of the parts is called *one ſecond*, or *half*; or it may be parted into three equal parts, and then each part is called *one third*, and two of them are called two thirds; and the like may be underſtood if it were parted into 4, 5, 6, 7, 8, 9, 20, 50, or 100, or how many ſoever.

Now to write theſe; do thus:

Write	One half	Thus	$\frac{1}{2}$
	One third		$\frac{1}{3}$
	One fourth		$\frac{1}{4}$
	One fifth		$\frac{1}{5}$
	One ſixth		$\frac{1}{6}$
	One ſeventh		$\frac{1}{7}$
	One eighth		$\frac{1}{8}$
	One ninth		$\frac{1}{9}$
	One tenth		$\frac{1}{10}$

In every one of theſe 10 *Fractions*, the Number below the line is called the *Denominator*, and it ſhews into how many parts the *Unite* is broken.

The number above the line ſhews how many of thoſe parts are taken, or contained in the *Fraction*, and is therefore called the *Numerator*: So in the

Fraction $\frac{3}{5}$: the *Denominator* 5 ſhews the *Unite* to be broken into 5 parts: and the *Numerator* 3 ſignifies 3 of ſuch parts to be contained in the *Fraction*; which Fraction therefore is called *three fifths.*

And here it is plain; that, As the *Numerator* is in proportion to the *Denominator*: ſo is the Fraction to 1, or Unity, for $\frac{3}{3}$ or $\frac{5}{5}$: or any the like, is equal to 1.

And therefore all Fractions are quotients of leſſer numbers divided by greater, as $\frac{4}{7}$ ſignifies 4 to be divided by 7, and as the divideud 4, is to the diviſor 7: ſo is the quotient $\frac{4}{7}$ to Unity.

And therefore this line of ſeparation which is drawn between the *Dividend* and *Diviſor*, doth properly ſignifie *Diviſion.*

Hitherto we have ſpoken only of ſuch Fractions as are leſs than 1, and thoſe are called *Proper Fractions*: but there are alſo $2\frac{1}{2}$, $3\frac{1}{4}$, $5\frac{1}{7}$, $6\frac{2}{5}$, and the like mixed Numbers; which ſo written ſignifie *two and an half*, 3 *and* 3 *quarters*, *five and a ſeventh*, 6 *and* 3 *fifths.* Theſe by multiplying the whole Numbers, by the Denominator, and to the product adding the Numerators reſpectively, are turned to $\frac{5}{2}$, $\frac{15}{4}$, $\frac{37}{7}$, $\frac{33}{5}$, which are called *Improper Fraction*, becauſe every one of them contains more than Unity.

Theſe, nevertheleſs may be *multiplied*, *divided*, *added*, or *ſubſtracted* in the ſame way as are proper Fractions. And this ſhall ſerve for *Numeration of Fractions.*

MULTIPLICATION.

THE RULE.

MUltiply all the Numerators *together*, *the laſt product ſhall be the* Numerator *of the product required*: *Likewiſe*

Likewiſe multiply all the Denominators *together, the laſt product ſhall be the* Denominotor *of the product ſought.*

Example 1.

If $\frac{3}{5}$ be to be multiplied by $\frac{4}{9}$, Multiply the *Numerator* 3 by the *Numerator* 4, the product is 12, for the *Numerator* of the new product. Alſo multiplying the *Denominator* 5, by the *Denominator* 9, they produce 45, for the *Denominator* of the deſired product, ſo that product which was required, is $\frac{12}{45}$.

Example 2.

If $\frac{1}{2}$, $\frac{1}{4}$, $\frac{4}{5}$, $\frac{5}{9}$, and $\frac{1}{11}$ were to be multipliedall together, begin with the *Numerators*, ſaying, once 3 is 3, and 3 times 4 is 12, and 12 times 5 is 60, and 60 times 3 is 180, for the *Numerator*: Then multiply the *Denominators*: ſaying, 2 times 4 is 8, and 8 times 5 is 40; and 40 times 9 is 360, and 360 times 11 is 3960, for the new *Denominator*. So that the product of all theſe is $\frac{180}{3960}$, that is 1644 to $\frac{1}{22}$, as ſhall be ſeen hereafter in *Reduction*.

And thus it appears, that proper *Fractions* being leſs than *One*, are ſtill made leſs by *Multiplying*: as here the product $\frac{1}{22}$ is much leſs than $\frac{1}{11}$, which is the leaſt Multiplier; and the reaſon hereof is plain, for ſeeing *Multiplication* is but the taking of a Number, a certain number of times, if that number of times be more than 1, then the Number to be taken is increaſed by being taken more than once; but if the Number of times be 1, it is not increaſed nor diminiſhed, but is ſtill the ſame; Laſtly, If that number of times be leſs than 1, as $\frac{1}{2}$, the number not being taken once, but half of once, produceth a number leſs by half; that is, the half of the number to be taken; and the like reaſon is of all others.

Example

Example 3.

Multiply the mixt Numbers, $3\frac{1}{2}$, $4\frac{1}{3}$, and $5\frac{3}{4}$: First (as hath been ſhewn already) turn them to improper Fractions: thus, firſt ſay, 2 times 3 is 6, and 1 is 7. So the firſt is $\frac{7}{2}$. Secondly, 3 times 4 is 12, and 1 is 13: ſo the Second is $\frac{13}{3}$. Laſtly, 4 times 5 is 20, and 3 is 23: ſo the laſt is $\frac{23}{4}$. Now the Fractions to be multiplied are $\frac{7}{2}$, $\frac{13}{3}$, and $\frac{23}{4}$; Firſt, for a new *Numerator*, ſay, 7 times 13 is 91, and 91 times 23 is 2093, for a new *Numerator*.

Then ſay, 2 times 3 is 6, and 6 times 4 is 24. So the new *Denominator* is 24.

And the product of all theſe Fractions is $2093\frac{23}{24}$, that is, if real diviſion be made, $87\frac{5}{24}$.

DIVISION.

Diviſion, To divide one Fraction by another, is but the croſs multiplication of them; that is, the *Numerator* of the one, by the *Denominator* of the other, and hereby the proportion of one Fraction to another is ſeen.

Example 1.

$$\frac{3}{4} \times \frac{6}{8} \quad \begin{matrix} 24 \\ \\ 24 \end{matrix}$$

Divide $\frac{3}{4}$ by $\frac{6}{8}$, to do it, ſet them thus: and multiply as the croſs leads; Saying, 3 times 8 is 24, which ſet over the Croſs for a new *Numerator*, and 6 times 4 is alſo 24: which ſet under the Croſs for a new *Denominator*; ſo the quotient is $\frac{24}{24}$, that is 1, which ſhews the Fractions to be equal one to another.

Example 2.

Example 2.

Divide $\frac{3}{5}$ by $\frac{4}{9}$. First, ſet them thus: And ſay, 3 times 9 is 27, for a *Numerator*, and 5 times 4 is 20, for the *Denominator*: ſo the quotient is $\frac{27}{20}$, and ſo many times is $\frac{4}{9}$, contained in $\frac{3}{5}$, that is, as 27 is to 20, ſo is $\frac{3}{5}$ to $\frac{4}{9}$, and ſo is $\frac{27}{20}$ to 1.

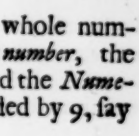
In *Diviſion* it is to be remembred, that the *Numerator* of the *quotient* ever ariſeth of the *Numerator* of the *Dividend*: And the *Denominator* of the *quotient* comes of the *Denominator* of the *Dividend*, each being croſs multiplied as before. And alſo remember always to ſet the Dividend on the left hand of the Croſs.

If a Fraction be to be divided by a whole number; Multiply the *Denominator* by that *number*, the product gives the new *Denominator*, and the *Numerator* remains the ſame. So if $\frac{1}{4}$ be divided by 9, ſay 9 times 4 is 36. So the quotient is $\frac{1}{36}$.

Or if $\frac{1}{4}$ were to be multiplied by 9, the product (by multiplying the Numerator by 9,) will be $\frac{2}{4}$: that is, 2 $\frac{1}{4}$.

Example 3.

Divide $\frac{320}{8}$ by $4\frac{1}{5}$, thus: ſay 320 times 9 is 2880, for a *Numerator*: And 8 times 45 is 360 for a Denominator. So the quotient is $\frac{2880}{360}$, or $\frac{8}{1}$.



For $\frac{320}{8}$ is equal to 40, and $4\frac{1}{5}$ equal to 5, but 40 contains 5 eight times.

And ſo in the ſecond example, it may be proved, that as 27 to 20, ſo is $\frac{3}{5}$ to $\frac{4}{9}$. For firſt, multiply the

two

two middlemoſt, then 20 times $\frac{1}{3}$ is $\frac{20}{3}$, that is 12.

Secondly, multiply the firſt and laſt, and then 27 times $\frac{4}{9}$ is $\frac{108}{9}$: that is alſo 12.

Wherefore by that which hath been ſaid in the *Golden Rule*, the four Numbers 27, 20, $\frac{1}{3}$, $\frac{4}{9}$, are proportional.

REDUCTION.

REDUCTION OF FRACTIONS is threefold.

1. *To reduce one Fraction* (which is not already in the leaſt) *to reduce a Denomination.*

2. *To reduce many Fractions of divers denominations, to one Denomination.*

3. *To reduce any Fraction from one Denomination* (as near as may be) *to any other Denomination deſired.*

I. For the firſt of theſe, *To reduce a* Fraction *to its leaſt terms.* Divide both the *Numerator* and the *Denominator* by the greateſt *Common Diviſion* that you can think of; the two *Quotients* being placed reſpectively in a Fraction, that Fraction ſhall be equal to the former Fraction, and in leſſer terms.

So (in the 3 Example of *Diviſion*) to reduce $\frac{1889}{3601}$ to $\frac{8}{1}$, divide 1889 by 360, the *Quotient* is 8: then divide 360 by 360, the *Quotient* is 1, and the new Fraction $\frac{8}{1}$ is equal to the former Fraction $\frac{1889}{3601}$, and in leſs terms, as you may ſee. But to find the greateſt common diviſor, this is

The Rule.

Divide the greater term *by the* leſſer (I mean by terms, the *Numerator* and *Denominator*) *and by the remainder*

mainder (if any be) *divide the* diviſor, *and if any thing ſtill remains, by that divide the laſt* diviſor, *continuing this courſe till nothing remain greater than* Unity; *that* diviſor *which is leaſt of all, is the greateſt* Common meaſure *of both terms, by which both being divided, and the* quotient *placed like a* Fraction, *that* Fraction *ſhall be equal to the former* Fraction, *and in the* leaſt terms.

Example.

Reduce $\frac{148}{16}$ to the leaſt terms; firſt divide 148 by 16, the quotient is 9, and 4 remains: again, divide 16 by 4, the quotient is 4, and nothing remains; wherefore taking 4, (the laſt diviſor) for the greateſt common diviſor, by it divide 148, the quotient is 37, and by it divide 16, the quotient is 4. Theſe two laſt quotients placed orderly in a Fraction, make $\frac{37}{4}$, which is equal to $\frac{148}{16}$, and in the leaſt terms, for no number greater than 1, will divide evenly both 37 and 4.

Other wayes there are of leſſening Fractions, as dividing the terms (if they be even numbers) by 2, and the quotients (if even) again by 2, or elſe by 3, or any other Number that will divide them both evenly, that is, leave nothing remaining, but the former Rule being general and eaſie, ſhall ſerve for all.

II. Now ſecondly, *To reduce many* Denominations *to one* common Denominator. Let the Fractions be $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{2}{10}$, to be reduced all to one denomination.

The Rule.

Multiply all the Denominators together, and the laſt Product ſhall be the common Denominator to all the Fractions — Then multiply every particular Numerator into all the Denominators *except his own, and the laſt* Product *ſhall be* Numerator *to that Fraction.*

Thus

Thus to reduce the forementioned Fractions $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{8}$, $\frac{9}{10}$, into one Denomination: Say, 2 times 4 is 8, and 8 times 5 is 40, and 40 times 8 is 320, and 320 times 10 is 3200, this last product 3200 shall be the common *Denominator*. Then to get Numerators for every one of them: As first, for the first, say 1 time 4 is 4, and 4 times 5 is 20, and 20 times 8 is 160, and 160 times 10 is 1600. For the first *Numeration*, so the first Fraction reduced is $\frac{1600}{3200}$. Then for the second *Numerator*: Say, 3 times 2 is 6, and 6 times 5 is 30, and 30 times 8 is 240, and 240 times 10 is 2400. So the second Fraction reduced, is $\frac{2400}{3200}$. After rhe same manner may the other three be reduced to $\frac{2560}{3200}$ for the third: $\frac{2720}{3200}$: for the fourth: and $\frac{2880}{3200}$ for the last: these are severally equal to the other, the first to the first, *&c.* as may be proved thus.

Let the Unity be a pound *Sterling*, then

	s.	
The $\frac{1}{2}$ of it is	10	
and $\frac{1}{4}$ is	15	
and $\frac{1}{5}$ is	16	*d.*
and $\frac{1}{8}$ is	17	6
and $\frac{1}{10}$ is	18	
In all	76 *s.*	6 *d.*

That is 3 whole Unites, and 16 *s.* 6 *d.* over; Turn 16 *s.* 6 *d.* all to six pences, it is 33, and because 6 *d.* is the fortieth part of a pound, therefore all the Fractions are equal to $\frac{33}{40}$.

Now add the new Fractions (which being all of one denomination) may be added like whole Numbers: thus,

1600

	1600
	2400
	2560
	2800
	2880
In all	12240

Which divided by the Denominator 3200, the quotient is 3 $\frac{1648}{3200}$. Now $\frac{1648}{3200}$, reduced to the leaſt terms, as hath been ſhewed how it may, will be $\frac{103}{200}$, ſo the ſum of theſe alſo is $3\frac{103}{200}$, which is equal to the ſum of the Fractions given to be reduced, and therefore they are equal in ſum, and might be thus proved equal ſeverally, that is, the firſt of them propounded to the firſt reduced. Divide the Numerator 1600 by the Numerator 1, the quotient is 1600. Alſo divide the Denominator 3200, by the Denominator 2, the quotient is alſo 1600; and ſo may any of the reſt be proved equal by the equality of quotients. But I leave it as plain enough already.

III. Thirdly, *Any Fraction being given, to change the* denomination *to any other more requiſite, retaining ſtill* (as near as may be) *the ſame value.*

THE RULE.

Multiply the Numerator *given, by the* Denominator *required, and divide the* Product *by the* Denominator *given; the* Quotient *ſhall be the* Numerator *required.*

Example.

Let the Fraction given be $\frac{7}{20}$ of a pound *Sterling*, what is that in the twentieth parts or ſhillings? Multiply 7 by 20, the Product is 140, which divided by 13, the quotient is $10\frac{10}{13}$, that is, 10 *s.* and $\frac{10}{13}$ of a ſhilling;

ling; which may be brought to pence thus, multiply 10 by 12, Product is 120, which divided by 13, quotient is $9\frac{3}{13}$ *d.* And again, multiply 3 by 4, the product is 12, which divide by 13, quotient is $\frac{12}{13}$ of a farthing, so seven thirteenths of a pound is 10 *s.* 9 *d.* and almost a farthing.

But he which is resolved to have it in the smallest coin, may do it at first work; for seeing a farthing is the 960 part of a pound, multiply 7 by 960, they produce 6720, which divided by 13, the quotient is 516 farthings, and $\frac{12}{13}$ of a farthing: these farthings may be turned to shillings, dividing by 48, or to pence by 4, as in *Reduction.*

This Rule though it be brief and plain, is of great use in *Arithmetick*; either for turning Natural and surd Fractions into Decimals; or any other desired *Denomination*, with such facility and speed as may be wished.

IV. *Fractions of Fractions.*

In reduction of Fractions, some make another, or more parts, as *Fractions of Fractions* for one: that is, when there is a part of a *Fraction*; or a part of a part of a *Fraction*, &c. to be valued in one *Fraction.*

THE RULE.

Multiply all the Numerators *together, the last* product *shall be the* Numerator *desired: Then multiply all the* Denominators *together, and this last product shall be the* Denominator *sought.*

Example.

Let the *Fractions of Fractions* propounded, be $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{1}{2}$, for so they are usually written; and let the *Numerators* be multiplied: saying, 4 times 3 is 12, and 12 times 1 is 12, the *Numerator* therefore required

quired is 12: then for the *Denominator*, say, 5 times 4 is 20, and 20 times 2 is 40, for the *Denominator* required; and $\frac{12}{40}$ is equal to $\frac{4}{5}$ of $\frac{1}{4}$ of $\frac{12}{20}$

The Proof.

Let the Unite be 40 *s.* one fifth of 40 is 8, and therefore $\frac{4}{5}$ is 32, of which one fourth is 8, and $\frac{1}{4}$ is 24, of which one half is 12, and therefore $\frac{12}{20}$ is the just sum of all the Fractions: This needs no farther exemplifying.

ADDITION.

TO add many Fractions into one sum, consider whether they be all of one Denomination or divers: if one, Then add all the *Numerators* together into one sum, that sum is the new Numerator: and the *Denominator*, in this case is not altered.

Example.

Let the Fractions to be added be $\frac{2}{4}$, $\frac{4}{4}$, $\frac{1}{4}$, $\frac{5}{4}$, Add the Numerators: saying 2 and 4 is 6, and 5 is 11, and 1 is 12. So the sum of them all is $\frac{12}{4}$, that is 3 Unites.

As, let the Unite be 20 *s.* one fourth is 5 *s.* and $\frac{2}{4}$ is 10 *s.* and $\frac{4}{4}$ is 20 *s.* which added to 10 *s.* is 30 *s.* then $\frac{1}{4}$ is 25 *s.* which added to thirty shillings gives 55 *s.* And lastly, $\frac{5}{4}$ is 5 *s.* which added to 55 *s.* makes 60 *s.* that is 3 times 20 *s.* that is 3 *l.* or 3 Unites.

But if the Fractions to be added, be of divers denominations; as let them be $\frac{1}{2}$, $\frac{1}{4}$, $\frac{4}{5}$, $\frac{2}{8}$; then (by the *Reduction* afore-going) they must be turned all into

one denomination, and then they will be $\frac{320}{480}$, $\frac{360}{480}$, $\frac{384}{480}$, and $\frac{420}{480}$, and may be added like those before: thus,

	320
	360
	384
	420
In all	1580

So the sum of all is $\frac{1580}{480}$, or $\frac{316}{120}$, or that is $3\ \frac{11}{120}$, which if it be money, and the Unite 1 *l.* it is then 3 *l.* 1 *s.* and 10 *d.* as may be tryed thus, First $\frac{2}{3}$ of a pound, is 13 *s.* and 4 *d.* and $\frac{1}{4}$ is 15 *s.* and $\frac{4}{5}$ is 16 *s.* Lastly, $\frac{7}{8}$ is 17 *s.* 6 *d.* These all added together, the the sum is 3 *l.* 1 *s.* 10 *d.*

SUBSTRACTION.

IN SUBSTRACION of one Fraction from another, if they be both of one denomination: It is done by taking the *Numerator* of one from the *Numerator* of the other, the remain is the new *Numerator*, and the *Denominator* the same as before.

So if $\frac{2}{9}$ be substracted from $\frac{8}{9}$, the remain is $\frac{6}{9}$, the like of all others.

But if they be not of one Denomination, they must first be reduced to be so; then that which is said before is sufficient.

Con-

Concerning the Golden Rule in Fractions.

THe *Golden Rule in Fractions* is the ſame as in whole Numbers, I will give you but one inſtance.

If $\frac{1}{4}$ *of a yard of Tape coſt* $\frac{1}{2}$ *of a peny, what ſhall one inch, that is,* $\frac{1}{36}$ *of a yard coſt?*

Multiply the ſecond by the third, the product is $\frac{1}{72}$, which divided by $\frac{1}{4}$, the quotient is $\frac{4}{216}$ of a penny, for the price of $\frac{1}{36}$ of a yard.

Otherwiſe.

Seeing $\frac{1}{4}$ of a yard may be turned to 27 inches: Say, if 27 coſt $\frac{1}{2}$, what 1? divide $\frac{1}{2}$ by 27, it makes $\frac{1}{54}$ for the anſwer: which is equal to $\frac{4}{216}$, and in the leaſt terms.

And whereſoever this may be done, to have the firſt and third Numbers of Fractions of one denomination, the beſt way is to work with their Numerators, not regarding their Denominators at all: As, *If* $\frac{2}{3}$ *coſt* $\frac{1}{4}$, *what* $\frac{7}{3}$? *Inſtead* thereof write, *If* 2 *coſt* $\frac{1}{4}$, *what* 7? Multiply $\frac{1}{4}$ by 7, it produceth $\frac{7}{4}$, which divided by 2, the quotient is $\frac{7}{8}$, and that is the anſwer in the leaſt terms.

And all this while it ſhould have been noted that the Fractions are ever written in a ſmaller figure then the whole Numbers.

The RULE of PRACTICE.

IN the *Golden Rule*, or *Rule of Three Direct*, I intimated; that if the first of the three Proportional Numbers given were *One*, that then the Product of the second and third numbers gives the fourth Proportional Number sought without using of any *Division*; — Also, that if the second or third of the Proportionals given were *One*, then there was no need of *Multiplication*; but dividing the greater of them by the first, the Quotient shall be the fourth Proportional sought for.

And from hence is framed this Rule of *Practice*, (by some called the *Merchants Rule*) which always hath *One*, an ingredient in the Question, and it is no other but an *Abridgement* or *Compendium* of the *Rule* of *Three*, when *One* is one of the three Proporotinals given.

And that such Questions that are to be resolved by this Rule may be the more readily and easily answered (Money commonly being one of the three Terms) it is expedient that he which intendeth to make much use of this Rule, should have readily in his mind the *Even* or *Aliquot* parts of a *Pound*, of a *Shilling*, and of a *Peny*. And also to have in *Memory* the several *Products* of 12 (the number of *Pence* in one *Shilling*) multiplied into 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12. All which are set down in the small *Tables* following, which ought first perfectly to be learned by heart, before farther Progress be made into this Rule.

TABLE

TABLE I. *The Aliquot or Even parts of a Pound or 20 Shillings.*

lb. — *d.*			
1 — 0	is the	One Twentieth $\frac{1}{20}$	of a Pound or 20 *s.*
2 — 0		One Tenth $\frac{1}{10}$	
2 — 6		One Eighth $\frac{1}{8}$	
3 — 4		One Sixth $\frac{1}{6}$	
4 — 0		One Fifth $\frac{1}{5}$	
5 — 0		One $\frac{1}{4}$	
6 — 8		One Third $\frac{1}{3}$	
10 — 0		One Half $\frac{1}{2}$	

TABLE II. *The Aliquot or Even parts of a Shilling.*

d. — *q.*			
1 — 0	is the	$\frac{1}{12}$ One Twelfth	of a Shilling.
1 — 2		$\frac{1}{8}$ One Eighth	
2 — 0		$\frac{1}{6}$ One Sixth.	
3 — 0		$\frac{1}{4}$ One Fourth	
4 — 0		$\frac{1}{3}$ One Third	
6 — 0		$\frac{1}{2}$ One Half	

TABLE III. *The several Pence in a Shilling multiplied by* 12.

2	Pence multiplied by 12 produceth	24
3		36
4		48
5		60
6		72
7		84
8		96
9		108
10		120
11		132
12		144

For the working of the *Rule of Practice*, when the Price given is of Equal parts of a *Shilling*, this is

THE RULE.

Knowing by your Table what part of a Shilling it is, (whether $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, *&c.) divide the sum propounded by it, and the Quotient shall be the number of Shillings answering your Question.*

Example.

At 6 d. the Ounce, what 7625 Ounces? Six Pence is (by your Table) $\frac{1}{2}$ of a Shilling, wherefore take one half of 7625, and it is 3812 *s.* and 1 remaining, which 1 is 6 *d.* So that 7625 Ounces, will cost 3812 *s.* 6 *d.* which is reduced into Pounds, by cuting off the last figure towards the right hand of 3812, and taking the half of the other figures, which will be Pounds, and if one remain, in taking of the half it is 10 *s.* — So the figure 2 being cut off from 3812, the half of 381 is 190 and 1 remaining, which is 190 *li.* 12 *s.* So the price of 7625 ounces will be 190 *li.* 12 *s.* 6 *d.* And so must you do for all others. As if the price be $\frac{1}{3}$, take $\frac{1}{3}$, if $\frac{1}{4}$ take $\frac{1}{4}$, as by the Examples following.

(1) At 6 *d.* the Ounce, what 7625 Ounces?

$\frac{1}{2}$ 3812 12 —— 6 *d.*

190 - 12 — 6 *d.*

(2) At 4 *d.* the yard, what 3621 yards?

$\frac{1}{3}$ 1207

60 *li.* 7 *s.* 0 *d.*

(3) At 3 *d.* the Gallon, what 989 Gallons?

$\frac{1}{4}$ 247 3 *d.*

12 *li.* 7 *s.* 3 *d.*

(4) At 2 *d.* the Pound, what 1760 Pounds?

$\frac{1}{6}$ 1126 8 *d.*

56 *li.* 6 *s.* 8 *d.*

293 --- 4 (5)

Fa: 14 13 : 4

(5) At 1 *d.* 2 *q.* the Ell, what 9623 Ells?

$\frac{1}{8}$ 12012 10 *d.* 2 *q.*
60 *li.* 2 *s.* 10 *d.* 2 *q.*

(6) At 1 *d.* the Ounce what 672 Ounces?

$1\frac{1}{12}$ 516
2 *li.* 16 *s.*

Thus have you *Examples* when the price is *even parts* of a *Shilling*, But when they are *uneven parts* of a *Shilling*, as 5 *d.* 7 *d.* or the like, then you must do the work at two or three Operations, though in the same manner, as Pence

Pence		*d.* *d.*
If the Price be 5	take for it	3 and 2
7		4 and 3
8		4 and 4
9		6 and 3
10		6 and 4
11		6 and 3 and 2

Examples of these uneven Parts of a Shilling.

(1) At 5 *d.* the Gallon, what 6254 Gallons?

$\frac{1}{4}$ is 3 *d.*	1563	6 *d.*
$\frac{1}{6}$ is 2 *d.*	1042	4 *d.*
5	26015	10 *d.*
	130 *li.* 5 *s.*	10 *d.*

(2) At 7 *d.* the Ounce, what 9271 Ounces?

$\frac{1}{3}$ is 4 *d.*	3090	4 *d.*
$\frac{1}{4}$ is 3 *d.*	2317	9 *d.*
7	54018	1 *d.*
	270 *li.*	8 *s.* 1 *d.*

(3) At

(3) At 8 *d.* the Yard, what 7952 Yards?

$\frac{1}{3}$ is 4 *d.*	2650	8 *d*
$\frac{1}{3}$ is 4 *d.*	2650	8 *d.*
8	53010	4 *d.*
	265 *li.* 1 *s.*	4 *d.*

(4) At 9 *d.* the Ell, what 3769 Ells?

$\frac{1}{2}$ is 6 *d.*	1884	6 *d.*
$\frac{1}{4}$ is 3 *d.*	942	3 *d.*
9	28216	9 *d.*
	142 *li.* 6 *s.*	9 *d.*

(5) At 10 *d.* the Dozen, what 625 Dozen?

$\frac{1}{2}$ is 6 *d.*	312	6 *d.*
$\frac{1}{3}$ is 4 *d.*	208	4 *d.*
10	5210	10 *d.*
	26 *li.* 0 *s.*	10 *d.*

(6) At 11 *d.* the Pound, what 6952 Pound?

$\frac{1}{2}$ is 6 *d.*	3476	
$\frac{1}{4}$ is 3 *d.*	1738	
$\frac{1}{6}$ is 2 *d.*	1158	8 *d.*
11	63712	8 *d.*
	318 *li.* 1 *s.*	8 *d.*

(7) At 12 *d.* or 1 *s.* the Ounce, what 9871 Ounces?
$\frac{1}{20}$ of 20 *s.* therefore $\frac{1}{2}$ 98712 is
(493 *li.* 12 *s.*

If the Price of the Commodity is in Farthings, or Half pence, bring the Sum into Pence, and work as in the preceding Questions, and according to the following Examples.

(1) At

(1) At 1 *q.* the Pound, what 6392 Pound?

	6392	
½	1598	
¼	1313	2 *d.*
	6 *li.* 13. *s.* 2 *d.*	

(2) At 2 *q.* the Ell, what 3625 Ells?

	3625	
½	1812	2 *q.*
¼	151	1
	7 *li.* 11 *s.* 0 *d.* 2 *q.*	

(3) At 3 *q.* the Ounce, what 7321 Ounces?

	7321	
½	3660	3 *q.*
⅛	305	
¼	152	6 *d.*
	4517	
	22 *li.* 17 *s.* 6 *d.* 3 *q.*	

This is the manner of working for the even parts of a Peny, but if they be uneven parts; As two pence 3 farthings, five pence 1 farthing or the like, *Work first for the even part of a Shilling, and then for the farthings, which added the work is done.* As in these Examples.

(1) At 3 *d.* 3 *q.* the Ell, what 817 Ells?

	817	
½	204	3 *q.*
¼	51	½
	2515	
	12 *li.* 15 *s.* 0 *d.* 3 *q.* ½	

(2) At

(2) At 4 *d.* 1 *q.* the Pound, what 7138 Pound?

$\frac{1}{6}$	1189	8
again	1189	8
$\frac{1}{8}$	148	$8\frac{1}{2}$
	25218	$0\frac{1}{2}$
	126 *li.* 8 *s.* 0 $d\frac{1}{2}$	

For the even parts of a *Pound*, you must take the parts as you find them expressed in the Table; as for 10 *s.* the $\frac{1}{2}$, for 5 *s.* the $\frac{1}{4}$, for 4 *s.* the $\frac{1}{5}$; as in Example,

(1) At 2 *s.* 6 *d.* the Ell, what 6294 Ells?

$\frac{1}{8}$	786 *li.*	15 *s.*

(2) At 4 *s.* the Ream, what 735 Reams?

$\frac{1}{5}$	147 *li.*	

If (in this Rule) at any time the Question consists of the parts of an *Ell*, *Yard*, *Pound*, *Ounce*, *Cross*, or the like; you must deal with the whole *Ells*, *Yards*, *Ounces*, &c. first; and afterwards add the price of the $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, or what other part soever it be. And thus much shall suffice for this *Rule* of *Practice*.

The RULE of FELLOWSHIP.

THis Rule is useful for *Merchants*, and all such as Trade in *Companies*, with a Joynt stock; and must share a proportional part of the gains, or loss; every one according to his stock which he laid in.

The Rule is two-fold, with *equal* time; or with *unequal* time.

That which is with *equal time*, is commonly called, the *Rule of Fellowship without time*.

Of this we will first speak. THE

THE RULE.

As the whole Joynt stock is to all the gain or loss: So is each mans particular stock, to his part of the gain, or loss.

Example 1.

Two Purchasers A. and B. buy 700 l. a year Land for ever, (when money is at 8 *per Cent.*) *for* 14000 *l. of which A. paid* 8000 *l. and B* 6000 *l. after* 5 *years* (money being fallen to 6 *per Cent.*) *they sell it for* 18700 *l. so there is gained* 4700 *l. how much of this must A. have.*

First for A.

Say, If 14000 gain 4700, what 8000? answer, 2685 $\frac{10000}{14000}$.

Then for B.

If 14000 gain 4700, what 6000? answer, 2014 $\frac{4000}{14000}$. As by the following operation doth appear.

(I) For A.

l. *l.* *l.*

If 14000 gain 4700, what 8000?

8

14000) 37600000 (2685 $\frac{10000}{14000}$

28000

96000

84000

120000

112000

80000

70000

10000 remainder.

(II) For

(II) For B.

l. *l.* *l.*

If 14000 gain 4700, what 6000?

6

14000) 28200000 (2014 $\frac{4000}{14000}$

28000

200000

140000

600000

560000

40000 remainder.

Here note, That this Work might have been much abreviated, if from each of the three numbers you had cut off two Cyphers towards the right hand, as hath been formerly shewed in the Compendiums of *Multiplication* and *Division*.

Now for the proof hereof,

If you add 2685 $\frac{10000}{14000}$

which is the sum that A. gained;

To —— 2014 $\frac{10000}{14000}$

The sum which B. gained; the sum of them is 4700,

Which is equal to the total gain.

And accordidg to the proportion of these two Numbers: that is, as 8 to 6, or 4 to 3. So they ought to have parted the yearly Rent also, all the time they received it: that is, A. ought to have 400 *l.* yearly; and B. 300 *l.*

Example 2.

A. B. and C. joyn their moneys to make a stock of 25000 *l. of which A. laid in* 10000 *l. B.* 8000 *l. and*

C.

C. put in 7000 *l. with this (after a certain time in trading) they gained* 7500 *l. how muſt this be parted?*

Firſt for A.

Say, if 25000 gain 7500, what 10000?
Or ſhorter, if 25 get $7\frac{1}{2}$, what 10? Multiply $7\frac{1}{2}$ by 10, it produceth 75, which divided by 25, the quotient is 3, that is, (reſtoring the three Cyphers) 3000 *l.* for A.

Then for B.

Say, if 25000 gain 7500, what 8000?
Or ſhorter, if 250 get 75, what 80?
Multiply and divide as the *Golden Rule* requires, to the quotient reſtore the two Cyphers, then it will be 2400 *l.* for B.

Laſtly, for C.

Say, if 250 give 75, what 70? anſwer 21, to which put the two Cyphers, it makes 2100 for C.
And theſe three 3000, 2400, and 2100, being added together, make 7500. And have that proportion as the particular ſtocks had: and therefore the Work is right.

(I) for A.

If 25 gain $7\frac{1}{2}$, what 10 *l.*

$$7\tfrac{1}{2}\ \tfrac{15}{2} \times \tfrac{10}{1} = \frac{150}{2}$$

(75

3000 *l.* for A.

(II for

(II) for B.

If 24000 gain 75, what 80?

80

250) 6000 (24

500
1000

1000

2400 for B.

(III) for C.

If 250 gain 75, what 70?

70

250) 5250 (21

500
250

250

2100 for C.

And if inſtead of gaining 7500 *l.* whereby every one is ſuppoſed to have his ſtock, and a part of the gains; they had loſt 7500 *l.* then their particular ſtocks had not been due to them, but ſo much as would be left after their proportional parts of the loſs were abated.

Example 3.

A. B. and C. with a joynt ſtock of 25000 *l. gain* 7500: *of which A. gets* 3000, *B.* 2400, *C.* 2100; *what was their ſtock?* This

This is but the Converſe of the former, therefore ſay, if 5700 require 25000, what doth 3000 require? 10000 for A, and ſo work for the other two.

Many examples are of little uſe (except to load the Readers memory) where the Rule is ſo ſhort and plain; I will therefore add no more to this part of the Rule, but immediately come to the Rule of Fellowſhip with time.

The RULE of FELLOWSHIP with time.

THis Rule is to be uſed when the times of the continuance of the particular ſtocks are unequal, and differ; ſo that here the difference of time, and alſo the difference of ſtock being both to be conſidered; it can be done no better way then by taking the *Power* of them both to be the particular ſtock; and all thoſe *Powers* added, to be the whole ſtock, that which I call the *Power* is the product of the money of every one multiplied by his time; And then

THE RULE.

As the ſum of thoſe Products, is to the whole gain; ſo is each particular product, to its part of the gain.

Queſtion 1.

Three Merchants A. B. C. make a ſtock of 10000 l. *of which A. layes in* 4000 *for* 3 *months, B.* 3000 l. *for* 6 *months; and C.* 3000 l. *for* 8 *months, with this they gain* 2000 l. *what is each mans ſhare?*

Firſt, for A. multiply 4000 by 3, it makes 12000, let that be accounted his particular ſtock.

Secondly,

Secondly, for B. multiply 3000 by 6, it makes 18000, his particular ſtock.

Laſtly, for C. multiply 300 by 8, it produceth 14000, for his ſtock, add theſe, they make 54000 l. for the general ſtock; then ſay,

For A.

If 54000 give 2000, what 12000? anſwer, 666 $\frac{24000}{54000}$.

Then for B.

If 54000 give 2000, what 18000? anſwer, 444 $\frac{16000}{54000}$.

Laſtly, for C.

If 54000 give 2000, what 24000? anſwer, 888 $\frac{48000}{54000}$.

The three Fractions may be reduced (by dividing each Numerator, and Denominator by 6000) and then the three ſhares will be 444 $\frac{4}{9}$, 666 $\frac{6}{9}$, and 888 $\frac{8}{9}$, which altogether make 2000, as they ought.

Queſtion 2.

Three Farmers, A. B. and C. lay out 1000 l. *to ſtock their grounds with Cattel, of which A. put in* 200 l. *for* 6 *years; B. had* 300 l. *going for* 4 *years; and C.* 500 l. *for* 2 *years; at the end (by unſeaſonable times) there was loſt* 200 l. *which made the remain of their ſtock but* 800 l. *what had each man left?*

Multiply 200 by 6, it gives 1200: Likewiſe, 300 by 4, it gives 1200. Laſtly, 500 by 2, the product is 1000: all theſe are are 3400 for the joynt ſtock.

Then firſt for A.

Say, if 3400 loſe 200, what 1200? anſwer, 70 $\frac{2000}{3400}$ for A. to which B. is equal, becauſe the power of his ſtock is ſo.

Therefore for C.

Say, if 3400 loſe 200, what 1000? anſwer, 58 $\frac{1813}{3400}$. So the 3 ſhares are 70 $\frac{20}{34}$, 70 $\frac{20}{34}$, and 28 $\frac{11}{34}$, equal to 200.

Now becauſe A. put in 200 *l.* and loſt 70 $\frac{20}{34}$, ſubſtract

ſtract the loſs from the ſtock, remains 129 $\frac{14}{34}$.
And ſo doing for B. his remain will be 289 $\frac{14}{34}$,
And for C. his remain is 441 $\frac{4}{34}$. Now theſe three remains, 129 $\frac{14}{34}$, 229 $\frac{14}{34}$, and 441 $\frac{4}{34}$, make up 800 *l.* which was the whole remain.

Queſtion 3.

A. rents a cloſe for a year, to pay 80 *l. he puts into it* 200 *ſheep: 2 months after B. puts* 40 *ſheep in, and* 5 *months after that C. puts in* 100 *ſheep; how muſt every one pay of the rent?*

Multiply 200 by 12, it produceth	2400
And 40 by 10, produceth.	400
Laſtly, 100 by 5, (which is C. time) produceth	500
In all	3300

Then for A.

If 3300 pay 80, what 2400? anſwer, 58 $\frac{192}{3300}$.

Then for B.

If 3300 pay 80, what 400? anſwer, 9 $\frac{256}{3300}$.

And for C.

If 3300 pay 80, what 500? anſwer, 9 $\frac{404}{3300}$.

The whole numbers make 79, and the broken numbers make 1. In all 80.

Note.

Whereas, hitherto we have conſidered onely difference of *time* and *money*; it may be noted, that there may be difference of other kinds, as *perſons* or *place*; but whatſoever they are, the power of all is found like theſe by multiplication; and are to be wrought like theſe, with ſo many Uſes of the *Golden Rule*, as the queſtion requires. I will therefore add but one queſtion more, which is this:

Queſtion.

One leaves a Legacy of 900 *l. among four Kinſfolk, A. B. C. D: ſo as B. may have twice as much as A. and C. thrice as much as B. and D. as much and half as much as C; what is every one to have?*

Say, if A. be 1, B. is 2, C. 6, and D. 9, add these Numbers: 1, 2, 6, 9, together, they give 18, then say, If 18 require 900, what 1? Answer is 50. So A. is to have 50 *l.* B. 100 *l.* C. 300 *l.* and D. 450 *l.* which are their just parts; and altogether are equal to 900 *l.* and the work right.

BARTER.

TO Barter is to exchange one Commodity for another, the nature whereof will best appear by the resolving of some Questions.

Question 1.

Two Merchants Barter, One hath Sugar at 4 l. *the* C. *ready money, but in Barter he will have* 4 l. 13 s. 4 d. *The other hath French Wine at* 13 l. *the Hogshead ready money; at what price must he rate his Wine to equalize the others advance of his Sugar in Barter?*

Say, by the Rule of Three direct,

If 4 *l.* in Barter require 13 *s.* 4 *d.* advance, what shall 13 *l.* in Barter require?

l.	s.	d.	l.
If 4	13	4	*what* 13?

1 2

3 0

1 3

1 6 0

1 3

4 8 0

1 6 0

2 0 8 0

s. d.

(illegible)

l. s. d.

Answer 2 – 3 – 4

Question

Question 2.

Two Barter, one hath 3 C. $\frac{1}{2}$ *of Ginger at* 13 d. $\frac{1}{2}$ per *pound. The other hath Sugar at* 15 d. $\frac{1}{4}$ per *pound. How much Sugar must be delivered for the* 3 C. $\frac{1}{2}$ *of Ginger.*

First, By the Rule of Three (or Practice) find what the 3 C. $\frac{1}{2}$ of Ginger comes to at 13 *d.* $\frac{1}{2}$ *per* pound, which will be found to be 22 *l.* 1 *s.* For

If 1 *l.* cost 13 *d.* $\frac{1}{2}$, what 3 C. $\frac{1}{2}$ cost?

Answer, 22 *l.* 1 *s.*

Secondly, Say, If 15 *d.* $\frac{1}{2}$ buy 1 *l.* of Sugar, what shall 22 *l.* 1 *s.* buy?

Answer, 347 $\frac{2}{61}$.

Question 3.

Two Barter, One hath Tobacco at 14 d. per *pound, which he will Barter for Sugar at* 10 d. *per* l. *how much Tobacco must be given for* 8900 l. *of Sugar?*

First, the 8900 *l.* of Sugar at 10 *d. per* pound, comes to 370 *l.* 16 *s.* 8 *d.*

Then, If 14 *d.* buy 1 *l.* of Tobacco, what number of pounds will 370 *l.* 16 *s.* 8 *d.* buy?

Answer, 6357 pound, and so many pounds of Tobacco at 14 *d.* must be given for 8900 pound of Sugar at 10 *d.*

Question 4.

Two Barter, One hath broad Cloth at 15 s. *the yard ready money, for which in Barter he will have* 16 s. 3 d. *The other hath Wooll at* 2 s. 10 d. per *pound ready money? what price must his Wooll be set at in Barter to equalize the advance which he puts upon his Cloth.*

Say by the Rule of Three direct.

If 15 *s.* ready money require 15 *s.* 3 *d.* in Barter; what shall 2 *s.* 10 *d.* ready money require?

Answer, 2 *d* — 3 *q.* $\frac{1}{3}$.

So that he must rate his Wooll at 3 *s.* 3 *q.* $\frac{1}{3}$ of a farthing *per* pound.

Of Interest Simple and Compound.

IN the *Appendix* to the *Second Part* of this *Book*, I have *Tables of Compound Interest*, *Rebate* or *Discount* of *Money*, *Purchase* of *Leases* and *Annuities*, whose *Construction* and *Use* are there *Exemplified* by Resolving of *Questions* suitable to each *Table*, as by having recourse thither will appear. But for that *Tables* may not alwayes be at hand, I thought it convenient here to shew how to resolve *Questions* both in *Simple* and *Compound Interest*; by which *Tables* of that nature may be *Calculated*, were there not enough already extant.

Question 1.

If 100 l. *in* 12 *months gain* 6 l. *what shall* 625 l. *gain in* 3 *years or* 36 *months?*

The Proportion is,

As 100 *l.* is to 6 *l.* in a year,
So is 625 *l.* to 112 *l.* 10 *s.* in a year.

Wherefore multiply 625 *l.* by 6 *l.* and divide the Product by 100, by cutting off two figures, the Quotient will be 37 $\frac{12}{100}$ that is, 37 *l.* 10 *s.* and this being multiplied by 3, giveth 112 *l.* 10 *s.* as by the Work appears.

l.	*l.*	*l.*
100	6	625
		6
		3715 0
l. *s.*		1 3
112—10		11215 0

Question 2.

If 160 l. *in* 12 *moneths gain* 6 l. *what will* 236 l. 10 s. 5 d. *gain in* 16 *months?*

The Proportion.

As 100 *l.* is to 6 *l.* in a year,

So is 236 *l.* 10 *s.* 5 *d.* to 14 *l.* 3 *s.* 9 *d.* 3 *q.* in a year.

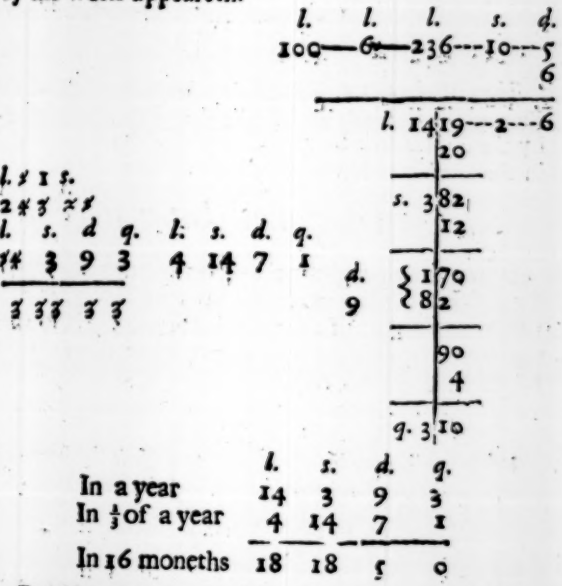
l. *s.* *d.*

Multiply 236—10—5 by 6,

The Product is 1419—2—6,

This divide by 100, which is done by cutting off two figures of the Integer, leaving 14 *l.* on the left hand of the line. The figures on the right hand multiplied by 20, and the figures (or remains) agin by 12; and lastly by 4; shall in all give 14 *l.* 3 *s.* 9 *d.* 3 *q.*

Which divide by 3, and add that third part to 14 *l.* 3 *s.* 9 *d.* 3 *q.* the sum will be 18 *l.* 18 *s.* 5 *d.* 0 *q.* as by the work appeareth.



	l.	*s.*	*d.*	*q.*
In a year	14	3	9	3
In ⅓ of a year	4	14	7	1
In 16 moneths	18	18	5	0

By this manner of Work, *If* 417 l. 11 s. 8 d. *be put out at Interest for* 2 *years at* 6 l. per *Cent. it will amount unto* (*I mean the Interest*) 50 l. 2 s. 2 d. *As by the Work appears.*

li.	li.	li.	s.	d.
100	6	417	11	08
				6
		25105	10	00

	li.	s.	d.	
				120
In one year	25	01	01	110
			02	112
In two years	50	02	02	1120
				14
				180

Thus these Questions are wrought by the *Single Rule of Three*; but they may be otherwise wrought by the *Golden Rule Compound of 5 Numbers*. Of which in that *Rule* you have an Example.

Of Compound Interest.

It may be wrought in the same manner that *Simple Interest* was, only add the increase every year as it riseth, to the sum of the year before-going, so continuing this course, till you have gone through the Numbers of Years required.

Question.

What will 500 li. *amount unto, if it be forborn* 4 *years, after the rate of* 6 per Cent. *Compound Interest.*

First Work.

li.	li.	li.
100	6	500
		6
		li. 30100

First year 30 *li.* Add

Add this 30 *li.* found at this first work to the 500 *li.* it makes 530 *li.* then for the

Second Work.

li.	*li.*	*li.*
100	6	530
		6
		li. 31180

Second year 31 *li.*
Which add to 530 *li.* it makes 561 *li.*

Third Work.

li.	*li.*	*li.*
100	6	561
		6
		li. 33166

Third year 33 *li.*
Which add to 561 *li.* it makes 594 *li.*

Fourth Work.

li.	*li.*	*li.*
100	6	594
		6
		li. 35164

For the fourth year 35 *li.*

The four *Products* of these four *Multiplications*, being added together (having respect to the figures cut off) do make 131 *li.* which added to the Principal, makes 631 *li.* $\frac{5}{10}$, and so much doth the Principal and Interest amount unto, being forborn four years, as here may be seen.

The Principal	500	
1 Product	30	00
2 Product	31	80
3 Product	33	66
4 Product	35	64
	131	10

And observing this *method*, you may resolve any *Question* for any *Number of Years*, and for any *Rate of Interest*, and by this *Rule* is the *first Table* in the *Appendix* of the second part of this *Book* made. And here it will not be improper to add two of those Tables in that *Appendix* (which are there in Decimal Numbers) reduced into *Pounds*, *Shillings*, *Pence*, and *q.*

The one *shewing what any sum of Money forborn, and number of Years under* 31, *will amount or be increased unto.*

The other *shewing the present worth of any* Annuity, Rent, *or* Portion, *for any number of years to come under* 31.

I. TABLE.

Years	*li.*	*s.*	*d.*	*q.*	Years.	*li.*	*s.*	*d.*	*q.*
1	1	1	2	2	16	2	10	9	2
2	1	2	5	2	17	2	13	10	1
3	1	3	9	3	18	2	17	1	0
4	1	5		30	19	3	0	6	0
5	1	6	9	0	20	2	4	1	3
6	1	8	4	2	21	3	7	11	3
7	1	10	0	3	22	3	12	0	3
8	1	1	10	2	23	3	16	4	3
9	1	13	9	2	24	4	0	11	3
10	1	15	9	3	25	4	5	10	0
11	1	17	11	2	26	4	10	11	3
12	2	0	3	0	27	4	16	5	2
13	2	2	7	3	28	5	2	2	3
14	2	5	2	2	29	5	8	4	2
15	2	7	1	1	30	5	14	10	1

The

The first Table, *Sheweth what any sum of Money being forborn any number of years (under 31) will be augmented unto, accounting Interest upon Interest at* 6 per Cent. per Annum.

The Use of the Table.

Question 1.

If 234 li. *be forborn for the term of* 18 *years, how much will it be increased unto, accounting* 6 li. per Cent. *Compound Interest?*

Look in the Table for 18 years, and right against it you shall find 2 *li.* 17 *s.* 1 *d.* And so much will 1 *li.* or 20 *s.* be increased unto in 18 years.

Then say by the Rule of three;

If 1 *li.* give 2 *li.* 17 *s.* 1 *d.* what will 324 *li.* give?

```
          20
        ------
          57 s.
          12
        ------
         115
         57
        ------
         685 d.
         324
        ------
        2740
       1370
      2055          1|s.
    ---------
12)221940 (1849|5
          924 li.
    12
    101
    ---
     96          li.    s.    d.
      59         924    15    00
     ---
      48
      114
     ---
      108
       60
      ---
       60
```

So

So that 324 *li.* being forborn 18 years, will be increaſed unto 924 *li.* 15 *s.*

Queſtion 2.

If 156 li. 15 *s.* 6 d. *be forborn* 20 *years, to what will it amount.*

Againſt 20 years in the Table is 3 *li.* 4 *s.* 1 *d.* 3 *q.* Wherefore work by the Rule of Three as followeth.

li.	*li.*	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*
1	3	4	1	3	136	15	6
	20 *s.*				20		
240					2735 *s.*		
	64				12		
	12				5476		
	129				2735		
	64				32826 *d.*		
	769 *d.*				3079		
	4				295434		
	3074 *q.*				229782		
					984780		
					10107254		

	q.	*d.*
438 *li.*	13 *s.*	6 *d.* 3 *q.*

So much will 136 *li.* 15 *s.* 6 *d.* be increaſed to in 20 years.

II. TA-

II. TABLE.

Years.	li.	s.	d.	q.	Years.	li.	s.	d.	q.
1	0	18	10	2	16	10	2	1	1
2	1	16	8	0	17	10	9	6	2
3	2	13	5	2	18	10	16	6	2
4	3	9	3	2	19	11	3	2	0
5	4	4	3	0	20	11	9	4	3
6	4	14	4	1	21	11	15	3	1
7	5	11	7	3	22	12	0	10	0
8	6	4	2	1	23	12	6	0	3
9	6	16	0	1	24	12	11	0	0
10	7	7	2	1	25	12	15	8	0
11	7	17	8	3	26	13	0	0	3
12	8	7	8	0	27	13	4	2	2
13	8	17	0	2	28	13	8	1	2
14	9	5	1	3	29	13	11	9	3
15	9	14	3	0	30	13	15	3	2

The second Table, *Shewing what any* Annuity, Rent, *or* Pension, *being forborn any number of* years *under* 31, rebating *or* discounting *yearly, after the rate of* 6 per Cent. *Compound Interest is worth in ready money.*

The use of the Table.

Question 1.

What is a Lease of 25 li. per Annum *payable yearly, and to continue* 21 *years, worth in present money?*

Look in the Table for 21 years, and against it you shall find 11 *li.* 15 *s.* 3 *d.* 1 *q.* Then say by the *Golden Rule.*

If

If 1 *li.* be worth 11 *li.* 15 *s.* 3 *d.* 1 *q.* what 25 *li.*

```
      20
     ----
     235 Shillings
      12
     ----
     473
    235
    ----
    2823 Pence.
       4
    -----
   11292 Farthings.
      25
   -----
   56460
  22584
  ------
  282300
     s s
   2     2252   (3 d.
282300 (22222 (58811
 22222  22222 (294    s.   3 d.
      222
   294 li.      1 s.     3 d.
```

And so much is 25 *li.* a year worth in ready Money to continue 21 years.

Question 2.

When is an Annuity *of* 75 li. *a year to continue* 30 *years, and to be paid yearly, worth in ready Money.*

Against 30 years in the Table you find 13 *li.* 15 *s.* 3 *d.* 2 *q.*

Then by the Rule of Three, say,

If

If 1 *li* 13 *li.* 15 *s.* 3 *d.* 2 *q.*

```
          20
        ----
         275  sh.
          12
        ----
         553
        275
        ----
        3303  d.
           4
        ----
       13212  q.
          75
        ----
       60060
      92484
      ------
      990900
          55  d.
 53252    554(9
 990900   (247725 (206413
 ******   ****** 1032-s.
          5555
```

li. *s.* *d.*

1032—3—9—And so much is 15 *li.* a year for 30 years worth in present money.

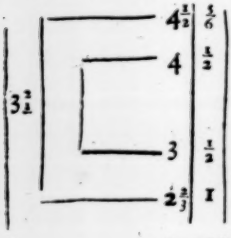
The Rule of Alligation.

THis hath its name from *binding*, *tying*, or *uniting* many particulars in one *Mass* or *Sum*, the nature of it will be understood in working some *Questions* or *Examples*.

Question

Question 1.

A Corn Master would mix 4 sorts of grains together, viz. *Wheat at* 4 s. *the bushel*, *Wheat at* 2 s. 6 d. *the bushel*; *Rie at* 3 s. *the bushel*, *and Barley at* 4 s. 8 d. *the bushel*; *so as to make* 15 *quarters in all*, *to be sold at* 3 s. 6 d. *the bushel*, *How much must he take of each?*



Place them as in the Margine, so as a greater and lesser may still be together, as $4\frac{1}{2}$, with $2\frac{2}{3}$ and 4 with 3, and place the price required by it self towards the left hand, as here you see $3\frac{2}{1}$; then in a seperated Column, note the difference between the price of a bushel of every one particular given, and a bushel of that required, as the difference betwixt $4\frac{1}{2}$ and $3\frac{2}{1}$ is 1, which must not be placed against $4\frac{1}{2}$, but against that number linked with $4\frac{1}{2}$, that is, against $2\frac{2}{3}$, and so must all the differences be ordered, as is easie to be seen in the Margine: then

THE RULE.

Multiply the whole Mass to be made, *by any particular* difference; *and divide the product by the sum of all the* differences, *the quotient shall be the just quantity of that* particular kind, *whose price standeth against the difference you wrought with.*

Example

First turn the quarters into bushels, by saying 8 times 15 is 120, then for the quantity of the first sort at $4\frac{1}{2}$: multiply 120 by $\frac{1}{6}$, the product is 100, which divided by $2\frac{1}{6}$, the quotient is 35 $\frac{1}{7}$ bushels

of

of that ſort at 4 *s.* 6 *d.* and working ſo for every of the other; they will be found to be thus

At 4 *s.* 6 *d.*	35 $\frac{5}{17}$	
At 4 *s.*	21 $\frac{3}{17}$	
At 3 *s.* 6 *d.*	21 $\frac{3}{17}$	buſhels.
At 2 *s.* 8 *d.*	42 $\frac{6}{17}$	
In all	120	

Now to prove this right; Firſt, multiply the whole Maſs 120 buſhels, by the deſired price 3 $\frac{1}{2}$ *s.* omitting the denominations, the ſum is 420 *s.*

Then ſecondly, multiply 38 $\frac{5}{17}$ by 4 $\frac{1}{2}$, it is,	158 $\frac{14}{17}$
And 21 $\frac{3}{17}$ by 4, it produces	84 $\frac{12}{17}$
And 21 $\frac{3}{17}$ by 3, it makes	63 $\frac{9}{17}$
And 42 $\frac{6}{17}$ by 2 $\frac{2}{3}$, is	112 $\frac{16}{17}$
In all	420

And ſo much all ſhould be worth in ſhillings.
And therefore the queſtion is rightly reſolved.

Queſtion 2.

One hath 6 ſorts of Fruits at ſeveral prices; Dates at 2 s. *Almonds at* 1 s. 4 d. *Currants at* 10 d. *Raiſins at* 5 d. *Prunes at* 4 d. *and Figs at* 3 d. *the pound; and would take of every ſort ſome to make a mixed quantity of* 30 l. *weight to ſell one with acother for* 9 d. *the pound, how much muſt he take of each?*

Having

Having placed the Numbers and their differences, and the sum of those differences distinctly as hath been shewed before, and may be seen by the Figure in the Margine; the Work is evermore like that in the former question. So 38 is the first number in the *Golden Rule*; 30, the second (which that it may not be forgotten, may be set at the right side of the figure) and every particular difference, as 6, 5, 4, *&c.* is the third in the Rule, to be repeated till all the differences have been employed.

So 30, multiplied by 6, produceth 80, which divided by 38, the quotient is $4\frac{22}{38}$ of a pound weight: and so much must be taken of Dates, at 24 *d.*

Secondly, 5 times 30 is 150, which divided by 38, the quotient is $3\frac{36}{38}$ for Almonds. And working after the same manner with 4, 1, 7, 15, their respective quantities will be found to be these;

	pounds	38*th. parts*
Dates	4,	28
Almonds	3,	36
Currants	3,	6
Raisins	0,	30
Prunes	5,	20
Figgs	11,	32
in all	26	152

That is $26\frac{152}{38}$, and the reduction of the fraction will make it 30, as it ought to be, and by comparing

ring the prices of these particulars added, with the price of 30 *li.* weight, at 9 *d. per l.* weight, which makes 470 *d.* this may be proved like the former.

But that the Reader may be perfect in it, I will do it here also as followeth:

Say first, 24 times 4 is 96, and 24 times 28 is 672: for the first set them thus: 96, 672

And 16 times 3 is 48 and 16 times 36 is 576	48, 576
And 10 times 3 is 30, and 10 times 6 is 60	30, 60
And 5 times 30 is 150	00, 150
And 4 times 5 is 20, and 4 times 20 is 80	20, 80
Lastly, 3 times 11 is 33, and 3 times 33 is 96	33, 96
In all	227, 1634

Now this 1634 being the sum of the Numerators of Fractions, whose common Denominator is 38, must be divided by 38, and the quotient will be 43, which added to the whole number 227; the sum is 270. And so much is 30 multiplied by 9, which shews the work to be right.

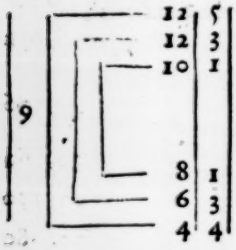
The Combination or linking of Numbers may be varied at pleasare, as whereas above I linked 24 and 3, also 16 with 4, and 10 with 3; it might have been 24 with 5, and 16 with 4, and 10 with 3. Or 24 with 4, and 16 with 3, and 10 with 4, of which diversity of linking would follow diversity of solutions, but all true, as the Reader may easily prove by himself.

Likewise, if the Numbers to be linked were 3, 5, 7, or any odd Numbers, one of them may be

linked to two ſeverally, to make the work even.

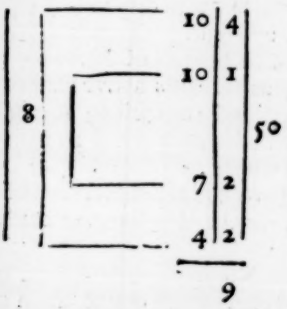
Example. 3.

If the Numbers were 12, 10, 8, 6, and 4, and the mean or common price required were 9, you might firſt link them as you ſee here, taking 12 twice, or elſe you might take any other twice as you ſhall like; and ſo the work will be every way right, though not the ſame, if the differences be rightly ſet off, and orderly uſed, as is taught before in the firſt queſtion.



Queſtion 3.

A Goldſmith would mix 3 *ſorts of Silver, A. B. C. A. is* 10 d. *weight better; B.* 7 d. *weight better; and C.* 4 d. *weight better, to make an Ingot of* 50 l. *weight, which ſhould be in fineneſs* 8 d. *weight better: How much muſt be taken of each?*



Set them their differences, and the ſum of their differences, as in the Margin.

1. Then

1. Then first 50 multiplied by 4 is $\frac{200}{9}$

and divided by 9, the quotient is

2. 50 multiplied by 2, $\frac{100}{9}$

and divided by 9, the quotient is

3. 50 multiplied by 2, $\frac{100}{9}$

and divided by 9, the quotient is

4. And again, the same $\frac{100}{9}$

In all $\frac{500}{9}$

Which is equal to 50, the quantity required.

Now the first Fraction multiplied by 10, (omitting the Denominator) is 2000

The second also by 10 gives 500

The third by 7 makes 700

The last by 4 makes 400

In all 3600

That is, $\frac{3600}{9}$; which is equal to 400, and if the whole Ingot 50, be multiplied by the betterness required, namely, by 8, they shall produce 400 also: So this is proved.

In every Alligation, or linking of two numbers, this is evident, that if the sum of the numbers linked be greater than the mean number required taken so many times as there are numbers to be linked, the question would be absurd; and the resolution thereof impossible. And this shall serve for the Rule of *Alligation.*

The Rule of False Position.

THis Rule serves to resolve Questions, which are not presently fit for the *Golden Rule*; and therefore instead of the true Number which is sought: Suppose any number great or small, and make trial of it, whether it resolve the Question without any error; if so, it is the true number: if not, note what error is at the end of the work, and whether that error be *too much*, or *too little*; if *too much*, mark it thus +, but if *too little*; thus —

Then suppose again another Number, (it imports not whether it be nearer or further off) and try as before, and mark that error also with + or —, according as you find it to be, either *more* or *less*.

And then this is

THE RULE.

Multiply the first Position *by the second* Error, *and the second* Position *by the first* Error, *and* (if the errors be both + or both —) Substract *the* lesser *product from the* greater, *and keep the remain for a* Dividend, *and the difference of the errors for the* Divisor; *the quotient of that division is the true Number required.*

But if the errors be one +, *the other* —, *then the sum of the product added together must be the* Dividend: *and the sum of the* Errors, *the* Divisor; the rest of the work is the same as before.

Question. 1.

A man is to drive 48 *young Turkies* 40 *miles, and for every*

every Turky which comes alive to the end of the Journey, he is to receive 3 d. *but for every one which dies by the way, he is to pay* 6 d. *At the end he received* 72 d. *How many died by the way?*

Let the first supposition be
That by the way there died 20
For them he was to pay 120 *d.* and for 28 which lived, he was to receive 84 *d.*
so he paid more than he received 36 *d.*
and should have got clear. 72 *d.*

Add 108
Whereof the first error is —108

Let the second supposition be 10
For these he paid 60 *d.* and for the rest he received 114 *d.* the difference is 54, and should be 72: so the second error is —18

Now 20 multiplyed by 18, produceth 360
and 10 by 108 produceth 1080
and the difference is 720 for the *Dividend*, likewise the difference of the errors is 90 for the *Divisor*, and the quotient is 8, which is the true Number of those he lost by the way: As may be proved by tryal.

Question 3.

If it were required to make up a pound Sterling *of Shillings and Groats only; and so as the number of* Groats *may be to the number of* Shillings, as 7 to 1; *How many Shillings must there be?*

First, suppose the Shillings 4
then the Groats must be equal to 16 *s.*
that is 48 Groats; but the Shillings
taken 7 times are 28, to which
48 should be equal, but is +20

Secondly, ſuppoſe the Shillings 3
then the Groats (making 18 *s.*) are 25
which ſhould be equal to 7 times 2, but is ✝ 40
Multiply 4 by 40, product is 160, then
Multiply 2 by 20, the product is 40, which taken from 160, reſts for the Dividend 120
And the difference of errors is 20

Laſtly, 120 divided by 20, the quotient is 6
The number of ſhillings therefore is 6
And the number of groats is 42
But as 7 to 1, ſo is 6 times 7 which is 42, to 6 times 1, which is 6: ſo the work is done.

Queſtion 3.

If there be 4 ſeveral weights, A. B. C. D. of which D. is 24 ounces, and C. is double to B. and triple to A. and D. with twice A. is double to C. and quadruple to B. How much doth every one of theſe Weights weigh?

Firſt, ſuppoſe A. to be 8
then D. with twice A. is 24, and 16, that is 40,
of which C. being the half is 20, and B 10.
Now thrice A. is 24, to which C. ſhould be equal, but is — 4
Secondly, let A. be ſuppoſed 4
then D. more, twice A. is 32, and C. 16.
and B. is 8, but thrice A. is 12, to which 16
ſhould be equal, but is — 4
Then 8 multiplied by 4, gives 32, and 4 by 4, produceth 16: both theſe Products give 48 for the Divid nd: and the ſum of the errors (becauſe the firſt is —, the other ✝) gives 8 for the Diviſor, and the quotient will be 6, to which A. is equal, and twice A. more D. is 36, of which C. being half is 18, and B. is 9, and thrice A. is equal to C. namely 18, and all right.

Whereas the firſt error is equal here to the ſecond,

it

it follows that the Positions were equally false; and therefore their difference which is 4, being parted into two equal parts, 2 and 2, if 2 be taken from 8, the remain is the true number 6, or if 2 be added to 4, (which was the second position) the sum will be also 6.

And further, whensoever the errors be one +, the other −, though they be not equal; yet then if the difference between the positions be parted into two parts, which are in proportion one to another, as the two errors are one to another respectively: then if the first part be taken from the first position (if that be the greater) or add to it (if it be the less) the same number required is thereby had.

As, let the last question be resumed,

And let the first position for A. be	15
Then the first error will be	−18
Then let the second position be	3
And so the second error will be	+6
And the difference of position is	12

Which divided into two parts 9 and 3, which have that proportion one to another as have the errors 18 and 6, then if the first part 9, be taken from the first position 15, there remains the true Number 6. Or else if the second part 3, be added to the second position 3: thereby also is made the true number 6.

The way of parting 12 (or any other) into two parts proportional with the errors, is easily done by the *Golden Rule*, thus:

As the sum of the errors 24,
is to the difference of position 12;
So is the greater error 18,
to the greater part required, namely 9.

Many other questions are in other Books exemplified and wrought by this Rule; but seeing I intend

not to write a great Book; and also because some of those questions may be resolved without this Rule, I will add no more: onely mention one of those questions.

If there be a Cistern with 4 Cocks, which holds 8 Barrels of Water, and the first cock will run it all out in 6 hours, the second in 4, the third in 3, and the last in 2 hours: in what time shall all of them run it out?

If the first in 6 hours run	8
the second in the same time would run	12
the third	16
the last	24
In all	60

Then say, if 60 require 6, what 8?

The answer $\frac{48}{60}$, that is $\frac{4}{5}$ of an hour; in which time all the 4 Cocks together would run out all the 8 Barrels of water.

The Rule of Ceres and Virginium.

THis is the most uncertain, and unnecessary Rule in Arithmetick; being seldome used except in sporting questions to puzzle young beginnners, with easie problems: such as follow.

Questions 1.

A Caterer bought 8 *birds of two sorts, as Geese and Hens for* 20 s. *the Geese cost* 4 s. *a piece, the Hens* 2 s. *a piece; How many did he buy of each sort.*

This

This may be done by the Rule of False; and also thus: multiply the whole number 8, into the least price 2, it produceth 16, which taken from the whole price 20, rests 4 for a *Dividend*; which divided by 2, which is the difference of the particular prices, the quotient is 2, for the number of *Geese*; and 6 must be for the *Hens*: the proof is easie.

Question 2.

If 21 *Persons*, Men, Women, *and* Children, *spend* 26 *shillings*; *so that every* Man *pays* 2 s. *every* Woman 1 s. *every* Child 6 d. *How many is there of each sort.*

THE RULE.

Multiply the number of Persons *by the* least expence, *and take the product of it from the* whole expence, *the rest shall be the* Dividend; *which divided by the* difference *betwixt the* greatest *and* least *particular* expences: *the quotient is a* Number, *which the Number of men (or they which spend most) comes near to; but cannot exceed: or if the said* Dividend *be divided by the sum of the* greatest *and* least *expences, the quotient is a Number, then which the Number of men (or those which spend most) cannot be much less.*

So here 21 multiplied by 6 *d.* that is, by $\frac{1}{2}$, the product is $10\frac{1}{2}$, which taken from 26, rests $15\frac{1}{2}$, for the Dividend: and then taking $\frac{1}{2}$ from 2, rests $1\frac{1}{2}$ for the Divisor, and the quotient is $2\frac{2}{3}$, which is something more than 10; the number of Men therefore must be but 9.

Then turn the Dividend, and the Divisor both into whole Numbers, by multiplying them by the common Denominator 2, so they reduced will be 31 and 3, as before is to be seen in the quotient.

Multiply the Divsor 3, by 9, (which is the Number

ber of *Men*) the product is 27, which taken from 31, (which is the reduced Dividend) the remain is 4, for the Number of *Women*; and the *Children* must be 8.

Example 1.

9 Men at 2 *s.* each	18 *s.*
4 Women at 1 *s.* each	4
8 Children ar 6 *d.* each	4
In all 21	In all 26

But the number of men may be also 8, which multiplied by the reduced Divisor 3; product i 24, which taken from 31, the remain is 7 for the Women; and then the Cnildren must be 6.

Example 2.

8 Men at 2 *s.* each	16
7 Women at 1 *s.* each	7
6 Children at 6 *d.* each	3
In all 21	In all 26

Or the number of Men may be 7, which multiplyed by 3, produceth 21, which taken from 31, remains 10 for the Women, and 4 Children.

Example 3.

7 Men at 2 *s.* each	14 *s.*
10 Women at 1 *s.* each	10
4 Children at 6 *d.* each	2
In all 21	In all 26

So is already seen 3 various solutions of this question, which makes this Rule the less to be regarded.

But

But further, the number of Men may be 10, and not more, for if you put them 11, that multiplyed by 3, produceth 33, which is greater than 31, from which it ſhould be taken, but I ſay it may be 10, and then there is only one Woman, and five Children: this confirms the former part of the Rule.

Now for the later part, if the Dividend 31 be divided by the ſum of the two extream expences (reduced by doubling as the Dividend is) 4, the quotient will be $7\frac{3}{4}$. And the Men may be 7, as hath been ſhewed; but they may be alſo but 6, and fewer they cannot be: as 6 Men, 13 Women, and 2 Children; for if you put them 5, that multiplied by 3, produceth 15, which taken from 31, there remains 16 for the Women, and ſo there ſhould be no Children: which is contrary to the ſuppoſition.

And further, becauſe the quotient was $7\frac{3}{4}$, the number of Men might be ſo, if pure Arithmetical diviſion be only regarded: and then the Women alſo are in number $7\frac{3}{4}$, and the Children $5\frac{1}{2}$, as may eaſily be tryed; I need not exemplifie it.

Queſtion 3.

If there be an Exhibition of 900 l. per Annum *to* 30 *perſons: ſome Clerks, ſome Meſſengers, and ſome Door-keepers, at* 61 l. *each Clerk*, 40 l. *each Meſſenger, and* 20 l. *each Door-keeper; How many muſt there be of each ſort?*

Multiply (according to the Rule) 30 by 20, the product is 600, which taken from 900, remains 300 for the Dividend; and 60 want 20, that is 40, for the Diviſor: and the quotient is $7\frac{1}{2}$, and more the Clerks cannot be; Alſo divide by 60 more 20, that is 80, quotient is $3\frac{3}{4}$, and much fewer the Clerks cannot be.

Not to ſtand upon the Fractions (in this caſe of dividing me) the Clerks may be 7, 6, 5, 4, 3: and the

the Messengers 1, 3, 5, 7, or 9, and the Door-keepers 22, 21, 20, 19, or 18, that the Clerks cannot (in whole numbers) be more than 7, or less than 3, may thus be proved; First, let them be 8, then 8 times 40 is 320, which is more than 300, out of which it should be taken: Secondly, let them be 2, then 2 times 40 is 80, out of 300 remains 220, which divided by 20, gives the quotient 11, for the Messengers; so the Clerks and Messengers being 13, the remain thereof to 30, namely 17, must be Door-keepers.

	but,	
2 Clerks at 60 *l.* each		120 *l.*
11 Messengers at 40 *l.* each		480
17 Door-keepers at 20 *l.* each		340
In all	30	In all 940

Which is 40 *l.* too much, therefore the Clerks cannot be two.

Note.

It may be asked, why the remain 220 should be divided by 20: whereas the like remain in the former Example, namely, 16, was taken (without any division) absolutely for the number of Women, or middle number? I answer, although the greatest or first Number being found, (as here to be 2) the residue of 2 to 30, might be rightly parted into two fit parts in the same manner as the first question of this Rule was resolved, or else by the Rule of False: yet to give further satisfaction, the cause of this is, the difference betwixt the two lesser expences, was there $\frac{1}{2}$, which (before the division was reduced to 1, which neither multiplies nor divides any Number, but leaves it the same: whereas, in this last, the middle

expence

expence (or exhibition) being 40, and the leaſt 20, the difference of them was 20, by which dividing the Remain of the laſt ſubſtraction: the quotient is ever the Number of the middle perſons. Which may ſerve as an addition to the Rule; where the ſorts things are but three.

Queſtion 4.

If there be 10 *perſons of four ſeveral Countries,* Engliſh, French, Dutch *and* Spaniſh, *to pay a Debt of* 1000 l. *So that every Engliſh Man payes* 50 l. *every French Man* 70 l. *every Dutch Man* 130 l. *and every Spaniard* 150 l. *How many is there of each?*

The Dividend (according to the former Rule) is 500.

Now to make the Diviſor, take his ſum that pays leaſt (namely 50) out of each of the other three 150, 130, and 70, and the Remains will be 100, 80, and 20.

Add the firſt and laſt for the Diviſor, it is 120.

And the quotient will be $4\frac{2}{12}$, and the Spaniards cannot be more.

Secondly, add the firſt and ſecond together for the Diviſor, it is 180, and the quotient is $2\frac{14}{18}$, and the Spaniards cannot be leſs.

I mean, they cannot be much more than 4, or leſs than 2: and therefore, ſeeing any one ſolution will ſerve, let them be 3, and by that multiply 100, and take the product out of 500, there remains 200 for a ſecond Dividend, which divided by (the ſecond remain) 80, the quotient is $2\frac{1}{2}$: therefore Dutchmen are 2; which multiplied by 80, make 160, take that out of 200, there remains 40 for a third Dividend: which divided by (the third remain) 20, the quotient is 2 for the French Men alſo; and conſequently the Engliſh muſt be 3, becauſe all of them are 10: But the Spaniards may be alſo 4 or 2.

Example

Example.

4 Spaniards at 150 *l.* each		600
1 Dutchman at 130 *l.*		130
1 Frenchman at 70 *l.*		70
4 Engliſh at 50 *l.* each		200
10	In all	1000
2 Spaniards at 150 *l.* each		300
3 Dutch at 130 *l.* each		390
3 French at 70 *l.* each		210
2 Engliſh at 50 *l.* each		100
10	In all	1000

The reaſon why the Spaniards and Engliſh, as alſo the Dutch and French are equal in number, is becauſe their payments differ equally from 100, which is the mean ſum with which 10 men ſhould pay 1000 *l.* and making it ſo, this queſtion, and many other of this nature, may be anſwered by the Rule of *Alligation:* thus,

If 160 give 10, what 50? Anſwer is $3\frac{20}{160}$, (that is in this caſe 3) for the Spaniards, and as many for the Engliſh, becauſe their reſpective differences from 100, the one 50 more; the Engliſh 50 leſs, are equal.

And alſo, becauſe the other two differences 30 and 30 are equal; the Number of the French is equal to the number of the Dutch.

But both thoſe numbers together are 4, becauſe 3 Spa-

3 Spaniards, and 3 Engliſh, taken out of 10, the remain muſt be 4.

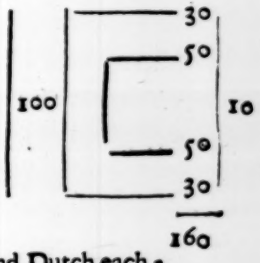
Wherefore the number of the French is 2, and the Dutch alſo are 2.

Or thus:

Accounting the men in the ſame order, as before.

If 160 require 10, what 30?

Anſwer is $1\frac{152}{160}$, (that is, in this caſe 2) for the Spaniards, and conſequently 2 Engliſh; and therefore the French and Dutch each 3.



But where any one of the particular ſums is equal to the mean ſum, there this cannot ſo well be done by *Alligation.*

Example.

If one ſhould buy 12 *Loaves of Bread for* 12 pence, *ſo that ſome might be* two peny, *ſome* peny, *ſome* half-peny, *and ſome* farthing *Loaves: and it be required to know how many he muſt buy of each?*

Then becauſe of 12 *loaves* for 12 pence, the mean price is 1, but one of the particulars being alſo 1, there ſhould be no peny loaves, becauſe there is no difference betwixt the mean *price*, and a *peny.*

But it may be found by the Rule of *Ceres* and *Virginum*, to be either

4 Two peny loaves	8 pence
2 Peny loaves	2 pence
2 Half-peny loaves	1 peny
4 Farthing loaves	1 peny
In all 12 loaves.	In all 12 pence.

Or else;

3 Two peny loaves	6 *d.*
4 Peny loaves	4
3 Half-peny loaves	$1\frac{1}{2}$.
2 Farthing loaves	$0\frac{1}{2}$.
In all 12	In all 12

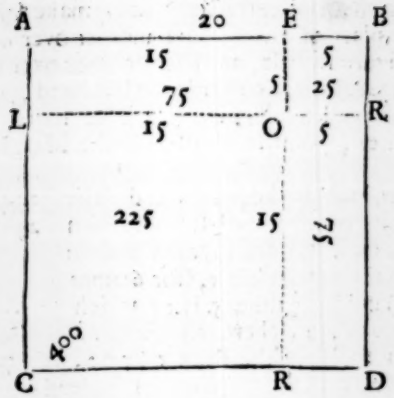
Extraction of Roots.

AND first for the *SQUARE ROOT*, that is, a ſquare Number being given, to find the Root or ſide of it in a Number: which root or ſide, being multiplied into it ſelf, muſt therefore produce that ſquare Number.

The doing of this is ſhewed in (almoſt) all Books of Arithmetick; and the reaſon of it (in ſome) which is taken from the fourth proportion of the ſecond Book of *Euclid*; which ſaith,

If a right line be divided by chance; the ſquares made of the parts, together with the Rectangle made of the parts twice, is equal to the ſquare of the whole.

Example.



Let the line A B, be divided by chance in the point E, it is manifeſt that the ſquare of A B, that is to ſay, the ſquare A B D C, is equal to the ſquare of L O, that is of A E, and to the ſquare of E B, and to the two rectangle figures A O and D O, that is the rectangle A O (which is made of the parts A E, and B E) twice; according to the Propoſition.

Now let A B, be ſuppoſed 20, A E 15, and B E 5.

Then the ſquare of A B, (which is made by multiplying the root 20 into it ſelf) is equal to 400.

And the ſquare A E, that is, 15 times 15, is equal to 225

And the ſquare of B E is 5 times 5 25

And the rectangle A O, 5 times 15 75

And the rectangle D O, 5 times 15 75

In all 400

which is equal to the ſquare of A B, as before.

If therefore a ſquare number as 334084 be given to have the root extracted: firſt, make a point over 4, (the place of Unity) and another over 0, the ſecond figure from it; and ſo to another over 3, the ſecond figure from 0, towards the left hand, obſerving the like order ſtill, if there were more ſecondary figures, point them all as in the Margine.

	. . .
Then look at the figures under the	334084
firſt point towards the left hand which	25
are 33, and take the greateſt ſquare root of them, which is 5, (for 6 times 6 is 36) and ſay 5 times 5 is 25, which	. . .
take out of 33, there remains . : . .	084084
Then multiply the Root 5 by 20, it	749
gives 100 for a Diviſor; and the Dividend is 840, namely, the figures which reach to the next point, and ſo the quotient might be 8; but muſt be but 7, becauſe the ſquare of the quotient, being now 49, muſt (together with 7 times the Diviſor) be taken out of 840.	
And the remainder will be	009184
Add this laſt quotient 7, being put to	9184
the former quotient 5, after the manner uſed in plain Diviſion, it will be 57,	0000
which multiplied ſtill by 20, the product is 1140 for a new Diviſor, and the Dividend (becauſe there remains but one point) is all the figures 9184.	

And the quotient can be but 8; and muſt be ſo much; for 8 times 1140 is 9120
and 8 times 8 is 64

In all 9184

Which taken out of the remain 9184, there now remains

remains nothing : which ſhews the ſquare 334084 is juſtly reſolved, and putting the laſt quotient 8 to the two former, 5 and 7, the whole will be 578, which is the true *Root* required.

And the parts of it, if they be 3, are 500, 70, and 8. Or if but two 570 and 8. And the truth may be either way proved, by adding the ſquares and twice the rectangles of the parts; for the ſum of them ſhall be equal to the whole ſquare 334084, as hath been ſhewed before.

It is ever certain that there ſhall be as many figures, or places of figures in the root, as there are points in the ſquare, ordered as before.

And by reading this little ſeriouſly, may any one be able to find the root of any other ſquare whatſoever: the firſt operation in all being to take the greateſt ſquare contained under that point next the left hand, and put the Root thereof for the firſt figure in a quotient.

The Secondary operation, muſt be repeated ſo often as there are remaining points; as hath been plainly ſhewed in the fore-going Example.

I will therefore add no more, but give you two other ſquares, and their roots, leaving the Reader to extract them himſelf.

So if there were given the ſquare 8836
The root of it according to former practice may be found to be 94

Or let there be given the ſquare 1522756
The root of that will 1234.

Here followeth a Table of Roots and their Squares from 1 to 1000.

1 Cent.		1 Cent.		1 Cent.	
R.	*Square*	*R.*	*Square*	*R.*	*Square*
1	1	31	961	61	3721
2	4	32	1024	62	3844
3	9	33	1089	63	3969
4	16	34	1156	64	4096
5	25	35	1225	65	4225
6	36	36	1296	66	4356
7	49	37	1369	67	4489
8	64	38	1444	68	4624
9	81	39	1521	69	4761
10	100	40	1600	70	4900
11	121	41	1681	71	5041
12	144	42	1764	72	5184
13	169	43	1849	73	5329
14	196	44	1936	74	5476
15	225	45	2025	75	5625
16	256	46	2116	76	5776
17	289	47	2209	77	5929
18	324	48	2304	78	6084
19	361	49	2401	79	6241
20	400	50	2500	80	6400
21	441	51	2601	81	6561
22	484	52	2704	82	6724
23	529	53	2809	83	6889
24	576	54	2916	84	7056
25	625	55	3025	85	7225
26	676	56	3136	86	7396
27	729	57	3249	87	7569
28	784	58	3364	88	7744
29	841	59	3481	89	7921
30	900	60	3600	90	8100

1 Cent.		1 Cent.		1 Cent.	
R.	*Square*	*R.*	*Square*	*R.*	*Square*
91	8281	121	14641	151	22801
92	8464	122	14884	152	23104
93	8649	123	15129	153	23409
94	8836	124	15376	154	23716
95	9025	125	15625	155	24025
96	9216	126	15876	156	24336
97	9409	127	16129	157	24649
98	9604	128	16384	158	24964
99	9801	129	16641	159	25281
100	10000	130	16900	160	25600
101	10201	131	17161	161	25921
102	10404	132	17424	162	26244
103	10609	133	17689	163	26569
104	10816	134	17956	164	26896
105	11025	135	18225	165	27225
106	11236	136	18496	166	27556
107	11449	137	18769	167	27889
108	11664	138	19044	168	28224
109	11881	139	19321	169	28561
110	12100	140	19600	170	28900
111	12321	141	19881	171	29241
112	12544	142	20164	172	29584
113	12769	143	20449	173	29929
114	12996	144	20736	174	30276
115	13225	145	21025	175	30625
116	13456	146	21316	176	30976
117	13689	147	21609	177	31329
118	13924	148	21904	178	31684
119	14161	149	22201	179	32041
120	14400	150	22500	180	32400

2 Cent.		2 Cent.		2 Cent.	
R.	*Square*	*R.*	*Square*	*R.*	*Square*
181	32761	211	44521	241	58081
182	33124	212	44944	242	58564
183	33489	213	45369	243	59049
184	33856	214	45796	244	59536
185	34225	215	46225	245	60025
186	34596	216	46656	246	60516
187	34969	217	47089	247	61009
188	35344	218	47524	248	61504
189	35721	219	47961	249	62001
190	36100	220	48400	250	62500
191	36481	221	48841	251	63001
192	36864	222	49284	252	63504
193	37249	223	49729	253	64009
194	37636	224	50176	254	64516
195	38025	225	50625	255	65025
196	38416	226	51076	256	65536
197	38809	227	51529	257	66049
198	39204	228	51984	258	66564
199	39601	229	52441	259	67081
200	40000	230	52900	260	67600
201	40401	231	53361	261	68121
202	40804	232	53824	262	68644
203	41209	233	54289	263	69169
204	41616	234	54756	264	69696
205	42025	235	55225	265	70225
206	42436	236	55696	266	70756
207	42849	237	56169	267	71289
208	43264	238	56644	268	71824
209	43681	239	57121	269	72361
210	44100	240	58600	270	72900

2 Cent.		3 Cent.		3 Cent.	
R.	*Rquare*	*R.*	*Square*	*R.*	*Square*
271	73441	301	90601	331	109561
272	73984	302	91204	332	110224
273	74529	303	91809	333	110889
274	75076	304	92416	334	111556
275	75625	305	93025	335	112225
276	76176	306	93636	336	112896
277	76729	307	94249	337	113569
278	77284	308	94864	338	114244
279	77841	309	95481	339	114921
280	78400	310	96100	340	115600
281	78961	311	96721	341	116281
282	79524	312	97344	342	116964
283	80089	313	97969	343	117649
284	80656	314	98596	344	118336
285	81225	315	99225	345	119025
286	81796	316	99856	346	119716
287	82369	317	100489	347	120409
288	82944	318	101124	348	121104
289	83521	319	101761	349	121801
290	84100	320	102400	350	122500
291	84681	321	103041	351	123201
292	85264	322	103684	352	123904
293	85849	323	104329	353	124609
294	86436	324	104976	354	125316
295	87025	325	105625	355	126025
296	87616	326	106276	356	126736
297	88309	327	106929	357	127449
298	88804	328	107584	358	128164
299	89401	329	108241	359	128881
300	90000	330	108900	360	129600

3 Cent.		4 Cent.		4 Cent.	
R.	*Square*	*R.*	*Square*	*R.*	*Square*
361	130321	391	153881	421	177241
362	131044	392	153664	422	178084
363	131769	393	154449	423	178929
364	132496	394	155236	424	179776
365	133225	395	156025	425	180625
366	133956	396	156816	426	181476
367	134689	397	157609	427	182329
368	135424	398	158404	428	183184
369	136161	399	159201	429	184041
370	136900	400	160000	430	184900
371	137641	401	160801	431	185761
372	138384	402	161604	432	186624
373	139129	403	162409	433	187489
374	139876	404	163216	434	188356
375	140625	405	164025	435	189225
376	141376	406	164836	436	190096
377	142129	407	165649	437	190969
378	142884	408	166464	438	191844
379	143641	409	167281	439	192721
380	144400	410	168100	440	193600
381	145161	411	168921	441	194481
382	145924	412	169744	442	195364
383	146689	413	170569	443	196249
384	147456	414	171306	444	197136
385	148225	415	172225	445	198025
386	148996	416	173056	446	198916
387	149769	417	173889	447	199809
388	150544	418	174724	448	200704
389	151321	419	175561	449	201601
390	152100	420	176400	450	202500

4 Cent.		5 Cent.		5 Cent.	
R.	*Square*	*R.*	*Square*	*R.*	*Square*
451	203401	481	231361	511	261121
452	204304	482	232324	512	262144
453	205209	483	233289	513	263169
454	206116	484	234256	514	264196
455	207025	485	235225	515	265225
456	207936	486	236196	516	266256
457	208849	487	237169	517	267289
458	209764	488	238144	518	268324
459	210681	489	239121	519	266361
460	211600	490	240100	520	270400
461	212521	491	241081	521	271441
492	213444	492	242064	522	272484
463	214369	493	243049	523	273529
464	215296	494	244036	524	274576
465	216225	495	245025	525	275625
466	217156	496	246016	526	276676
467	218089	497	247009	527	277729
468	219024	498	248004	528	278784
469	219961	499	249001	529	279541
470	220900	500	250000	530	280900
471	221841	501	251001	531	281961
472	222784	502	252004	532	283024
473	223729	503	253009	533	284089
474	224676	504	254016	534	285156
475	225625	505	255025	535	286225
476	226576	506	256036	536	287296
477	227529	507	257049	537	288369
478	228484	508	258064	538	289444
479	229441	509	259081	539	290521
480	230400	510	260100	540	291600

3 Cent.		4 Cent.		4 Cent.	
R.	*Square*	*R.*	*Square*	*R.*	*Square*
361	130321	391	153881	421	177241
362	131044	392	153664	422	178084
363	131769	393	154449	423	178929
364	132496	394	155236	424	179776
365	133225	395	156025	425	180625
366	133956	396	156816	426	181476
367	134689	397	157609	427	182329
368	135424	398	158404	428	183184
369	136161	399	159201	429	184041
370	136900	400	160000	430	184900
371	137641	401	160801	431	185761
372	138384	402	161604	432	186624
373	139129	403	162409	433	187489
374	139876	404	163216	434	188356
375	140625	405	164025	435	189225
376	141376	406	164836	436	190096
377	142129	407	165649	437	190969
378	142884	408	166464	438	191844
379	143641	409	167281	439	192721
380	144400	410	168100	440	193600
381	145161	411	168921	441	194481
382	145924	412	169744	442	195364
383	146689	413	170569	443	196249
384	147456	414	171306	444	197136
385	148225	415	172225	445	198025
386	148996	416	173056	446	198916
387	149769	417	173889	447	199809
388	150544	418	174724	448	200704
389	151321	419	175561	449	201601
390	152100	420	176400	450	202500

4 Cent.		5 Cent.		5 Cent.	
R.	*Square*	*R.*	*Square*	*R.*	*Square*
451	203401	481	231361	511	261121
452	204304	482	232324	512	262144
453	205209	483	233289	513	263169
454	206116	484	234256	514	264196
455	207025	485	235225	515	265225
456	207936	486	236196	516	266256
457	208849	487	237169	517	267289
458	209764	488	238144	518	268324
459	210681	489	239121	519	266361
460	211600	490	240100	520	270400
461	212521	491	241081	521	271441
492	213444	492	242064	522	272484
463	214369	493	243049	523	273529
464	215296	494	244036	524	274576
465	216225	495	245025	525	275625
466	217156	496	246016	526	276676
467	218089	497	247009	527	277729
468	219024	498	248004	528	278784
469	219961	499	249001	529	279541
470	220900	500	250000	530	280900
471	221841	501	251001	531	281961
472	222784	502	252004	532	283024
473	223729	503	253009	533	284089
474	224676	504	254016	534	285156
475	225625	505	255025	535	286225
476	226576	506	256036	536	287296
477	227529	507	257049	537	288369
478	228484	508	258064	538	289444
479	229441	509	259081	539	290521
480	230400	510	260100	540	291600

5. Cent.		5 Cent.		6 Cent.	
R.	Square	R.	Square	R.	Square
541	292681	571	326041	601	361201
542	293764	572	327184	602	362404
543	294849	573	328329	603	363609
544	295936	574	329476	604	364816
545	297025	575	330625	605	366025
546	298116	576	331776	606	367236
547	299209	577	332929	607	368449
548	300304	578	334084	608	369664
549	301401	579	335241	609	370881
550	302500	580	336400	610	372100
551	303601	581	337561	611	373321
552	304704	582	338724	612	374544
553	305809	583	339889	613	375769
554	306916	584	341056	614	376996
555	308025	585	342225	615	378225
556	309136	586	343396	616	379456
557	310249	587	344569	617	380689
558	311364	588	345744	618	381924
559	312481	589	346921	619	383161
560	313600	590	348100	620	384400
561	314721	591	349281	621	385641
562	315844	592	350464	622	386884
563	316969	593	351649	623	388129
564	318096	594	352836	624	389376
565	319225	595	354025	625	390625
566	320356	596	355216	626	391876
567	321489	597	356409	627	393129
568	322624	598	357604	628	394384
569	323761	599	358801	629	395641
570	324900	600	360000	630	396900

6 Cent.		6 Cent.		7 Cent.	
R	*Square*	*R.*	*Square*	*R.*	*Square*
631	398162	661	436921	691	477481
632	399424	662	438244	692	478864
633	400684	663	439569	693	480249
634	401959	664	440896	694	481636
635	403226	665	442225	695	483025
636	404495	666	444556	696	484416
637	405769	667	444889	697	485809
638	407044	668	446224	698	487204
639	408321	669	447561	699	488601
640	409600	670	448900	700	490000
641	410881	671	450241	701	491401
642	412164	672	451584	702	492804
643	413449	673	452929	703	494209
644	414736	674	454276	704	495616
645	416025	675	455625	705	497025
646	417316	676	456976	706	498436
647	418609	677	458329	707	499849
648	419904	678	459684	708	501264
649	421201	679	461041	709	502681
650	422500	680	462400	710	504100
651	423801	681	463761	711	505521
652	425104	682	465124	712	506944
653	426409	683	466489	713	508369
654	427716	684	467856	714	509796
655	429025	685	469225	715	511225
656	430336	686	470596	716	512656
657	431649	687	471969	717	514089
658	432964	688	473344	718	515524
659	434281	689	474721	719	516961
660	435600	690	476100	720	518400

7 Cent.		7 Cent.		8 Cent.	
R.	*Square*	*R.*	*Square*	*R.*	*Square*
721	519841	751	564001	781	609961
722	521284	752	565504	782	611524
723	522729	753	567009	783	613089
724	524176	754	568516	784	614656
725	525625	755	570025	785	616225
726	527076	756	571536	786	617796
727	528529	757	573049	787	619369
728	529984	758	574564	788	620944
729	531441	759	576081	789	622521
730	532900	760	577600	790	624100
731	534361	761	579121	791	625681
732	535824	762	580644	792	627264
733	537289	763	582169	793	628849
734	538756	764	583696	794	630436
735	540225	765	585225	795	632025
736	541696	766	586756	796	633616
737	543169	767	588289	797	635209
738	544644	768	589824	798	636004
739	546121	769	591361	799	638401
740	547600	770	592900	800	640000
741	549081	771	594441	801	641601
742	550565	772	595984	802	643204
743	552049	773	597529	803	644809
744	553536	774	599076	804	646416
745	555025	775	600625	805	648025
746	556516	776	602176	806	649636
747	558009	777	603729	807	651249
748	559504	778	605284	808	652864
749	561001	779	606841	809	654481
750	562500	780	608400	810	656100

8 *Cent.*	
R.	*Square*
811	657721
812	659344
813	660969
814	662596
815	664225
816	665856
817	667489
818	669124
819	670761
820	672400
821	674041
822	675684
823	677329
824	678976
825	680625
826	682276
827	683929
828	685584
829	687241
830	688900
831	690561
832	692224
833	693889
834	695556
835	697225
836	698896
837	700569
838	702244
839	703921
840	705600

8 *Cent.*	
R.	*Square*
841	707281
842	708964
843	710649
844	712336
845	714025
846	715716
847	717409
848	719104
849	720801
850	722500
851	724201
852	725904
853	727609
754	729316
855	731025
856	732736
857	734449
858	736164
859	737881
860	739600
861	741321
862	743044
863	744769
864	746496
865	748225
866	749956
867	751689
868	753424
869	755161
870	756900

8 *Cent.*	
R.	*Square*
871	758641
872	760384
873	762129
874	763876
875	765625
876	767376
877	769129
878	770884
879	772641
880	774400
881	776161
882	777924
883	779689
884	781456
885	783225
886	784996
887	786769
888	788544
889	790321
890	792100
891	793881
892	795664
893	797449
894	799236
895	801025
896	802816
897	804609
898	806404
899	808201
900	810000

9 Cent.		9 Cent.		9 Cent.	
R.	*Square*	*R.*	*Square*	*R.*	*Square*
901	811801	931	866761	961	923521
902	813604	932	868624	962	925444
903	815409	933	870489	963	927369
904	817216	934	872356	964	929296
905	819025	935	874225	965	931225
906	820836	936	876096	966	933156
907	822649	937	877969	967	935089
908	824464	938	879844	968	937024
909	826281	939	881721	969	938961
910	828100	940	883600	970	940900
911	829921	941	885481	971	942841
912	831744	942	887364	972	944784
913	833569	943	889249	973	946729
914	835396	944	891136	974	948676
915	837225	945	893025	975	950625
916	839056	946	894916	976	952576
917	840889	947	896809	977	954529
918	842724	948	898704	978	956484
919	844561	949	900601	979	958441
920	846400	950	902500	980	960400
921	848241	951	904401	981	962361
922	850084	952	906304	982	964324
923	852929	953	908209	983	966289
924	853776	954	910116	984	968256
925	855625	955	912025	985	970225
926	857476	956	913936	986	972196
927	859329	957	915849	987	974169
928	861184	958	917764	988	976144
929	863041	959	919681	989	978121
930	864900	960	921600	990	980100

9 Cent	
R.	*Square*
991	982081
992	984064
993	986049
994	988036
995	990025

9 Cent.	
R.	*Square*
996	992016
997	994009
998	996004
999	998001
1000	1000000

Extraction of the Cube Root.

A *Cube* is a Solid, or a Body contained within six equal squares; and may be fitly represented by a Die.

When a Cube is given, as is 435512; point the number as you see in the place of Unity, and every third figure after; then see what is the root of the greatest Cube contained under the first point toward the left hand; that is, in 435, it will be found 7 (for 8 times 8. taken 8 times, is 512, which is too much) put this 7 for the first figure in the quotient, having the Cube thereof, which is 343

out of 435: thus.

435512
343

And the remain will be 092512
And so the first work is done.

For

For the second, take the square of the quotient, that is 49, which multiply by 300, the product is 14700
to which add 30 times 7, that is :: 230

which makes the Divisor 14910
And the Dividend is 92519
So the quotient might be 6, but must be but 5, because the Cube of the new quotient, and 210 times the square of the said quotient must be allowed in this work, as followeth:

The remain is		92519512
Multiply the first product	147000 by	78875
the second quotient 5, and they produce	73500	
then multiply 210 by the square of 5, it makes	: 5250	
to which add the Cube of 5,	:: 125	
It makes in all	78875	
Which taken from 92519 there remains		13644512
and the quotient is 75, whose squre 6625, multiplied by 300		
the product is	1687500	
secondly, 30 times 75 is	:: 2250	
And the new Divisor is	1710000	
And the third quotient figure is 8, by which multiply 1687500, it is		1350000
Likewise, 2250, multipled by the square of 8 (64) is		144000
And the Cube of 8 is		::: 512
Altogether are		13644512

Which

Which taken from the second remain, there remains now thirdly nothing. And therefore the last quotient 8, put to the former two, it is 758 for the whole Root, as may be tryed by multiplying 758 into it self, and the product again by 758; then the last product shall be equal to the whole Cube, which was given at first to be resolved.

This one Example is sufficient for the general understanding of the manner of working, wherefore I will add no more Examples, but (as I did in the square Root, so in this I will) give you two or three numbers with their Roots, leaving the practice of them to the Learner, Thus,

Of { 389017 / 6440125 / 961504803 / 12895213625 } { 73 / 405 / 987 / 1345 } is the Root.

Here followeth a Table of Cube Roots, from 1 *to* 1000.

1 Cent.		1 Cent.		1 Cent.	
R.	Cube	R.	Cube	R.	Cube
1	1	31	29791	61	216981
2	8	32	32768	62	238328
3	27	33	35937	63	293047
4	64	34	39304	64	262144
5	125	35	42825	65	274625
6	216	36	48656	66	287496
7	343	37	50653	67	300753
8	512	38	54872	68	314432
9	729	39	55419	69	329199
10	1000	40	64000	70	333000
11	1331	41	68921	71	357911
12	1728	42	74088	72	373348
13	2197	43	79507	73	389017
14	2744	44	85184	74	405224
15	3375	45	91125	75	411875
16	4096	46	97336	76	438976
17	4913	47	103823	77	456533
18	5832	48	110592	78	474552
19	6859	49	117649	79	493039
20	8000	50	125000	80	512000
21	9261	51	135651	81	531441
22	10648	52	140608	82	550428
23	12167	53	148877	83	571787
24	13824	54	157464	84	592604
25	15625	55	167375	85	614125
26	17576	56	175616	86	636056
27	19683	57	185193	87	648303
28	21972	58	195112	88	681472
29	24389	59	205379	89	705669
30	27000	60	216000	90	729000

1 Cent.		1 Cent.		1 Cent.	
R.	*Cube*	*R.*	*Cube*	*R.*	*Cube*
91	753571	121	1771561	151	3442951
92	778688	122	1815848	152	3511808
93	804357	123	1860867	153	3581577
94	830584	124	1906624	154	3652264
95	857375	125	1953125	155	3723875
96	884736	126	2000376	156	3796416
97	925673	127	2048383	157	3869893
98	941192	128	2097172	158	3944312
99	970299	129	2146689	159	4019679
100	1000000	130	2197000	160	4096000
101	1030301	131	2248091	161	4173281
102	1061208	132	2299968	162	4251528
103	1092727	133	2352637	163	4330744
104	1124856	134	2406104	164	4410944
105	1157625	135	2460375	165	4492125
106	1191016	136	2515856	166	4574296
107	1225043	137	2570353	167	4657463
108	1259712	138	2628072	168	4741632
109	1295029	139	2685619	169	4826809
110	1231000	140	2744000	170	4913000
111	1367631	141	2803221	171	5000211
112	1404928	142	2864288	172	5088448
113	1442897	143	2924207	173	5177717
114	1481544	144	2985984	174	5268024
115	1520875	145	3027525	175	5359375
116	1560896	146	3112136	176	5451776
117	1601613	147	3176523	177	5545233
118	1643032	148	3241792	178	5639752
119	1685159	149	3307949	179	5735339
120	1728000	150	3375000	180	5832000

2 Cent.		2 Cent.		2 Cent.	
R.	*Cube*	*R.*	*Cube*	*R.*	*Cube*
181	5929741	211	9393931	241	13997521
182	6028568	212	9528128	242	14172448
183	6128487	213	9663597	243	14348907
184	6229504	214	9800344	244	14526684
185	6331625	215	9938375	245	14705125
186	6434856	216	10077696	246	14886936
187	6539203	217	10218313	247	15069223
188	6644672	218	10360232	248	15252992
189	6751269	219	10503459	249	15438249
190	6859000	220	10648000	250	15625000
191	6967871	221	10793861	251	15813251
192	7077888	222	10941048	252	16003008
193	7189057	223	11089567	253	16194277
194	7301384	224	11239424	254	16387064
195	7414875	225	11390625	255	16581375
196	7529536	226	11543176	256	16777216
197	7645373	227	11697083	257	16974593
198	7762392	228	11852452	258	17173512
199	7882599	229	12008989	259	17473979
200	8000000	230	12167000	260	17576000
201	8120601	231	12326391	261	17779581
202	8242408	232	12487168	262	17984728
203	8365427	233	12649337	263	18191447
204	8489664	234	12812904	264	18399744
205	8615125	235	12977875	265	18609625
206	8741816	236	13144256	266	18821096
207	8869743	237	13312053	267	19034063
208	8999912	238	13481272	268	19248832
209	9129329	239	13651919	269	19465109
210	9261000	240	13824000	270	19683000

2 Cent.		3 Cent.		3 Cent.	
R.	Cube	R.	Cube	R.	Cube
271	19902511	301	27270901	331	36264691
272	20123648	302	27543608	332	36594368
273	20346417	303	27818127	333	36926037
274	20571024	304	28094464	334	37259704
275	20796875	305	28372625	335	37595375
276	21024576	306	28652616	336	37933056
277	21253933	307	28934443	337	38272753
278	21484952	308	29218112	338	38614472
279	21717639	309	29503629	339	38958219
280	21952000	310	29791000	340	39304000
281	22188041	311	30080231	341	39651821
282	22425768	312	30371328	342	40001688
283	22665187	313	30664297	343	40353607
284	22906304	314	30959144	344	40707584
285	23149125	315	31255875	345	41063625
286	23393656	316	31554496	346	41421736
287	23539903	317	31855013	347	41781923
288	23893872	318	32157432	348	42144192
289	24137569	319	32461759	349	42508549
290	24389000	320	32768000	350	42875000
291	24642171	321	33076161	351	43243551
292	24897088	322	33386248	352	43614208
293	25143757	323	33698267	353	43986977
294	25412184	324	34012224	354	44361864
295	25672363	325	34328125	355	44738875
296	25934336	326	34645976	356	45118016
297	26198073	327	34965783	357	45499293
298	26353592	328	35287152	358	45882712
299	26730899	329	35611286	359	46268279
300	27000000	330	35937000	360	46656000

3 Cent.		4 Cent.		4 Cent.	
R.	*Cube*	*R.*	*Cube*	*R.*	*Cube*
361	47045881	391	59776471	421	74618461
362	47439928	392	60236288	422	75151448
363	47832147	393	60698457	423	75686967
364	48228544	394	61162984	424	76225024
365	48627125	395	61629875	425	76765625
366	49027896	396	62099136	426	77308776
367	49430863	397	62570773	427	77854483
368	49836032	398	63044792	428	78402752
369	50243409	399	63521199	429	78953589
370	50653000	400	64000000	430	79507000
371	51064811	401	64481201	431	80062991
372	51478848	402	64964808	432	80621568
373	51895117	403	65450827	433	81182737
374	52313624	404	65939264	434	81746504
375	52734375	405	66430125	435	82312875
376	53157376	406	66923416	436	82881856
377	53582633	407	67419143	437	83453453
378	54010152	408	67917312	438	84027672
379	54439939	409	68417929	439	84604519
380	54872000	410	68921000	440	85184000
381	55306341	411	69426531	441	85766121
382	55742968	412	69934528	442	86350888
383	56181887	413	70444997	443	86938307
384	56623104	414	70957944	444	87528384
385	57066625	415	71473375	445	88121125
386	57512456	416	71991296	446	88716536
387	57960603	417	72511713	447	89314623
388	58411072	418	73034632	448	89915392
389	58863869	419	73560059	449	90518849
390	59319000	420	74088000	450	91125000

4 Cent.		5 Cent.		5 Cent.	
R.	*Cube*	*R.*	*5 Cent.*	*R.*	*Cube*
451	91733851	481	111284641	511	133432821
452	92345408	482	111980168	512	134217728
453	92959077	483	112678587	513	135005697
454	93576664	484	113379904	514	135796744
455	94196375	485	114084125	515	136590875
456	94818816	486	114791256	516	137388096
457	95443993	487	115501303	517	138188413
458	96071912	488	116214272	518	138991832
459	96702579	489	116930169	519	139798359
460	97336000	490	117649000	520	140608000
461	97972181	491	118370771	521	141420761
462	98611128	492	119095488	522	142246648
463	99252847	493	119823157	523	143055667
464	99897344	494	120553784	524	143877824
465	100544625	495	121287375	525	144703125
466	101194696	496	122023936	526	145531576
467	101847563	497	122763473	527	146362183
468	102503232	498	123505992	528	147197952
469	103161709	499	124251499	529	148035889
470	103823000	500	125000000	530	148877000
471	104487111	501	125751501	531	149721291
472	105154048	502	126506008	532	150568768
473	105823817	503	127263527	533	151419437
474	106497424	504	128024064	534	152273304
475	107171875	505	128787625	535	153130375
476	107850176	506	129554216	536	153990656
477	108531333	507	130323843	537	154854153
478	109215352	508	131096512	538	155720872
479	109902239	509	131872229	539	156590819
480	110592000	510	132651000	540	157464000

5 Cent.		5 Cent.		6 Cent.	
R.	*Cube*	*R.*	*Cube*	*R.*	*Cube*
541	158340421	571	186169411	601	217081801
542	159220088	572	187149248	602	218167208
543	160103007	573	188132517	603	219256227
544	160989184	574	189119224	604	220348864
545	161878625	575	190109375	605	221445125
546	162771336	576	191102976	606	222545016
547	163667323	577	192100033	607	223648543
548	164566592	578	193100552	608	224755712
549	166569149	579	194104539	609	225866529
550	166375000	580	195112000	610	226981000
551	167284151	581	196122941	611	228099131
552	168196608	582	197137368	612	229220928
553	169112377	583	198155287	613	230346397
554	170031464	584	199176704	614	231475544
555	170953875	585	200202625	615	232608375
556	171879616	586	201230056	616	233744896
557	172808683	587	202262003	617	234885113
558	173741112	588	203297472	618	236029032
559	174676879	589	204336469	619	237176659
560	175616000	590	205379000	620	238328000
561	176558481	591	206425071	621	239483061
562	177504328	592	207474688	622	240641848
563	178453547	593	208527857	623	241804367
564	179306144	594	209584584	624	242970624
565	188262125	595	210644875	625	244140625
566	181221496	596	211708736	626	245314376
567	182154263	597	212776173	627	246491883
568	183150432	598	213847192	628	247673152
569	184220009	599	214921799	629	248858189
570	185193000	600	216000000	630	250047000

6 Cent.		6 Cent.		7 Cent.	
R.	Cube	R.	Cube	R.	Cube
631	251239591	661	288047811	691	339939371
632	252435968	662	290117528	692	331373888
633	253636137	663	291434247	693	332812557
634	254840104	664	292754944	694	334255384
635	256047875	665	294079625	695	335702375
636	257259456	666	295408296	696	337155396
637	258474853	667	296740963	697	338608873
638	259694072	668	298077632	698	340068392
639	260917119	669	299418309	699	341532099
640	262144000	670	300763000	700	343000000
641	263374721	671	302111711	701	344472101
642	264609288	672	303464448	702	345948408
643	265847707	673	304821217	703	347428927
644	267089984	674	306182024	704	348913664
645	268336125	675	307546875	705	350402625
646	269586136	676	308915776	706	351895816
647	270840023	677	310288733	707	353393243
648	272097792	678	311665752	708	354894912
649	273359449	679	313046839	709	356400829
650	274625000	680	314432000	710	357911000
651	275894451	681	315821241	711	359425431
652	277167808	682	317214568	712	360944128
653	278445077	683	318611987	713	362467097
654	279726264	684	320013504	714	363994344
655	281011375	685	321419115	715	365525875
656	282300416	686	322828856	716	367061696
657	283593393	687	324242703	717	368601813
658	284890312	688	325660672	718	370146232
659	286191179	689	327082769	719	371694959
660	287496000	690	328509000	720	373248000

7 Cent.		7 Cent		8 Cent.	
R.	*Cube*	*R.*	*Cube*	*R.*	*Cube*
721	374805361	751	423564751	781	476379541
722	376367048	752	425259008	782	478211768
723	377933067	753	426957777	783	480048687
724	379503424	754	428661064	784	481890304
725	381078125	755	430368875	785	483736625
726	382657176	756	432081216	786	485587656
727	384240583	757	433798093	787	487443403
728	385828352	758	435519512	788	489303872
729	387420489	759	437245479	789	491169069
730	389017000	760	438976000	790	493039000
731	390617891	761	440711081	791	494913671
732	392223168	762	442450728	792	496793088
733	393832837	763	444194947	793	498677257
734	395446904	764	445943744	794	500566184
735	397065375	765	447697125	795	502459875
736	398688256	766	449455096	796	504358336
737	400315553	767	451217663	797	506261573
738	401947272	768	452984832	798	508169592
739	403583419	769	454756609	799	510082399
740	405224000	770	456533000	800	512000000
741	406869021	771	458314011	801	513922401
742	408518488	772	460099648	802	515849608
743	410172407	773	461889917	803	517781627
744	411830784	774	463684824	804	519718464
745	413493625	775	465484375	805	521660125
746	415160936	776	467288576	806	523606616
747	416832723	777	469097433	807	525557943
748	418508992	778	470910952	808	527514112
749	420189749	779	472729139	809	529475129
750	421875000	780	474552000	810	531441000

8 Cent.		8 Cent.		8 Cent.	
R.	*Cube*	*R.*	*Cube*	*R.*	*Cube*
811	533411731	841	594823321	871	660776311
812	535387328	842	596947688	872	663054848
813	537367797	843	599077107	873	665338617
814	539353144	844	601211184	874	667627624
815	541343375	845	603351125	875	669921875
816	543338496	846	605495736	876	672221376
817	545338513	847	607645423	877	674526133
818	547343432	848	609800192	878	676836152
819	549353259	849	611960049	879	679151435
820	551368000	850	614125000	880	681472000
821	553387661	851	616295051	881	683797841
822	555412248	852	618470208	882	686128968
823	557441767	853	620650477	883	688465387
824	559476224	854	622835864	884	690807104
825	561515625	855	625026375	885	693154125
826	563559976	856	627222016	886	695506456
827	565609283	857	629422793	887	697864103
828	567663552	858	631628712	888	700227072
829	569722789	859	633839779	889	702595369
830	571787000	860	636056000	890	704969000
831	573856191	861	638277381	891	707347971
832	575930368	862	640503928	892	709732288
833	578009537	863	642735647	893	712121957
834	580093704	864	644972544	894	714516984
835	582182875	865	647214625	895	716917375
836	584277056	866	649316896	896	719323136
837	586376253	867	651714363	897	721734273
838	588480472	868	653972032	898	724150792
839	590589719	869	656234909	899	726572699
840	592740000	870	658503000	900	729000000

9 Cent.		9 Cent.		9 Cent.	
R.	*Cube*	*R.*	*Cube*	*R.*	*Cube*
901	731432701	931	806954491	961	887503681
902	733870808	932	802557569	962	890277218
903	736314327	933	812166237	963	893056347
904	738763264	934	814780504	964	895841344
905	741217625	935	817400375	965	898632125
906	743677416	936	820025856	966	901428696
907	746142643	937	822659953	967	904231063
908	748613312	938	825293672	968	907039232
909	751089429	939	827936019	969	909853209
910	753571000	940	830584000	970	912673000
911	756058031	941	833237621	971	915498611
912	758550528	942	835896888	972	918330048
913	761048497	943	838561807	973	921167317
914	763551944	944	841232384	974	924010424
915	766060875	945	843908625	975	926859375
916	768575296	946	846590536	976	929714176
917	771095213	947	849278123	977	932574833
918	773610632	948	851971392	978	935441352
919	776151559	949	854670349	979	938313739
920	778688000	950	857375000	980	941192000
921	781229961	951	860085351	981	944076141
922	783777448	952	862801408	982	946966168
923	786330467	953	865523177	983	949862087
924	788889024	954	868250664	984	952763904
925	791453125	955	870983875	985	955671625
926	794022776	956	873722816	986	958585256
927	796597983	957	876467493	987	961504873
928	799178752	958	879217912	988	964430272
929	801765089	959	881974079	989	967361669
930	804357000	960	884736000	990	970299000

9 Cent.	
R.	Cube
991	973242271
992	976191483
993	979146657
994	982107784
995	985074875

9 Cent.	
R.	Cube
996	988047936
997	991026973
998	994011992
999	997002999
1000	1000000000

Some

Some Uſes of the Square and Cube Root.

Uſes of the Square Root.

WHat the *Square* and *Cube Root* are, and how to *extract* them, hath already been taught; and for more eaſe and expedition, there are *Tables* ready calculated, both of the *Square* and *Cube Roots*, from 1 to 1000: We come now to ſhew ſome Uſes thereof, which in ſome meaſure will appear in the *Propoſitions* following:

PROPOSITION. I.

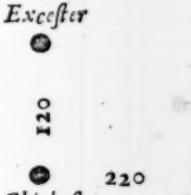
Admit the height of the Wall of a Fort or Caſtle to be ſcaled, be 30 Foot, and the breadth of the Trench about the Fort be 40 Foot; I demand of what length a Scaling Ladder ſhall be, juſtly to reach from the edge or brow of the Trench, to the top of the Wall?

By the 47th of the firſt Book of *Euclids Elements*, it is demonſtrated; that, *the ſquare of the Hypotenuſal of all right angled plain Triangles is equal to the ſquares of the 2 other ſides*; I therefore to reſolve this Propoſition, ſquare the height of the *Wall*, which is 30, *facit* 900; alſo I ſquare the breadth of the *Trench* which is 40 *facit* 1600; theſe two added together make 2500, the ſquare root whereof is 50: and ſo long muſt a Scaling Ladder be made to reach from the edge of the *Trench* to the top of the *Wall*.

PRO-

PROPOSITION. II.

There be two Towns, as Chiceſter *and* York, *which lie North and South one from another, and their diſtance is* 220 *miles, and* Exceſter *lieth directly weſt from* Chicheſter, 120 *miles; I deſire to know the diſtance of* York *from* Exceſter?



Square 120, the diſtance of *Exceſter* and *Chicheſter*, it maketh 14400, likewiſe ſquare 220, the diſtance of *York* and *Chicheſter*, *facit*, 48400; theſe two numbers added together make 62800, whoſe ſquare root extracted (or found in the Table) will be 250$\frac{3}{4}$ near, and ſo many miles is *Exceſter* diſtant from *York*.

Uſe of the Cube Root.

ONe chiefuſe of the *Cube Root*, is to find out a proportion between like Solids; ſuch are Spheres, Cubes, and ſuch like; as in the Propoſition following:

PROPOSITION. I.

If a Bullet of Braſs of 4 *inches Diameter, weigh* 9 *pound, what ſhall a Bullet of Braſs weigh, whoſe Diameter is* 8 *inches?*

Cube 4, the Diameter of the leſſer Bullet, makes 64, likewiſe Cube the Diameter of the greater Bullet 8, makes 4608. This done, ſay by the Rule of Proportion; If the Cube 64 give 9 *li.* weight, what ſhall

ſhall the Cube number 4608 give? Multiply and divide, you ſhall find 72; and ſo many pounds will a Bullet of Braſs weigh, whoſe Diameter is 8 inches.

PROPOSITION II.

If a Fathom of Rope of 10 inches compaſs about, do weigh 17 pound, how much ſhall a Fathom of Rope weigh, which is but 8 inches compaſs about?

The ſquare of 10 is 100, the ſquare of 8 is 64; wherefore by the Rule of Proportion, ſay

As 100 (the ſquare of 10)
Is to 64 (the ſquare of 8)
So is 17 (the weight of the fathom of Rope of 10 inches)
To $10\frac{88}{100}$ pounds (the weight of the fathom of Rope of 8 inches about.)

PROPOSITION III.

If a Ship of 100 Tun be 20 foot broad at the Mid ſhip Beam, of what breadth at the Beam ſhall a Ship (of the like building) be that ſhall be 200 Tun?

The Cube of 20 is 8000, then by the Rule of Proportion ſay,

As 100 Tun (the burthen of the Ship given)
Is to 200 Tun (the burthen of the Ship required)
So is 8000 (the Cube of the given Ships Beam)
To 16000 (the Cube of the required Ships Beam.)

Now the Cube Root of 16000 is $25\frac{1}{5}$ almoſt, and ſo long at the Mid-ſhip Beam muſt a Ship of the ſame Model be, whoſe burthen is 200 Tun.

The end of the Firſt Part.

DECIMAL ARITHMETICK.

The Second Part.

CONTAINING

The Grounds and Reason thereof; The Use of this kind of Artificial (or Decimal) way of working, illustrated by divers Examples, in all the most usual Rules of Arithmetick.

By *Will. Leybourn.*

LONDON,
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Decimal ARITHMETICK.

The Second Part.

HAving in the first Part of this Book exemplified the Art of *Vulgar Arithmetick* both in *whole Numbers* and *Fractions*; We come now to treat of DECIMAL ARITHMETICK, which teacheth how to perform (as it were) in *Whole Numbers*, all that the former did effect by *Fractions*. I will not insist upon the *Antiquity* or *Excellency* of this kind of *Arithmetick*, but come immediately to the *practice* thereof, and shall therefore premise these Propositions following.

Proposition 1.

A vulgar Fraction being given, how to reduce the same into a Decimal.

The Rule.

To the Numerator *of the Fraction given, add what number of* Cyphers *you please; then divide the* Numerator *by the* Denominator, *the quotient shall be the* Decimal Fraction *required.*

Example 1.

Let it be required to reduce $\frac{4}{17}$ *into a Decimal:* First, to the Numerator 4, add five Cyphers, so will it be 400000, divide this number by the Denominator 17, and the quotient will be 23529, which is the Decimal required.

¶ And here Note, that *Decimal Fractions* are not written in a smaller figure with a line between them, as *vulgar Fractions* are, but of the same figure only there must be a Comma or point put between the whole Number and the Fraction, and that is the distinction.

Example 2. *If you would express* 235 $\frac{4}{17}$ *in a decimal way,* it must be written as followeth.

By the last example you find that $\frac{4}{17}$ reduced to a Decimal, was 23529, therefore 235 $\frac{4}{17}$ must be written thus: 235,23529. In Decimal Fractions the Numerator is only expressed, and the Denominator only intimated; for this RULE is general, *Of how many figures soever the* Numerator *of a decimal fraction doth consist, of so many* Cyphers *with a* Unite *before them, doth the* Denominator *of the same Fraction consist.* So this Decimal 12,625, if it were written in a vulgar way, would be $12.\frac{625}{1000}$, but in a Decimal, only 12,625, the Comma or point between 12 and 625 distinguisheth the

the *whole number* from the *Fraction*, and the fraction 625 consisting of three figures, intimates that the Denominator thereof must consist of three Cyphers and an Unite before them; so the Decimal before expressed, 235,235$\frac{29}{100000}$, if it were written in the vulgar way, would be 235 $\frac{23529}{100000}$.

But it sufficeth to express in Decimals the Numerators only, and omit the Denominators, the Denominators of all Decimal Fractions being either 10, 100, 1000, 10000, 100000, &c. according to the number of figures contained in the Numerators,

According to this Rule, you shall find that

$\frac{4}{5}$ will be in Decimals by adding 5 Cyphers. .80000.

$12\frac{1}{7}$ will be in Decimals by adding 5 Cyphers. 12.42857.

$132\frac{29}{17}$ will be in Decimals by adding 5 Cyphers. 132.52941.

And by this means, all manner of Fractions of *Coins*, *Weights* and *Measures*, may be reduced from vulgar Fractions, to Decimal Fractions; as by the next Proposition will appear.

Proposition 2.

How to express English-Coin *in Decimal Numbers.*

Let it be required to express 9 shillings (which is $\frac{9}{20}$ of a pound *Sterling*) in a Decimal; To the Numerator 9 add two Cyphers, making it 900, which divide by twenty, the quotient is 45, for the Decimal of 9 *s.* So the Decimal of 13 *s.* will be 65, and so for any number of shillings.

¶ Here note, that in the Reduction of Vulgar Fractions into Decimals, that many times the first, second or third places of the Decimal Fractions are Cyphers, as in the following Table the Decimal of one Farthing is .0104167, and the reason is, because if you

reduce $\frac{1}{2\,8\,8\,0}$ into a Decimal (for one farthing is the 960 part of a pound *Sterling*) you ſhall by adding of Cyphers to the Numerator find the Quotient to be 104167, but two Cyphers muſt be placed before it; becauſe dividing 1000000 by 960, the place of Unites in the Diviſor at the firſt demand extendeth unto the third Cypher in the Dividend, for in reducing of Vulgar Fractions to Decimals, this is

A general Rule.

That if the place of Unites *in the* Diviſor, *at the firſt demand extend but unto the firſt of the* Cyphers *annexed to the* Numerator *of the Fraction, there muſt be no* Cypher *put before in the Quotient, but if the place of* Unites *extend unto the ſecond* Cypher *added, then one* Cypher *muſt be placed before in the Quotient, if unto the third* Cypher, *then two Cyphers muſt be placed before in the Quotient, &c.*

According to which Rule, if you make tryal you ſhall find that the Decimal of 7 *s.* will be .35, the Decimal of 5 *d.* will be .02083333, the Decimal of two farthings will be .00208333, as in the Table.

By theſe Rules laſt delivered are the enſuing Tables of *Engliſh Money*, *Weight* and *Meaſure* compoſed, and the like may be done for a forreign *Coin*, &c. according as every mans occaſions ſhall require.

The

The TABLE of *English Coin* in Decimals.

English Coin.	
Sh. 19	.95
18	.9
17	.85
16	.8
15	.75
14	.7
13	.65
12	.6
11	.55
10	.5
9	.45
8	.4
7	.35
6	.3
5	.25
4	.2
3	.15
2	.1
1	.05

D. 11	.04583333
10	.04166667
9	.0375
8	.03333333
7	.02916667
6	.025
5	.02083333
4	.01666667
3	.0125
2	.0833333
1	.04166667
F. 3	.003125
2	.00208333
1	.00104167

Troy Weight in Decimals.	
O. 11	.91666667
10	.83333333
9	.75

	8	.66666667
	7	.58333333
	6	.5
	5	.41666667
	4	33333333
	3	25
	2	16666667
	1	08333333
P	19	07916667
	18	075
	17	07083333
	16	06666667
	15	0625
	14	05833333
	13	05416667
	12	05
	11	04583333
	10	04166667
	9	0375
	8	03333333
	7	02916667
	6	025
	5	02083333
	4	01666667
	3	0125
	2	00833333
	1	00416667

Gr.	23	.00399395
	22	.00381944
	21	00364583
	20	00347222
	19	00329861
	18	003125
	17	.00295139
	16	00277778
	15	00260417
	14	00243056
	13	00225694
	12	00208333
	11	00190972
	10	.00173611
	9	0015625
	8	00138889
	7	00121528
	6	00104166
	5	00086805
	4	.00069444
	3	00052083
	2	00044722
	1	00017311

Aver-

Averdupois great Weight in DECIMALS.

3 *qu.*		.75
2 *qu.*		5
1 *qu.*		25
	27	24107142
	26	23214285
	25	.22321428
	24	21428571
	23	20535714
	22	19642857
	21	1875
	20	.17857143
	19	16964286
	18	16071428
	17	15178571
	16	14285714
	15	13392857
	14	125
	13	11607143
	12	.10714286
	11	09821428
	10	08928571
	9	08035714
	8	07141857
	7	0625
	6	.05357143
	5	04464286
	4	03571428
	3	02678571
	2	01785714
	1	.00892857
Oun.	15	.00837053
	14	0078125
	13	00725446
	12	00669643
	11	00613839
	10	.00558035
	9	00502232
	8	00446429
	7	00390625
	6	00334821
	5	00279018
	4	00223214
	3	.00167411
	2	00111607
	1	00055804
3 *qu.*		00041853
half		00027902
1 *qu.*		00013951

Aver-

Averdupois little Weight in DECIMALS.

Ounces	15	.937
	14	875
	13	8125
	12	75
	11	6875
	10	.625
	9	5625
	8	5
	7	4375
	6	375
	5	3125
	4	25
	3	1875
	2	125
	1	.0625
Drams.	15	.05859375
	14	0546875
	13	05078115
	12	046875
	11	04296875
	10	.039c625
	9	.03515625
	8	03125
	7	02734375
	6	0234375
	5	01953125
	4	015625
	3	01171875
	2	0078125
	1	.00390625
3 *qu.*		00292969
half		00195312
1 *qu.*		00097656

Liquid Measures in Decimals.

P.	7	.875
	6	75
	5	625
	4	5
	3	375
	2	25
	1	125
3 *qu.*		09375
half.		0625
1 *qu.*		.03125

Dry Measures in Decimals.

Bushels.	7	.875
	6	75
	5	625
	4	5
	3	.375
	2	25
	.1	.125
Pecks.	.3	.09375
	2	0625
		03225
3 qu.		.0234375
half.		015625
1 qu.		.0078125
Pints.	3	0058594
	2	0039063
	.1	.0019531

Long Measures, the Integars being Yards and Ells in Decimals.

Qua.	3	.75
	2	5
	1	.25
Naile.	3	.1875
	2	125
	1	.0625
3 qu.		046875
half.		03125
1 qu.		.015625

Time in DECIMALS.

Mo.	11	.916667
	10	833333
	9	75
	8	666667
	7	833333
	6	5
	5	416667
	4	333333
	3	25
	2	166667
	1	.083333
Da.	30	082193
	29	097454
	28	076714
	27	073973
	26	071233
	25	068495

24	.065755
23	063016
22	060274
21	057536
20	054795
19	052055
18	049316
17	.046577
16	043837
15	041097
14	038357
13	035617
12	032877
11	030137
10	027397
9	024657
8	.021918
7	029178
6	016438
5	013698
4	010959
3	0082192
2	0054795
1	.0027397

Dozens in Decimals.

De.	11	.9166667
	10	8333333
	9	75
	8	6666667
	7	5833333
	6	5
	5	4166667
	4	3333333
	3	25
	2	1666667
	1	.0833333
Pa.	11	.076388
	10	0694444
	9	0625
	8	0555555
	7	0486111
	6	.0416667
	5	0347222
	4	0277778
	3	0208333
	2	0138889
	1	.0069444

The

The uſe of the fore-going Tables.

THe *Tables* preceding are in number nine; The firſt being of *Engliſh Coin*; The ſecond of *Troy Weight*; The third of *Averdupois great Weight*. The fourth of *Averdupois little Weight*. The fifth of *Liquid Meaſures*. The ſixth of *Dry Meaſures*. The ſeventh of *Long Meaſures*. The eighth of *Time*, and the ninth of *Dozens*. Theſe ſeveral Tables are made by the Rules immediately going before them, and their uſe is to expreſs in *Decimal numbers* either *Money*, *Weight*, or *Meaſure*, as by the following *Propoſitions* will appear.

PROP. I.

How by the Table *to expreſs* Engliſh-Coin *in Decimals.*

The firſt of the nine Tables is for this purpoſe; therefore if you would expreſs either *ſhillings*, *pence*, or *farthings* in Decimal numbers, you muſt repair to the firſt Table, which is of *Engliſh Coin*, and there againſt 13 ſhillings you ſhall find .65, which is the Decimal of 13 ſhillings, alſo againſt ſeven pence you ſhall find .02916667, which is the Decimal repreſenting 7 pence: Alſo againſt 2 farthings you ſhall find .00208-333, which is the Decimal anſwering 2 farthings, and the like is to be done for any other number of ſhillings, pence, or farthings.

But if it be required to find the Decimal of divers Denom-

Denominations of Coin in one Sum, as of ſhillings, pence, and farthings together, you muſt add the Decimals of all the particulars together, and the Sum of them ſhall be the Decimal ſought.

Examples.

If you would know the Decimal of 13 *s.* 7 *d.* 2 *q.* in one number, you muſt firſt look in the Table for the Decimal of 13 *s.* .65 which is .65, and ſet that down, then .02916667 look for the Decimal of 7 *d.* which is .00208333 .00916667, and ſet that down alſo : ———— Laſtly, ſeek the Decimal of 2 *q.* which .68125000 is .00208333, ſet that down alſo; then if you add theſe three numbers together, as in common Addition, you ſhall find the ſum of them to be .6812-5000, which is the Decimal belonging to 13 *s.* 7 *d.* 2 *q.* as by the work in the Margine appeareth.

PROP. II.

How by the Table *to expreſs* Troy-Weight *in Decimals.*

The ſecond Table is of *Troy Weight*, the ſeveral Denominations whereof are *Ounces*, *Peny-weights*, and *Grains:* So that by the Table you ſhall find that the Decimal belonging to five ounces is .41666667, the Decimal belonging to 17 peny-weight is ..07083333, and the Decimal belonging to 13 Grains is .00225694, and ſo of any other number of ounces, peny-weights, and grains ſeverally.

But if it were required to expreſs theſe (or any other) ſeveral

ſeveral Denominations in one Decimal Fraction, then you muſt (as before you did for money) take out of the Table the ſeveral Decimals belonging to the reſpective quantities, and add them together, ſo ſhall the Sum of that Addition be the Decimal ſought.

Example.

If it were required to find a Decimal which ſhould repreſent 5 ounces, 17 peny-weight, 13 grains, you muſt firſt look in the Table for the Decimal belonging to five ounces, which is .41666667, and write it down, then look the Decimal belonging to 17 grains, which is .07083333, and write that down, then look for the Decimal of 13 grains, which is .00225694, and write that down, then adding theſe three numbers together, you ſhall find the ſum of them to be .48975694, which is the Decimal repreſenting 5 ounces, 17 grains, 13 peny-weight, as by the operation in the Margent appeareth.

.41666667
.07083333
.00225694
.48975694

PROP. III.

How by the Table *to expreſs* Averdupois great weight *in Decimals.*

The third Table is of *Averdupois* great weight, the ſeveral Denominations whereof are *Quarters* of *Hundreds*, *Pounds*, *Ounces*, and *Quarters* of *Ounces*; thus you ſhall find in the Table, that the Decimal of 3 Quarters of a Hundred is .75, the Decimal of 22 pounds is .19642857, the Decimal of 7 ounces is .00390625, and the Decimal of 3 quarters of an ounce is

is .00041853, in this manner by the Table you may find the correspondent Decimal belonging to any number of quarters, pounds, ounces, and parts of ounces severally.

But if it be required to find one Decimal number which shall represent divers Denominations, you must first find the Decimal belonging to the several particulars, and add them together, the sum whereof shall be the entire Decimal required.

Example.

Let it be required to find a Decimal which shall represent 3 quarters, 22 pounds, 7 ounces ¼ of an ounce. First, look in the Table for the decimal of three quarters of a hundred, which is .75, and write it down, then look for the decimal of 22 pound, which is .19642857, and write that down, also look the Decimal belonging to seven ounces, which is .00390625, and write that down: Lastly, seek the decimal of three quarters of an ounce, which is .00041853, and write that down, then adding these four numbers together, you shall find their sum to be .95075335, which is the Decimal representing 3 qu. 22 *lib.* 7 oun. ¼ of an ounce.

.75	
.19642847	
.00390625	
.00041853	
.95075335	

PROP. IV.

How by the Table *to express* Averdupois little weight *in Decimals.*

The fourth Table is of *Averdupois* little weight, the Denominations whereof are *Ounces*, *Drams* and *Quarters*

ters of drams, ſo that the Decimal of 11 ounces is .6875, the Decimal of five drams is .01953125, and the Decimal of one quarter of a dram is .00097656

But if it be required to find one decimal number, which ſhall repreſent 11 ounces, 5 ¼ then you muſt firſt look for the decimal belonging to 11 ounces, which is .6875, and ſet it down; then look for the decimal anſwering to 5 drams, which is .01953-125, and ſet that down. Laſtly, look the decimal belonging to a quarter of a dram, which is .00097656, and ſet that down; theſe three numbers being added together, produce .70800781, which is the correſpondent decimal belonging to 11 ounces, 5 drams, and a quarter of a dram.

.6875	
.01953125	
.00097656	
70800781	

PROP. V.

How by the Table *to expreſs* Liquid Meaſures *in Decimals.*

Becauſe there is ſo great variety of Liquid Meaſures, that hardly any two commodities are ſold by the ſame, the difference of the Gallon continually making alteration, we have therefore in this fifth Table made the greateſt denomination to be one Gallon, the next leſs denomination being Pints and quarters of Pints, ſo that in the Table you ſhall find the decimal belonging to three pints to be .375, and the decimal belonging to two quarters, or half a pint, to be .062.5, and ſo for any other.

But for to expreſs Pints and parts of Pints in one entire decimal number, you muſt add the Decimals of the ſeveral denominations together, and their Sum ſhall be the entire Decimal.

So if you were to expreſs 3 pints and an half in one entire decimal number, add the decimal of three pints, which is .375, to the decimal of two quarters which is .0625, and their ſum .4375, ſhall be the decimal of three pints and an half.

.375	
.0625	
.4375	

PROP. VI.

How by the Table *to expreſs* Dry Meaſures *in Decimals.*

The ſixth Table is of *Dry Meaſures*, the ſeveral denominations whereof are *Buſhels*, *Pecks*, *quarters of Pecks* and *Pints*, ſo may you find the decimal of five buſhels to be .625, the decimal of two pecks to be .0625, the decimal of three quarters of a peck to be .023437, the decimal of two pints to be 0039063. Thus are the correſpondent decimals belonging to the ſeveral denominations found.

But if you would have one number to expreſs 5 buſhels, 2 pecks, three quarters of a peck, and 2 pints: you muſt firſt find the decimal belonging to 5 pecks, which is .625, and write it down, then find the decimal of two pecks, which is ,0625, then ſeek the decimal of three quarters of a peck, which is .0234375, and write that down. Laſtly, ſeek the decimal of two pints, which is .0039063, which numbers being added together, produce .7148438, which is the decimal belonging (or expreſſing) 5 buſhels, 2 pecks, three quarters of a peck, and 2 pints.

.625	
.0625	
.0234375	
.0039063	
.7148438	

PROP

PROP. VII.

How by the Table *to expreſs* Long Meaſures *in Decimals.*

The ſeventh Table is of *Long Meaſures*, the Integers being *Yards* and *Ells*: and the leſſer denominations are *quarters* of *Yards* or *Ells*, *Nailes*, and quarters of *Nailes*. So you may find in the Table that the decimal of three quarters of a Yard, or an Ell, is .75, the decimal of two Nailes, is .125, and the decimal of one quarter of a Naile is .015625.

But if you would have one number to expreſs 3 quarters of a Yard, or an Ell, two Nailes, and one quarter of a Naile; you muſt ſeek the decimal of three quarters of a Yard or Ell, which is .75, and write it down, likewiſe ſeek the decimal of two Nailes, which is .125, and write that down. Laſtly, ſeek the decimal of one quarter of a Nail, which is .015625, and write that down, theſe three numbers added together, make .890625, which is the decimal belonging to 3 quarters of a Yard or Ell, 2 Nailes, and one quarter of a Naile.

.75	
.125	
.015625	
.890625	

PROP. VIII.

How by the Table *to expreſs the parts of* Time *in Decimals.*

Time is uſually divided into *Years*, *Months* and *Dayes*: ſo the eighth Table which is of *Time*, conſiſteth of theſe two denominations, *Months* and *Dayes*, you may

find that the decimal of 5 months is .416667, the decimal of 26 dayes is .071233. These are the principal decimals, but the compound decimal number representing 5 months, 26 dayes, is .487900, as you shall find, it you add .071233, which is the decimal of 26 dayes, to .416667, which is the decimal of five Months.

PROP. IX.

How by the Table *to expresse* Dozens *in Decimals.*

The last Table is of *Dozens*, the Integer being a *Grosse*, and the smaller denominations are Dozens, and parts of Dozens, so may you find the decimal of seven dozen to be .5833333, and the decimal of five parts of a dozen to be .0347222, and these two numbers added together, make .6220555, which is the number which representeth 7 dozen, and $\frac{5}{12}$ parts of a dozen.

In the setting down of Decimal Fractions, to add them together, you must alwayes observe to set *Primes* under *Primes*, *Seconds* under *Seconds*, &c. which the points before the several Fractions will direct you to do.

Hitherto we have shewed the use of the foregoing Tables in expressing of Fractions in decimal numbers. It resteth now to shew the use of them in finding what Fraction either of Money, Weight or Measure, any decimal number given doth represent, and that shall be made evident by the ensuing Proposition.

PROP.

PROP. X.

A Decimal number being given, how to find what Fraction it doth repreſent.

Let .02916667 be a decimal number, repreſenting ſome Fraction-part of *Engliſh Coin*: Becauſe it is required to find the value of this Fraction in *Engliſh Coin*, you muſt therefore repair to the Table of *Engliſh Coin*; in the ſecond Column of which Table ſeek for the number given (*viz.* 02916667) which you ſhall find to ſtand againſt 7 pence, and ſo much is the value of the decimal Fraction 02916667, in Engliſh Coin.

Alſo if the decimal Fraction .75 were given, you ſhall find the value thereof to be 15 ſhillings, and the value of .003125 to be three farthings.

Likewiſe in the Table of *Troy weight*, if .41666667 were given, it would ſignifie five ounces, and .05416667 would expreſs 13 peny weight, and .00173611 will expreſs ten grains, &c.

After this manner may you find the value of any decimal number given, either in *Money*, *Weight* or *Meaſure*, when the number given may be exactly found in the Table: But if the number given cannot be found exactly in the Table unto which it is directed at one entrance. Then you muſt, *Find in the ſame Table, the neareſt number you can, leſs than the given number, and take the number that anſwers unto it in the firſt Column, which will be the greateſt Fraction of the number required; then ſubſtracting the decimal thus found, out of the decimal given, you ſhall have a remainder, which remainder ſeek alſo in the ſecond Column of the Table, if it may be found, if not, ſeek the neareſt leſs, and the number anſwering thereunto in the firſt Column ſhall be the next greateſt Fraction;*

then ſubſtracting this decimal found out of the former remainder, there will be another remainder, which alſo ſeek in the Table, and proceed as in the former: An Example or two will make all plain.

Example. 1.

Let .68125000 be a Decimal given, repreſenting ſome part of *Engliſh Coin.* If you look in the Table of Engliſh Coin for .68126000, you cannot find it, but the neareſt number in the Table leſs than it, is .65, againſt which I find .13 *s.* ſo that 13 *s.* is the greateſt fraction-part of *Engliſh Coin* agreeing to this number.

This done, ſubſtract .65 out of .68125000, and there will remain .03125000, which number alſo you muſt ſeek in the Table of *Engliſh Coin*, but being you cannot find it there, you muſt take the neareſt number leſs than it, which is .02916667, againſt which I find 7 pence, which is the next greateſt Fraction-part of *Engliſh Coin* agreeing to this number.

Again, ſubſtract .02916667, out of .03125000, and there will remain .00208333, which number ſeek in the Table, and you ſhall find it to ſtand againſt 2 farthings, and ſo much doth this laſt remainder ſignifie in *Engliſh Coin*, and the whole given number .68125000 doth repreſent in *Engliſh Coin* thirteen ſhillings ſeven pence two farthings, as by the operation following doth appear.

.68125000

.68125000 number given.
.65..... the next leſſer number in the Table repreſenting 13 *s.*

.03125000 firſt remainder.
.02916667 the next leſſer number in the Table, repreſenting 7 *d.*

.00208333 ſecond remainder, which repreſents two farthings.

So doth the whole number repreſent 13 *s.* 7 *d.* 2 *q.*

Example 2.

Let the Decimal .87426934 repreſenting ſome fraction of a pound ſterling be given. If you look in the Table of *Engliſh Coin* for .87426934 you cannot find it; but the neareſt number in the Table leſs than it, is .85, againſt which I find 17 ſhillings, ſo that 17 ſhillings is the greateſt fraction-part of *Engliſh Coin*, agreeing to this number.

Then ſubſtracting .85 out of .87426934 there will remain .02426934, which number alſo you muſt ſeek in the Table of *Engliſh Coin*, but ſeeing you cannot find it there, you muſt take the neareſt number leſs than it, which is .02083333, againſt which I find five pence, which is the next greateſt Fraction-part of *Engliſh Coin*.

Laſtly, ſubſtract .02083333, out of .02426934, and there will remain .00343601, which number you muſt alſo ſeek in the Table of *Engliſh Coin*; but not finding it exactly there, you muſt take the neareſt number leſs, which is .003125, againſt which you ſhall find 3 farthings, which is the next greateſt fraction-part of *Engliſh Coin*, and the Decimal .87426934, doth in value ſignifie 17 ſhillings 5 pence 3 farthings, and ſomething

more, for .003125 is the decimal of 3 farthings; and the number you are to look for in the Table is .00343601, greater than the decimal of 3 farthings; wherefore, if you ſubſtract .003125 out of .00343601, there will remain .31101, which is the $\frac{31101}{100000}$ part of a farthing, which is inconſiderable. See the following operation.

87426934 Decimal given.
85..... Decimal of——————17 *s.*

92426934 Firſt remainder.
02083333 Decimal of——————5 *d.*

00343600 Second remainder.
003125.. Decimal of——————3 *q.*

00031101 Decimal part of a Farthing.

¶ And here note, that whatſoever hath been here ſaid concerning the uſes of the Table of Engliſh Coin, the ſame order is to be obſerved in the uſe of the other Tables of *Weight*, *Meaſure*, *Time*, &c. as by the following Examples (if you make trial) will appear.

Examples.

1 If this decimal 48975694, were given to know the value thereof in *Troy weight*, you ſhall find it to contain 15 ounces, 17 peny weights, and 13 grains.

2 Alſo if .95075335 were a Decimal given, and it were required to find the value thereof in *Averdupois great weight*, you ſhall find it to contain 3 quarters of a hundred, 22 pound, 7 ounces, and 3 quarters of an ounce.

3 Like-

3 Likewiſe, if .70800781 were a decimal Fraction given, you ſhall find the value thereof in *Averdupois little weight* to be 11 ounces, 5 drams, and one quarter of a dram.

4 If .4375 were a Decimal, whoſe value were required in *Liquid Meaſure*, you ſhall find it to contain 3 pints and an half.

5 Let .7148438 be a Decimal given, whoſe value is required in *Dry Meaſures*, you ſhall find it to contain 5 buſhels, 2 pecks, 3 quarters of a peck, and 2 pints.

Thus have I ſhewed you the uſe of theſe decimal Tables in expreſſing of the fraction-parts of *Money*, *Weight*, *Meaſure*, &c. But becauſe theſe Tables may not be alwayes at hand, when there is need of them, I will here ſhew you how the value of any decimal given may be known by multiplycation onely; and this is

THE RULE.

Multiply the Decimal *given, by the number of known parts of the next inferiour* Denomination, *which are equal to the* Integer, *the* Product *is the value of the* Decimal *propoſed in that inferiour* Denomination; *and if there happen to be any* Decimal *in the* Product, *you may in like manner find the value thereof in the next inferiour* Denomination, *and ſo proceed till you come to the leaſt known parts of the Integer.*

Example.

Let .67395834 be a Decimal given, repreſenting the fraction of a *Pound ſterling*. Firſt multiply .67395834 by 20, (the number of ſhillings in a pound ſterling) and the product will be 13479166680, from which cutting off the laſt eight figures with a point, or daſh

of

of the pen) becauſe there were eight figures in the given Fraction) there will ſtand before the point (towards the left hand) 13, which are ſhillings, and the remainder .47916680 ſtanding behind the point, will be the fraction-part of one ſhilling ſterling, which number .47916680, you muſt multiply by 12 (the number of pence in one ſhilling) and the Product will be 575000160, from which number cut off the laſt eight figures as before, and there will be 5 left to the left hand, which are 5 pence, and the figures on the right hand of the point, *viz.* .75000160 are the fraction-part of one peny ſterling, which therefore multiply by 4 (the number of farthings in one peny) and the Product of that multiplication will be 300000640, from which cut off the laſt eight figures to the right hand, and there will be left 3 towards the left hand, which repreſenteth 3 farthings, and the remaining figures towards the right hand are but the fraction-part of a farthing, which we therefore reject. And thus you find by *Multiplication* only, that this fraction .67395834 doth repreſent in the known parts of *Engliſh Coin*, 13 ſhillings, 5 pence, 3 farthings, as by the following operation appeareth.

	.67395834
	20
Shillings	13,47916680
	12
	95833360
	47916680
Pence	5,75000160
	4
Farthings	3,00000640

In

In like manner, if this fraction .94809028 were given, repreſenting ſome fraction-part of Troy weight, you ſhall find the value thereof to be 11 ounces, 7 peny weight, 13 grains, as by the operation following appeareth.

	.94809028
	12
	189618056
	94809828
Ounces.	11,37708336
	20
Peny-weight.	7,54166720
	24
	216666880
	208333440
Grains.	13|00001280

In this manner may any Decimal given be reduced into the known parts of the Integer by *Multiplication* only. And

¶ Here note, that whereas in the preceeding Tables the Decimal fractions conſiſt of *ſeven* or *eight* Figures, we ſhall in the proſecution of our work make uſe only of *four* or *five* of the firſt of them, which will be ſufficient in ordinary practice, and come near enough to the truth in any ordinary queſtion whatſoever.

So if in ſtead of .02916667, which is the fraction-part of 7 pence, you take out only .02916, it will be ſufficient.

Alſo

Also for { .05833333, .0058594, .5833333 } take { .05833 In *Troy Weight.*, .0058 In *Dry Measure.*, .5833 In *Time.* }

Thus much concerning the construction and use of the decimal *Tables*, we shall now come to the practice of *Decimal Arithmetick*, which shall be taught in the Rules following.

Of Notation of Decimals.

NOTATION of Decimals is contrary to that of whole numbers: for whereas in whole numbers the values of figures are increased ten-fold by continual addition of Cyphers towards the right hand: so, on the contrary, the values of the places of Decimals do decrease in the same proportion.

And whereas in whole number, Cyphers in the first place towards the left hand are unnecessary, yet in Decimals, they are absolutely necessary to discover the true denominator. Also Cyphers at the end (or towards the right hand) of decimal numbers are of no value, for one single Figure in decimals signifies as much as the same Figure would do, if there were Cyphers placed behind it, so 7 is equivalent unto 70, 700, or 7000, &c. For the denominators of decimal Fractions are alwayes Cyphers with a Unite towards the left hand, as hath been already intimated. So $\frac{70}{100}$ being reduced to its least terms will be $\frac{7}{10}$, and $\frac{7000}{10000}$ will be reduced to $\frac{7}{10}$ also, and so of any other, as by the Table following doth evidently appear.

987654321 | 12345678

100000000	.00000001
10000000	.0000001
1000000	.000001
100000	.00001
10000	.0001
a thouſand 1000	.001 or $\frac{1}{1000}$
a hundred 100	.01 or $\frac{1}{100}$
Ten 10	.1 or $\frac{1}{10}$

Addition of DECIMALS.

IN Addition of Decimals, the ſame order is to be obſerved as in Addition of numbers of one Denomination before taught in the firſt part, in which there is no difficulty: But in Decimal numbers the chief care to be taken is in placing your whole numbers and Fractions in their due order, which you ſhall eaſily and certainly do, if you obſerve this general Rule, *viz.* to *place your whole numbers and Fractions one under another, ſo that the points of ſeparation which* (in decimal numbers) *diſtinguiſh the whole numbers from the Fractions, ſtand directly one under the other*, then are you to proceed in the addition of them in all reſpects, as you did in whole numbers.

Example 1.

Let it be required to add together in one ſum theſe ſeveral ſums following, in a decimal way, *viz.* 36 *li.* 2 *s.* 8 *d.* 29 *li.* 0 *s.* 2 *d.* 31 *li.* 16 *s.* 9 *d.* and 6 *li.* 2 *s.* 5 *d.*

Firſt, ſet down 36 *li.* and a point or Comma after it, then

then for the fraction-part of 2 *s.* 8 *d.* look in your Table of Engliſh Coin, where you ſhall find the decimal fraction of 2 *s.* 8 *d.* to be, .1333 therefore for 36 *li.* 2 *s.* 8 *d.* ſet down 36. 1333.

Secondly, for your 29 *li.* 0 *s.* 2 *d.* ſet down 29. 0083.

Thirdly, for your 31 *li.* 16 *s.* 9 *d.* ſet down 31. 8375.

Laſtly, for your 6 *li.* 2 *s.* 5 *d.* ſet down 6. 1208 as you ſee done in the operation following.

	li.	*s.*	*d.*		
	36	02	8		36, 1333
	29	08	2	ſet down	29, 0083
For	31	16	9		31, 8375
	6	02	5		6, 1208
	103	02	0		103, 0999

Your decimal numbers being thus placed in due order one under another, proceed to the adding of them together, as if they were whole numbers, and you ſhall find the ſum or total of them to be 103. 0999.

Now the 103 which ſtands towards the left hand, are 103, pounds, and the .0999 which ſtands towards the right hand of the Comma, is the fraction-part of one pound ſterling, the value whereof you may find (by the Propoſition before going) to be two ſhillings *fere*, which ſhould be two ſhillings exact, but it wanteth ſomewhat, *viz.* the $\frac{1}{1000}$ part of a farthing, which is inſenſible; for if by the afore mentioned rule you ſeek the value of the decimal fraction, .0999, you ſhall find it to be 1 ſhilling, 11 pence, 3 farthings, and the $\frac{2}{1000}$ part of a farthing, which you may call in all 2 ſhillings, for decimal numbers will ſeldom happen to give the exact value of fractions, but will be either greater or leſ-

ſer

ſer than they ought to be; but in ſuch a ſum as this is, the thouſandth part of a farthing is not to be regarded.

Example 2.

Let it be required to add together in a decimal way theſe ſums following, *viz.* 29 *li.* 18 *ſ.* 7 *d.* 3 *q.* 63 *li.* 11 *ſ.* 2 *d.* 1 *q.* 129 *li.* 4 *ſ.* 0 *d.* 2 *q.* and 3 *li.* 7 *ſ.* 10 *d.* 1 *q.*

Firſt, for 29 *li.* 18 *ſ.* 7 *d.* 3 *q.* ſet down 29. 93229.

Thirdly, for 129 *li.* 4 *ſ.* 2 *q.* ſet down 129. 20208.

Laſtly, for 3 *li.* 7 *ſ.* 10 *d.* 1 *q.* ſet down 3. 39271 as you ſee here down in the Margine.

Your decimal numbers thus placed in order, add them together, as if they were whole numbers, and you ſhall find the ſum of them to contain 226. c8645.	29.93229 63.55937 129.20208 3. 39371
Now the 226 which ſtands towards the left hand of the Comma,	226.08645

are 226 pounds, and the other figures towards the right hand, *viz.* 08645 are the fraction-parts of a pound ſterling, which if you reduce by the fore-mentioned Propoſition, you ſhall find the value thereof to be 1 ſhilling, 8 pence, 3 farthings, ſo the whole ſum is 226 *li.* 1 *ſ.* 8 *d.* 3 *q.*

And here note, that what hath been ſaid, as concerning *Money*, the ſame is alſo to be underſtood of *Weight*, *Meaſure*, *Time*, &c. as by the following Examples will appear.

Other

Other Examples for Practice.

Example 1.

In Money.

135.8833
95.5583
3.2875

234.7291
234 *li.* 14 *s.* 7 *d.*

Example 2.

In Troy-weight.

7.97413
6.65330
3.62187

18.24930
18 *li.* 2 *on.* 19 *p.* *w.* 20 *gr.*

Example 3.

In *Averdupois little w.*

12.7227
76.3594
32.625
91.4883
32.8398

246.0398
246 *li.* 00 *oun.* 9 *dr.*

Example 4.

In *Averdup. great weight.*

37.9442
9.3053
33.6786
10.0000
12.8142

103.7423
103 *C.* 2 *q.* 27 *lb.* 3 *oun.*

Substraction of Decimals.

THE Substraction of Decimals differeth nothing from the Substracting of one whole number from another, and the decimal numbers to be substracted one from another, must be placed in the same order, as in *Addition* of Decimal numbers, the practice of Substraction

ſtraction ſhall be ſeen in the following Examples.

Example. 1.

Let it be required to ſubſtract 31 *li.* 16 *ſ.* 9 *d.* out of 36 *li.* 2 *ſ.* 8 *d.*

Firſt, for your 36 *li.* 2 *ſ.* 8 *d.* ſet down the decimal thereof, which is 36. 1333.

Secondly, for your 31 *li.* 16 *ſ.* 9 *d.* ſet down the Decimal thereof 31, 8375.

This done draw a line under them, and ſubſtracting the leſſer from the greater, you ſhall find the remainder to be 4.2958 the 4 on the left ſide of the Comma are four pounds, and the .2958 which ſtandeth towards the right hand, is the fraction-part of a pound, the value whereof being ſought, will be found to be 5 *ſ.* 11 pence. So that if you ſubſtract 31 *li.* 16 *ſ.* 9 *d.* out of 36 *li.* 2 *ſ.* 8 *d.* there will remain 4 *li.* 5 *ſ.* 11 *d.*

36,	1333
21,	8374
4,	2958

But if divers ſums be to be ſubſtracted out of one greater ſum, then you muſt firſt add all the ſeveral ſmaller ſums together, and ſubſtract the ſum of them from the greater given ſum, ſo ſhall the reſidue be the ſum deſired.

Examples for Practice.

Example 1.
In Money.

Lent	2784. 8375
Paid at several times.	36. 1333 29. 0083 31. 8375 6. 1208
paid in all	103. 0999
rests to pay	2581. 7376 2581 *li.* 14 *s.* 0 *d.*

Example 2.
In *Averdupois* great weight.

Bought	103. 7423
Sold	37. 9442
Unsold	65. 7981

65 *C.* 3 *q.* 5 *l.* 7 *ou.*

Example 3.
In Troy weight.

Delivered to a Goldsmith of old Plate	7. 97413
Received of new Plate	5. 59670
Rests in the Goldsmiths hands	2. 7743

2 *li.* 4 *oun.* 10 *p. w.* 14 *gr.*

Multiplication of Decimals.

MULTIPLICATION *of Decimals* differeth nothing at all from the *Multiplication* of whole numbers, for making the greater number the *Multiplicand*, and the lesser number the *Multiplier*, the number issuing from that Multiplication shall be called the *Product*.

Now in the Multiplication of decimal numbers one by another, if there be any Fraction either in the *Multiplicand* or *Multiplier*, or Fractions in both: So many figures

figures as the Fractions contain, ſo many figures muſt be cut off from the *Product* towards the right hand, which ſhall be the Fraction of the *Product*, and the figures towards the left hand of the Comma in the Product, ſhall be the Integers of the Product.

Example 1.

Let it be required to multiply 34 pounds, five ſhillings, three pence, by 16 pounds, ſix ſhillings, ſix pence.

Firſt, ſeek the Decimal of 34 *li.* 5 *ſ.* 3 *d.* which you ſhall find to be 34. 2625, make this your Multiplicand, then ſeek the Decimal of 16 *li.* 6 *ſ.* 6 *d.* which you ſhall find to be 16. 325, make this decimal number your Multiplier; then draw a line, and Multiply theſe two numbers together, as if they were whole numbers, and you ſhall find the Product of them to be 559. 3353125. Now becauſe there are four figures in the Multiplicand which are Fractions, namely, theſe four towards the right hand, *viz.* 2625, and there are alſo three figures in the multiplier, which are Fractions, namely, theſe three towards the right hand, *viz.* 325, that is in all ſeven figures, repreſenting Fractions, I therefore cut off from the product the ſeven figures towards the right hand, by making of a Comma there, to diſtinguiſh the whole number from the fraction: So is 559 the integer or whole number, and .3353125, the Fraction of this Multiplication.

Multiplicand	34. 2625
Multiplier	16. 325
	1713125
	685250
	1027875
	2055750
	342625
Product	559. 3353125

Example 2.

If there be Fractions in the multiplicand, and none in the multiplier, yet the work is ſtill the ſame, for you muſt cut off only ſo many figures from the product, as there are Fractions either in multiplicand, multiplier, or both: So if it were required to multiply 5767 yards, and 3 quarters of a yard, by 235 yards, you muſt firſt ſet down 5767. 75 for your 5767 yards, and three quarters, which number muſt be your multiplicand: And alſo ſet down 235 yards for your multiplier, then multiplying them together, as if they were whole numbers you ſhall find the product to be 1355421. 25, and becauſe there are only two Fraction figures, both which are in the multiplicand, namely, the two laſt thereof .75, and none in the multiplier. I therefore cut off only two figures of the product, namely, the two laſt, which are .25, ſo is the product of this multiplication 1555421. 25 which is 1355421 ſquare yards, and one quarter of a yard. And ſo if a Garden or other piece of Land, lying ſquare, ſhould contain in length 5767 yards, and three quarters, and in breadth 235 yards, the whole piece would contain 1355421 ſquare yards, and one quarter of a yard.

	5767,75
	235
	2883875
	1730325
	1153550
	1355421.25

Example 3.

If decimal Fractions be to be multiplied by decimal Fractions, you muſt then (as before) multiply them as whole numbers, and from the product cut off ſo many Figures towards the right hand, as there are Figures in

in the multiplicand and the multiplier: So if it were required to multiply .953 by .782, you ſhall find their product to be .745246, which being but ſix figures in all, I cut them off and that fraction .745246 is the product of the multiplication of the two given Fractions.

	.953
	.782
	1906
	624
	6671
	.745246

Example 4.

If any two Decimal fractions being multiplied together, the product thereof doth not conſiſt of ſo many places as are required (by the former rules) to be cut off, you muſt then ſupply that defect by prefixing a Cypher, or Cyphers before the product towards the left hand: So if theſe Decimal fractions .063 and .0752 were to be multiplied, their product would be 47376. Now (by the former rules) you ſhould cut off ſeven figures of the product towards the right hand, but this product 47376 conſiſteth but of five figures; wherefore to make it ſeven figures, I prefix two Cyphers before the product on the left hand, making it .0047376, and that is the true product produced by this multiplication.

	.0752
	.063
	2256
	4512
	.0047376

Example 3.

If you would multiply any Decimal (either Fraction only, or whole number and fraction together) by 10, 100, 1000, *&c.* You muſt add ſo many Cyphers

phers to the multiplicand, as there are Cyphers in the multiplyer, and cut off ſo many Figures as there are fractions in the multiplicand, and that number ſhall be the product required: So if 7,856025 were a Decimal given to be multiplied by 100, add two Cyphers to the number given, making it 785602500, then becauſe there were ſix figures of this number towards the right hand, it will be 785,602500, which is the true product required.

Examples for Practice.

Example 1.

7,432
2,61

7432
44592
14864

1939752

Example 2.

22,358
32

44716
67074

715.456

Example 3.

·352
·24

1408
604

·7448

Example 4.

375.6218
100

375.621800

Diviſion

Diviſion of Decimls.

AS *Diviſion* of whole numbers is the hardeſt of the four *Species* of *Vulgar Arithmetick*, ſo the *Diviſion* of *Decimals* is the moſt difficult of the four kinds of *Decimal Arithmetick*, but I hope to make it plain, to the underſtanding of the meaneſt capacity.

The ſeveral varieties that may happen in Diviſion, are principally (if not only theſe) *four*. Namely, Firſt, *To divide whole numbers and fractions.* Secondly, *To divide whole numbers by mixt, or mixt numbers by whole.* Thirdly, *To divide a greater fraction by a leſs*; and Laſtly, *To divide a leſſer fraction by a greater.*

In Diviſion of Decimals this Rule is general, *If the* Dividend *be greater than the* Diviſor, *the* Quotient *will be either a* whole number *or* a mixt, *but if the* Dividend *be leſs than the* Diviſor, *the* Quotient *will be* a Decimal. And (for convenience in working, if there be need) any number of Cyphers may be annexed to the Dividend, that thereby the Quotient may extend to as many places as the tenour of the queſtion ſhall require.

The manner of the working of *Diviſion* in Decimals, is the ſame with that before delivered in whole numbers in the firſt part of Vulgar Arithmetick, as will appear by the Examples following, in every of the four premiſed varieties.

The Rule for the firſt variety.

The Dividend *and the* Diviſor, *being both* mixt numbers, *or one of them being a* whole number *and the other*

a mixt; *or the* Dividend *being a* Decimal, *and the* Diviſor *a* whole number *or* a mixt, *the firſt figure in the quotient will be of the ſame* place *or* degree, *with that Figure or Cypher of the* Dividend, *which at the firſt demand ſtandeth, or (at leaſt) is ſuppoſed to ſtand directly over the place of* Unites *in the* Diviſor.

Example 1. *Where the terms given are both mixt numbers.*

Let it be required to divide 659. 3354125 by 16. 325. Here the terms given are both of mixt numbers, which being placed according to the Rules delivered before, for the Diviſion of whole numbers, the figure in the Dividend, which at the firſt demand, ſtandeth over 6, the place of Unites in the Diviſor is 5, and becauſe this ſtandeth in the place of tenths, therefore the firſt figure in the quotient is in the place of tenths alſo, and the whole number conſiſteth of two of the foremoſt places, and the reſt is a Decimal, thus the quotient ſought in our preſent example is 34. 2625, of which 34 the two firſt figures is the Integer or whole number, 2625 the Decimal fraction.

Diviſor

Diviſor	*Dividend*	*Quotient.*
16.325)	559.3353125	(34.2625
	.....	
	489.75	
	69.585	
	65.300	
	4.2853	
	3.2650	
	1.02031	
	97950	
	040812	
	32650	
	c81625	
	81625	

Example 2. *One of the terms given, being a whole number, the other mixt.*

The mixt number 1375421.26 being divided by the whole number 235, the quotient will be 5767.75, and the firſt figure in the place of Thouſands, as by the operation doth appear.

Diviſor

```
26)    .35673    (001372
        ...
       ------
        26
        096
       ------
         78
         187
       ------
         182
         0053
       ------
           52
```

The Rule for the second variety.

When the Dividend *is a* whole *or* mixt *number, and the* Divisor *a* Decimal, *add as many Cyphers to the* Dividend *as there are places in the* Divisor; *for the integral part of the quotient will consist of as many places as the* Divisor, *and the places arising from the integral parts of the* Dividend *added together.*

Example. 1.

Let 348.75 be the mixt number given, to be divided by the decimal .25, to the number given, I add two Cyphers, the number of places in the divisor, and then it will be 348.7500, which being divided by .25; the integral part of the quotient will be 1395. because the whole part of the dividend 348, being divided by .25 giveth two places, and the number of places in the Divisor being two, giveth two more; and so the Integral part consisteth of four figures, as by the operation.

Divisor,

Diviſor	*Dividend*	*Quotient.*
235)	135542125	(5767·75
	.....	
	1175	
	1804	
	1645	
	1592	
	1410	
	1821	
	1645	
	1762	
	1645	
	1275	
	1175	

Example 3. *The Dividend being a Decimal, and Diviſor a whole number.*

The Decimal fraction .35673 being divided by the whole number 26, the quotient will be .002372, and the firſt ſignificant figure in the place of thouſands, or fourth place from Unity, as by the operation it doth appear.

Diviſor	*Dividend*
.25)	348.7500 (1395
	...
	25
	98
	75
	237
	225
	125
	125

Example 2.

Let the mixt number 72. 5 be divided by .075, the number of places in the integral part of this Quotient will be 966, becauſe there are 3 places in the Diviſor; and but 3, becauſe the integral part of the Dividend is leſs than the ſignificant figures in the Diviſor, as by the operation it doth appear

Diviſor	*Dividend*
075)	72.5000 (966
	675
	500
	450
	500
	450
	500
	450

The

The Rule for the third Variety.

When the Terms given are both Decimals, *the* Dividend *being the greater, the integral part of the quotients will conſiſt of as many places as the* Diviſor *doth.*

Example.

Let the Decimal .73658 be divided by the Decimal .32 the integral part of the Quotient will be 23, becauſe the Diviſor doth conſiſt of two places, as by the operation in the Margine doth appear.

Diviſor	*Dividend.*
.32)	.73958 (23.11
	....
	64
	99
	96
	35
	32
	38
	32
	6

The Rule for the fourth variety.

When the terms given are both Decimals, *conſiſting of equal places, the* Dividend *being the* leſſer term, *place the* Dividend *as a* Numerator, *and the* Diviſor *as* Denominator; *ſo is ſuch vulgar fraction the quotient ſought: But if the terms given conſiſt not of equal places, ſupply the place or places wanting in either of the terms, by annexing a Cypher*

a Cypher or Cyphers on the right hand, and then proceed as before. Thus if .27 be given to be divided by .93, the quotient will be $\frac{27}{93}$. Also if .35 be given to be divided by .78563, the quotient by annexing 3 Cyphers to .35, the leſſer decimal given, will be $\frac{350000}{78563}$, which vulgar fractions may be reduced into decimals if need be, by the firſt Propoſition in this Second part of decimal Arithmetick.

Examples for Practice.

44) .35673 (.0081,&c.

352
47

44
3

.25) 2481.00 (9924

225
0231

225
60

50
100

100
000

Having given you Examples of the four foregoing Rules in the ſeveral caſes of Diviſion in Decimals, according to the third way of Diviſion taught in the Firſt Part: I will now bring all the forementioned four Rules into one general Rule, and give you Examples of all the Varieties, that can poſſibly ariſe in Decimal Diviſion, ready wrought according to the firſt (or moſt common) way of Diviſion, taught in the Firſt Part.

The

The general Rule is this.

When the Dividend and Divisor are placed orderly one under another (as in the First Part is directed) Observe this general Rule.

When the place of Primes in the Divisor, of any Decimal Fraction, comes to be under the place of Primes in the Dividend; The demand that is then made shall be the last figure of the whole number (or Integer) that is to be placed in the Quotient: all after it being Primes, Seconds, Thirds,&c.

Examples of all Varieties

Example 1.

To divide a mixt number by a mixt number, as 172.5 by 3.746.

```
      13 (4
      2675
    4 4848(4
   22 62610
   54 96446(6
  172. 500000 (46.049
   3.7 466666
    3.74444
     3.777
      33
```

Example

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50
100
100
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   22 62610
   54 96446(6
  172. 500000 (46.049
   3.7 466666
    3.74444
      3.777
        33
```

Example

Example 2.

Of a mixt Number by a mixt Number; the Divisor being greater than the Dividend. As 2. 34 by 52. 125.

```
      (1
       3(0
       81
      492(8
      6509
      47621
      55000(7
     2662242(5
  2.34000000 (.04489
     52.125555
      52.1222
       52.11
        52.
```

Example 3.

To divide a Whole Number by a Decimal Fraction, As 82, by .056.

```
       3   (2
      26 325
      36 650(2
      82.0000 (1465.3
    .056 6666
       5 555
```

Exam-

Example 4.

To divide a Decimal Fraction by a Decimal Fraction, As .0125 by .5.

.0x z y (.025
0.y.y

Example 5.

A Decimal Fraction by a Decimal Fraction, As .8564 by .008.

000.8 y 6 4 8 (107.05
.00 8 8 8 8 8

Example 6.

A Decimal Fraction by a Decimal Fraction. As .73952 by .32,

x y z z
0.7 3 9 5 z (2.311
0.3 z z z z
3 3 3

Example 7.

A Mixt Number by a Whole Number. As 32.959 by 27.

x x
x y y (1
3 x . 9 y (9 (1.220
x 7 . 7 7 7
x x x

Example 8.

A Mixt Number by a Whole Number. As 36.7-875 by 243.

x
z
6
x3 7z
z5 xox
63.7875 (0.2625
z4 3333
z 444
zz

The

The Rule of Three in Fractions *Vulgar* and *Decimal.*

WHat the Rule of Three is, and the manner of working, is already ſhewed in the firſt part, that which we here intend is only to add ſome Examples in Fractions Vulgar, as well as Decimal; that by comparing the work in both, the Excellent uſe of Decimal Arithmetick might the better appear.

And how to convert the known parts of *Money*, *Weight*, or *Meaſures Engliſh*, into *Decimals* hath been already ſhewed, both *Arithmetically* and by *Tables*; yet to prevent the ſeveral *additions* and *ſubſtractions* in thoſe *Tables*, I have here annexed another *Decimal Table*, for the more ſpeedy Reduction of *Engliſh money* under two ſhillings, all ſums of money above, not having pence or farthings annexed, being as eaſily reduced by *memory* as by *Tables*; and this I have the rather done, becauſe the ſame *Table* will alſo reduce the *Coins* of *France*, and the parts of *Troy-weight*, if an ounce be made the Integer, which in point of practice is much more uſeful than the pound.

The Table of REDUCTION.

Penny							
	001042				026014		
	002083	Gr. 1			027083	13	
	003125				028125		
1	004166	2		7	029166	14	
	005208				030208		
	006250	3			031250	15	
	007291				032291		
2	008333	4		8	033333	16	
	009375				034375		
	010416	5			035416	17	
	011458				036458		
3	012500	6		9	037500	18	
	013541				038541		
	014583	7			039583	19	
	015625				040625		
4	016666	8		10	041666	20	
	017708				042708		
	018750	9			043750	21	
	019791				044791		
5	020833	10		11	045833	22	
	021874				046875		
	022916	11			047916	23	
	023958				048958		
6	025000	12		12	050000	24	

The Table of REDUCTION.

	051042				075014		
	052083	G. 1			077083	13	
s. d.	053125				078125		
1. 1	054166	2		7	079166	14	
	055208				080208		
	056250	3			081250	15	
	057292				082921		
2	058333	4		8	083333	16	
	059375				084375		
	060416	5			085416	17	
	061458				086458		
3	062500	6		9	087500	18	
	063542				088541		
	064583	7			089583	19	
	065625				090625		
4	066666	8		10	091666	20	
	067708				092708		
	068750	9			093750	21	
	069792				094791		
5	070833	10		11	095833	22	
	071874				098675		
	072916	11			099716	23	
	073958				099858		
6	075000	12		12	000000	24	

These things premiſed, we will now ſhew the uſe of the Table in ſome practical queſtions belonging to the Rule of Three direct.

1 *Queſtion.*

If $\frac{7}{8}$ of a yard of Cloth, coſt $\frac{9}{12}$ of a pound: what ſhall 17 yards coſt at the ſame rate?

If $\frac{7}{8}$ coſt $\frac{9}{12}$, what ſhall 17 coſt? Anſ. 14 *li.* $\frac{4}{7}$.

Firſt, multiply $\frac{9}{12}$ by $\frac{7}{8}$ the product is $\frac{63}{96}$, then divide $\frac{63}{96}$ by $\frac{7}{8}$, the quotient is $\frac{1224}{84}$: again, if you divide 1224 by 84, the quotient is $14\frac{8}{84}$, or in the leaſt terms 14 pound $\frac{4}{7}$ of a pound.

And the value of this fraction $\frac{4}{7}$ of a pound, will be found by the third Rule of Reduction of Fractions to be 11 ſhillings 5 pence, and $\frac{4}{7}$ of a peny, which is ſomewhat above two farthings: for it is 2 farthings, and $\frac{3}{7}$ of a farthing.

The ſame Queſtion in Decimals.

If $\frac{7}{8}$ of a yard of Cloth coſt $\frac{9}{12}$ of a pound, what ſhall 17 yards coſt at the ſame rate?

To anſwer this queſtion $\frac{7}{8}$ of a yard, and $\frac{9}{12}$ of a pound muſt firſt be reduced into decimals, either by Diviſion, or by the Tables of Reduction: by both which wayes of Reduction the decimal of $\frac{7}{8}$ will be .875, and the decimals of $\frac{9}{12}$ will be .75, and then the terms of the queſtion will ſtand thus;

If .875 parts of a yard coſt .75 parts of a pound, what ſhall 17 yards coſt at the ſame rate?

If 0.875——0.75——17. Here if you multiply the ſecond term 0.75 by 17 the third term given, the product will be 12.75. and this product divided by .875, gives in the quotient 14.57142, that is, 14 pound .57142 parts of a pound, or 145 Decades, that is 14 pound

pound 10 ſhillings, and .7142 parts of a Decade (or two ſhillings) which by the preceeding Tables is 1 *s.* 5 *d.* 2 farthings, and .0059 parts of a farthing.

2 *Queſtion.*

If a piece of Gold plate weighing 19 ounces 3 peny weight and 5 grains, be worth 62 pounds 10 ſhillings 6 pence; what is one ounce of the ſame Gold worth?

This queſtion in vulgar fractions muſt be expreſſed thus.

If 1 *l.* $\frac{3417}{5760}$ Troy-weight, coſt 62 *l.* $\frac{126}{240}$, what ſhall $\frac{1}{12}$ of a pound Troy coſt at the ſame rate?

To anſwer this queſtion, the fractions 1 *l.* $\frac{3417}{5760}$ and 62 *l.* $\frac{126}{240}$, muſt be firſt reduced into improper fractions, and the fraction $\frac{1}{12}$ into the leaſt known parts of a pound Troy, and then the queſtion will ſtand thus.

If $\frac{9197}{5760}$ give $\frac{15006}{240}$, what ſhall $\frac{480}{5760}$ give?

Now becauſe it is neceſſary the terms given be reduced into their leaſt Denominations, before the queſtion be reſolved, therefore the anſwer may be found, by uſing the terms given thus reduced as whole numbers, not having any regard to the Denominators of theſe fractions; Saying thus,

If 9197 grains, coſt 15006 pence, what ſhall 480 grains coſt.

And here if you multiply 15006 by 480, the product will be 7202880, which being divided by 9197, the quotient will be 783 pence $\frac{1619}{9197}$ parts of a peny, and dividing 783 by 12, it will be 65 ſhillings 3 pence $\frac{1619}{9197}$, or 3 *l.* 5 *s.* 3 *d.* $\frac{1619}{9197}$. And although this queſtion is thus more eaſily anſwered than it would have been, if the terms had been wrought as vulgar fractions, yet the ſame terms being reduced to Decimals, the anſwer of the queſtion will yet be found with more eaſe, as ſhall appear by the operation following.

The same question in Decimals.

If a piece of Gold plate weighing 19 ounces 3 peny weight and 5 grains, be worth 26 l. 10 s. 6 d. what is an ounce of the same Gold worth?

The Decimal of 19 ounces 3 peny weight and 5 grains, making an ounce the Integer, is by this Table 19. 16041, for that 19 ounces are 19 Integers, 2 peny weight is one tenth of an ounce, and the Decimal of 1 *d. w.* 5 grains is by this Table .06041; and the decimal of 62 *l.* 10 *s.* 6 *d.* by the same Table is 62.525, and because an Unite or Integer is the third term given, there needs no multiplication, if therefore you divide 62. 525 the second term, by 16. 16041 the first term propounded, the quotient will be 3. 2632, that is 3 pounds 5 shilling 3 pence, and somewhat more, as by the operation in the margine it doth appear.

19.16041) 62.52500000 (3.2632
...

57.48123
5.043770

3.832082
1.2116880

1.1496246
6206340

5748123
4582170

3732082
750088

3 Que-

3 *Queſtion,*

If 5 *Ells and a quarter of linnen Cloth coſt* 2 l. 16 s. 8 d. 3q. *what ſhall* 278 *Ells and a half coſt at the ſame rate?*

If you would work this Queſtion by whole numbers, your eaſieſt way is firſt to reduce all the terms into their leaſt Denominations, that is to ſay, the Ells into quarters, and the pounds, ſhillings, pence, and farthings all into farthings, ſo ſhall your 5 Ells and a quarter be 21 quarters, and your 278 Ells and an half will be 1114 quarters, and your 2 *l.* 26 *s.* 8 *d.* 3 *q.* will be 2723 farthings, and then will your queſtion ſtand thus in whole numbers.

quarters *farthings* *quarters.*
If 21—coſt——2723—what will——1114 coſt?

Then multiplying the ſecond number by the third, that is, 2723 by 1114, the product will be 3033422, which divided by 21, the quotient will be 144448 farthings, which being again reduced into pounds, ſhillings, and pence, giveth 150 *l.* 9 *s.* and 4 pence, as by the operation following doth appear.

The

The Operation.

li.	*s.*	*d.*	*q.*	
$5\frac{1}{4}$	2	16	8	3	$278\frac{1}{2}$
21	20				4
	40				1112
	16 *shillings.*				2
	56				1114
	12				
	112				
	56				
	672				
	8 *pence.*				
	680				
	4				
	2720				
	3 *farthings.*				
	2723				
	1114				
	10892				
	2723				
	2723				
	2723				
	3033422				

21)

21) 3033422 (144448

21
93
84
93
84
94
84
102
84
182
168
14

		d.			
z		*z* (4	*x*	*s.*	*d.*
xxxxx	(*xxxxx*	(*xxx*	(9	(150
xxxxx	*xxxxx*	*xxx*	0	
	xxx			

But if you would work the ſame queſtion by Decimal numbers, you may ſave the labour of reducing the terms to their leaſt Denominations, for 5 Ells and a quarter is in decimal numbers 5.25, and 278 Ells and an half is 278.5, and 2 pound 16 *s.* 8 *d.* 3 *q.* is in decimals 2.8364, and then your queſtion in decimals, will ſtand thus:

Ells.

Ells. *pounds.* *Ells.*
If 5.25 coſt 2.836, what 278.51.

If you multiply (according to the Rule) the ſecond term by the third, that is 2.8364 by 278.5, the product of that multiplication will be 789.93400, which divided by the firſt term 5.25, the quotient will be 150.4642, which decimal repreſenteth 150 *l.* 9 *s.* 4 pence, and ſo much in money will 278 Ells and a half coſt.

The Operation.

Ells. *pounds.* *Ells.*
5.25 —— 2.8364 —— 278.5
27.85

14182
226912
198548
56728

789.93740

525)

```
5.25) 789.937400 ( 150.4642
      ......          or
      ----------
      525            ( 150 li. 9 s. 4 d.
      2649
      ----
      2625
       2437
      ----
       2100
        3374
      -----
        3150
         2240
      -----
         2100
          1400
      -----
          1050
           350
```

I have been the larger in this Rule, and especially in this Example, which is incumbred with fractions sufficient, because I would have the Reader the better discern the difference between the *Vulgar* and the *Decimal* way, and also to see how expeditious the one is over the other. Now this example being thus largely explained, I shall with the more brevity pass over the Rules following, giving one Example, or two at the most in each Rule. And thus much shall suffice for the Golden Rule, or Rule of Three direct in Fractions.

The

The Rule of Three Reverse.

A *Lends* B. 233 l. 6 s. 8 d. *for a year without Interest, upon condition that* B. *should do the like courtesie for* A. *when required.* A. *hath occasion for money* 7 *months; how much money ought* B. *to lend* A. *to requite his courtesie, and save himself harmless?*

I will not in this place tell you what the Rule of Three reverse is, nor the manner of working thereof, that being already sufficiently declared in the first part, but give you the Example, and the working thereof, which take as followeth: So will the Question be thus stated:

months	*li.*	*s.*	*d.*	*months.*
12	233	6	8	7

Which in Decimals stands thus,

months	*li.*	*months.*
12	233.33	7

```
   233.33
       12
   ------
    46666
   23333
   ------
  2799.96
   6666(3
  279996 (39999 3/4
   77777
```

Here you see that 12 months and 7 months are whole numbers, and so we let them alone without any Reduction,

Reduction, but the Decimal of 233 *l.* 6 *s.* 8 *d.* will be found by the fore-mentioned Tables and Rules to be 233.33, which is the middle term in the queſtion, and of the ſame quality with that, muſt the fourth term ſought be, therefore if (according to the Rule delivered in the firſt part) you multiply 233.33 by 12, the product will be 2799.96, which divided by 7, giveth in the quotient 39999, which is the Decimal of 400 *l.* and ſo much money ought *B.* to lend *A.* for 7 months.

The Rule of Proportion, conſiſting of five Numbers.

Queſtion 1.

If 100 li. *in* 12 *moneths yields* 6 li. *intereſt, what intereſt ſhall* 264 li. 16 s. 5 d. *yield in* 15 *months at the ſame rate?*

Set down your numbers in Decimals, as in the Example following appeareth, ſo ſhall you find the Decimal of 264 *li.* 16 *s.* 5 *d.* to be 264.8208, all the reſt being whole numbers, having no fractions joyned with them we neglect, and work with them as they are, ſo will the ſeveral numbers of your queſtion (if rightly diſpoſed) ſtand as followeth:

li.

li.	*mo.*	*li.*	*li.*	*mon.*
100	12	6	264.8208	15
	100		6	
	1200		1588.9248	
			15	
			79446240	
			15889248	
		1200)	23833.8720	(19.8615

1200
11833
10800
10338
9600
7387
7200
1872
1200
6720
6000
720

or *li.* 19 *s.* 17 *d.* 2 *q.* 3 *fere.*

Your numbers being thus orderly disposed, you must according to the Rule before delivered in the first Part, multiply the first and second terms together, which in this Example are 100 and 12, whose product

duct is 1200, which is your Diviſor; Then multiply the three laſt terms one into another, as 264, 8208 (which is the Decimal of 264 *li.* 16 *ſ.* 5 *d.*) by 6, and the product thereof will be 1588.9248, which number again multiplied by 15, (which is the laſt term) the product will be 23833. 8720 which is your Dividend, and this number being divided by your former product, giveth in the quotient 19.8615, which is the Decimal of 19 *li.* 17 *ſ.* 2 *d.* 3 *q. fere*, and ſo much doth the ſimple Intereſt of 264 *li.* 16 *ſ.* and 5 *d.* amount unto in 15 months, after the rate of ſix *per centum* for a year.

Queſtion 2.

If the carriage of 23 *hundred and* 3 *quarters of any thing* 127 *miles, coſt* 4 li. 13 s. 6 d. *what ſhall the carriage of* 47 *hundred and an half of ſuch like commodity coſt, being carried* 381 *miles.*

Place your numbers in order as in the following Example doth appear, then multiply the firſt and ſecond terms together for your Diviſor, and the three laſt one into another for your Dividend, and ſo will the quotient of this diviſion anſwer the queſtion demanded, and the work will ſtand as followeth.

C.	*miles*	*li.*	*C.*	*miles*
23.75	127	4675	47.50	381

127 — 47.50

16625 — 23375
4750 — 32725
2375 — 18700

3016.25 — 222.06250
381

22206250
177650000
66618750

3016.25) 84605 81250 (28.050
.....

603250
2426081

2413000
15081250

15081250
00000000

Here you see that the first and second terms multiplied together produced 3016.25, which must be your Divisor, and the three last terms being multiplied one into another, produce 84605.81250, which number divided by 3016.25, giveth in the quotient 28.050, which Decimal representeth 28 *l.* one shilling, and so much will the carriage of 47 hundred and a half cost, being carried 381 miles.

Thus

Thus have I ſhewed the uſe of decimal Arithmetick in ſuch queſtions as concern the *Golden Rule*, or *Rule of Three*; both Direct, Reverſe, and Compounded, by an Example or two in each Rule, and thoſe compounded of Fractions ſufficient, I ſhould now proceed to queſtions in *Fellowſhip*, with and without Time, as alſo *Barter*, *Alligation*, the Extraction of the Square and Cube Roots, &c. but foraſmuch as theſe laſt mentioned Rules depend only upon the *Rule of Three*, as by Examples in the firſt part doth plainly appear, I ſhall therefore ſave that labour, and ſpare my Reader the pains of practiſing queſtions which wholly depend upon that which I (by this time) ſuppoſe him perfect in; Yet if the Reader be deſirous to make tryal of any ſuch queſtion for his own ſatisfaction, he may either make tryal of thoſe queſtions in the former part of this Book in thoſe ſeveral Rules, reducing the numbers there given into Decimals, or if he pleaſe, he may frame queſtions according to his own fancy. And thus I ſhall conclude this Second Part.

The End of the Second Part.

AN APPENDIX, To the *Second Part.*

Section 1.

Of Exchange *of the* Coins, Weights, *and* Measures *of one* Country, *with the* Coins, Weights *and* Measures *of another* Country.

TO perform this work, there is nothing required more than the *Golden Rule*, if first the *Rate* or *Proportion* between the *Coins*, *Weights*, and *Measures* of any two *Countrys* be first known, which is best obtained by experience, rather than taken upon trust, all that in this place I shall do, is only to instruct the ingenious in the manner of Work, and make use of such Rates or Proportions as I find set down by Mr. *Lewis Roberts* Merchant, in his *Map of Commerce*.

Question 1.

How many Riders (each Rider containing 1 l. 1 s. 2 d. 2 q. *Sterling) shall I receive for* 125 l. 6 s. 4 d. 2 q. *Sterling?*

Facit 237 Riders.

l.	*s.*	*d.*	*Rider*	*li.*	*s.*	*d.*
If 1	1	2½	*give* 1	*how many Riders ſhall give?* 251	6	4½

Here if you reduce your numbers to their leaſt Denominations, or ſet them down in Decimals, and multiply and divide according to the Golden Rule, you ſhall find in your quotient 237, and ſo many *Riders* ought to be received for 251 *l.* 6 *s.* 4 *d.* 2 *q.* Sterling.

Queſtion 2.

How many French-Crowns (*each French Crown being valued at* 6 *s. Sterling*) *ſhall I receive for* 492 *l.* 18 *s. Sterling?*

Facit 1643 French Crowns.

s.	*F.C.*	*li.*	*s.*
If 6	*give* 1,	*what ſhall* 492	18 *give.*

Multiply and divide according to the Golden Rule, and you ſhall have in your quotient 1643, and ſo many French Crowns are to be received for 492 *l.* 18 *s.* Sterling.

Queſtion 3.

A Merchant delivered at Paris 1643 *Crowns of* 6 *s. Sterling the piece, how many pounds Sterling ought to be received at* London?

Anſwer 492 *l.* 18 *s. Sterling.*

Crown	*s.*	*Crowns*
If 1	*give* 6,	*what ſhall* 1643 *give?*

Multiply and divide, and you ſhall have in your quotient 492 *l.* 18 *s.* and ſo much Sterling Money ought to be delivered at *London*, for 1643 French Crowns, of 6 *s.* the Crown Sterling.

Queſtion 4

If 3 *yards at* London, *be* 4 *Ells at* Antwerp, *how many yards at* London *make* 84 *Ells at* Antwerp?

Ells Antwerp	*Yards* London	*Ells Ant.*
4	3	84

Facit 63.

And ſo many yards at *London*, are equal to 84 Ells at *Antwerp*.

Queſtion 5.

How many Yards of London *make* 27 *Ells of* Antwerp, *when* 100 *Ells of* Antwerp *make* 60 *Ells of* Lions, *and* 20 *Ells of* Lions *make* 25 *yards of* London?

The firſt Work.

Ells Lions	*Yards* London	*Ells* Lions
20	25	60
	60	
	150|0	
	75 *facit* 75	

That is 75 Yards of *London* is equal to 100 Ells of *Antwerp*.

The

The Second Work.

Ells Antwerp,	*Yards* Lon.	*Ells* Antwerp.
100	75	27
	27	
	525	
	150	
	20|25	*Yards of London facit* 20. 25.

Question 6.

If 100 li. *Sterling be* 104 li. 6 s. 4 d. *Flemish, what is one pound Sterling worth?*

li. Sterl.	*li.*	*s.*	*d.*	*li. Ster.*
100	134	6	4	1
	20			
	2686			
	12			
	5372			
	26864			
	322|36			

Facit 322 *pence* $\frac{36}{100}$ of a peny.

Question 7.

How many Ells of Franckford *make* $42\frac{1}{4}$ *Ells of* Vienna *in* Austria, *when* 35 *Ells of* Vienna *make* 24 *at* Lions, 3 *Ells of* Lions, 5 *Ells of* Antwerp, *and* 100 *Ells of* Antwerp, 125 *Ells at* Franckford.

First Work.

Ells Ant.	*Ells* Franck.	*Ells* Ant.
100	1.25	5
	5	
	625	

Facit 6.25 or 6 Ells and a quarter of *Franckford* equal to 3 Ells of *Lions*.

Second Work.

Ells Lions,	*Ells* Franck.	*Ells* Lions.
3	6.25	24
	24	
	2500	
	1250	
	3)150.00(50	
	15	
	0	

Facit 50 Ells of *Franckford*, equal to 35 Ells of *Vienna*.

Third

Third Work.

Ells Vien.	*Ells* Franck.	*Ells* Vien.
35	50	42.2

35) 2112.50 (60.35

210

125

105

200

175

25

2112,50

Facit 60.35. or $60\frac{7}{20}$ Ells of *Franckford*, equal to 35 Ells of *Vienna*.

Thus have I given you a few Examples of Exchanges, I will now insert some few Tables derived from Mr. *Lewis Roberts* his Map of Commerce afore-said, of the truth of which I am not a competent Judge, but shall leave that to the scrutiny of such as have occasion to trade into Forreign Countries.

A TABLE shewing what one pound of *Averdupois* weight at *London* maketh in divers eminent Cities, and other remarkable places.

One pound of Averdupois *weight at* London *makes at*

	lb.
Antwerp	.9615
Amsterdam	.9
Abbeville	.91
Ancona	1.282
Avignon	1.12
Burdeaux	.91
Burgoyne	.91
Bolonia	1.25
Bridges	.98
Calabria	1.3698
Calais	1 07
Constantinople	.8474 Loder;
Deep	.91
Dantsick	1.16
Ferrara	1.3333
Florence	1.282
Flanders in generel	1.06
Geneva	.9345

Genoa

One pound of Averdupois *weight at* London, *makes at*

	lb.
Genoa	1.4084 ſuttle 1.4285 groſſe
Hamburg	.92
Holland	.95
Lixborne	.881
Lions	1.07 common weight .98 ſilk weight .9 cuſtomers weight
Legorn	1.3333
Millan	1.4285
Mirandola	1.3333
Norimberg	.88
Naples	1.4084
Paris	.89
Prague	.83
Placentia	1.3888
Rochell	1.12
Rome	1.27
Rovan	.875 by Vicont .9017 common weight
Sivill	1.08
Tholouſa	1.12
Turin	1.2195
Venetia	1.5625 ſuttle 9433 groſs
Vienna	.813

The

The use of the preceeding Table.

How much weight at Bolonia, *will* 655 l. *Averdupois make?*

Look in the Table for *Bolonia*, and right against it you shall find 1. 25, which sheweth that one pound Averdupois at *London* is equal to 1 .25 *l.* at *Bolonia*, Therefore say by the Rule of Three:

If 1 *l.* Averdupois give 1. 25 *l.* at *Bolonia*, what shall 655 *li.* Averdupois give? *Answer* 818. 75. As by the operation following doth appear.

li.	*li.*	*li.*
1	125	655
	655	
	625	
	625	
	750	
	81875 or 818¾	

A TABLE shewing what one pound Weight in divers Forreign Cities, and remarkable Places, maketh at *London* of *Averdupois* Weight.

One pound weight in	*makes at* London *of* Averdupois *weight.*
	lb.
ANtwerp	1.04
Amsterdam	1.1111
Abeville	1.0989
Ancona	.78
Avignon	8928
Burdeaux	1.0989
Burgoyne	1.0980
Bolonia	.8
Bridges	1.0204
Calabria	.73
Calais	.9345
Deep	1.0989
Dantsick	.862
Ferrara	.75
Florence in general	.78
Geneva	.9433
Genoa subtle	1.07
Genoa groſs	.71
	.7

Ham-

One pound weight in		makes at London of Averdupois weight.	lb.
	Hamburg		1.0865
	Holland		1.0526
	Lisbone		1.135
	Lions: common weight		.9345
	Lions: silk weight		1.0204
	Lions: custom weight		1.1111
	Legorne		.75
	Millan		.7
	Mirandola		.75
	Norimberg		1.136
	Naples		.71
	Paris		1.1235
	Prague		1.2048
	Placentia		.72
	Rochel		.8928
	Rome		.7874
	Rovan: by Vicount,		1.1428
	Rovan: common weight,		1.1089
	Sivil		.9259
	Tholousa		.8928
	Turin		.82
	Venetia: suttle,		.64
	Venetia: groſs:		1.06
	Vienna		1.23

The

The use of the foregoing Table.

In 7652 *li. weight at* Mirandola, *how many pound Weight of* Averdupois ?

Look in the Table for *Mirandola*, and right against it you shall find .75, which sheweth that one pound *Averdupois* is equal to 75 or $\frac{3}{4}$ of a pound at *Mirandola*, wherefore say by the Rule of Three,

If 1 *l.* at *Mirandola*, gives .75 or $\frac{3}{4}$ of a pound *Averdupois*, what shall 7652 *l.* of *Mirandola* give? *Answer* 5739, as by the operation following doth appear.

li.

1 ——— .75 ——— 7652

75

38260

53564

573900

A Table

A TABLE reducing Engliſh Ells to the Meaſures of divers forreign Cities and remarkable places.

One Ell at London *makes at*

Amſterdam		1.6949	Ells.
Antwerp		1.6666	
Bridges		1.64	
Arras		1.65	
Norimberg		1.74	
Colen		2.08	
Liſle		1.66	
Maſtricht		1.57	
Frankford		2.0866	
Dantſick		1.3833	
Vienna		1.45	
Paris		.95	Aulns.
Rovan		1.03	
Lions		1.0166	
Calais		1.57	
Venice	Linnen	1.8	Braces.
	Silk	1.96	
Lacques		2.	
Florence		2.04	
Milan		2.3	
Legorn		2.	
Madera Iſles		1.0328	
Sivill		1.35	

Lisbone

One Ell at London makes at:			
	Lisbone	1.	Vares
	Castilia	1.3875	
	Andaluzia	1.4625	
	Granado	1.3625	
	Genoa	4.8083	Palmes
	Saragosa	.55	Canes
	Rome	.56	
	Barselona	.7225	
	Valentia	1.2125	

The use of this Table.

In 632 *Ells at* London, *how many Braces at* Florence?

Look in the Table for *Florence*, and right against it you shall find 2. 04, which sheweth, that one Ell at *London*, maketh at *Florence* 2.204 Braces; wherefore say by the Rule of Three.

If one Ell at London *give* 2. 04 *Braces at* Florence, *how many Braces shall* 632 *Ells give?* Answer 1289.28, as by the operation following doth appear.

1 — 2.04 — 632

632

408
612
1224

1289|28

A TABLE reducing the Measures of divers Forreign Cities and Remarkable places, to *Engliſh* Ells.

One Ell at	Amſterdam	makes at London	.59
	Antwerp		.6
	Bridges		.6097
	Arras		.606
	Norimberg		.5474
	Colen		.4807
	Liſle		.6024
	Maſtricht		.6369
	Frankford		.4792
	Dantſick		.7228
	Vienna		.9896
One Aulne at	Paris		1.0526
	Rovan		.9768
	Lions		.9836
	Calais		.6369
One Brace at	Venice Linnen		.5555
	Venice Silk		.5102
	Lucques		.5
	Florence		.4901
	Milan		.4347
	Legorn		.5
	Madera Iſles		.9681

Ells

Sivil

One Vare at	Sivil	*makes at* London	.7409
	Lisbone		1.
	Caſtilia		.7207
	Andaluſia		.7339
	Granado		.7339
One Palm at Genoa			.2079
One Cane at	Saragoſa		1.8181
	Rome		1.7857
	Barſelona		1.4035
	Valentia		.8247

The uſe of this Table.

In 5727 *Braces at* Legorn, *how many Ells* Engliſh.

Look in the Table for *Legorn*, and right againſt it you ſhall find ,5, which ſheweth that one Brace at *Legorn* maketh at *London* .5 or half an Ell, wherefore ſay by the Rule of Three.

If one Brace at Legorn *give* ,5 *Ells at* London, *what ſhall* 5727 *Braces give?* Anſwer 2863.5, as by the work appeareth.

1 ——— .5 ——— 5727

.5

2863.5

Section 2.

Concerning Intereſt and Annuities.

The First TABLE, ſhewing what One Pound being forborn any number of Years under 31, will amount unto, accounting Intereſt upon Intereſt, after the Rate of 6 *per cent.*

Y.	6 per cent.	Y.	6 per cent.	Y.	6 per cent.
1	1,06	11	1,89829	21	3,89956
2	1,1236	12	2,01219	22	3,60353
3	1,19101	13	2,13292	23	4,81975
4	1,26247	14	2,26090	24	4,04893
5	1,33822	15	2,39655	25	4,29187
6	1,41851	16	2,54035	26	4,54938
7	1,50363	17	2,69277	27	4,82234
8	1,59384	18	2,85433	28	5,11168
9	1,68947	19	3,02559	29	5,41838
10	1,79084	20	3,20713	30	5,74349

The

The first Column of this Table having *T* at the top thereof, beginning at 1, and so proceeding to 30, signifies years, and the number in the next Column answering thereunto does shew what one pound is worth, being forborn any number of years under 31, which Table is made according to this proportion.

As 100 to 106, so is 1 to 1.06

and again,

As 100——106——106——1.1236

and thirdly.

As 100——106——1.1236——1.19101

Et sic ad infinitum.

The use of this Table.

What 136 l. 15 s. 6 d. *will amount unto, being forborn* 20 *years, after the rate of* 6 per centum, *interest upon interest.*

Look in the Table for 20 years, and right against it in the broader Column, you shall find 3.20713; which shews that one pound being forborn 20 years will be augmented to 3.20713. Then if you reduce your 136 *li.* 15 *s.* 6 *d.* into a Decimal, either by the Tables in the Second part, or by the Scales in the Third Part of this Book, you shall find it to be 136,775. Wherefore say by the Rule of Three Direct.

If one pound being forborn 20 *years will be augmented to* 3.20713, *to how much will* 136.775 li. *be augmented to in the ſame time.* Anſwer, *to* 438 *li.* 13 *s.* 1 *d.* 1 *q.* as by the operation following doth appear.

li.	*li.*	*li.*
1	3.20713	136.775

136.775

1603565
2244991
2244991
1924278
962139
320713

438.65520575

Or

438 *li.* 13 *s.* 1 *d. q.*

The

The Second TABLE ſheweth, what one pound will amount unto, being forborn any number of Years under 31, at 6 *per cent.* Intereſt upon Intereſt, the Annuity being to be paid Yearly.

Y.	6 per cent.	Y.	6 per cent.	Y.	6 per cent
1	1,00000	11	14,97164	21	39 99272
2	2,06000	12	16,86994	22	43,39228
3	3,18360	13	18,88213	23	46 99582
4	4,37461	14	21,01506	24	50,81557
5	5,63709	15	23,27596	25	54,86451
6	6,97531	16	25,67252	26	59,15638
7	8,39383	17	28,21287	27	63,70576
8	9,89746	18	30,90565	28	68,52810
9	11,49131	19	33.75999	29	73,63979
10	13,18079	20	36,78559	30	79,05818

The uſe of this Table.

What will an Annuity of 20 li. *payable Yearly, be augmented unto in* 12 *years, being all that time forborn, accounting intereſt upon intereſt at* 6 per cent. per annum.

Look in the first column of the Table for 12 years, and right against it in the next column you shall find 16, 86994, which shews, that 1 *li.* Annuity, payable yearly, being forborn 12 years, will amount unto 16, 86994, wherefore say by the Rule of Three Direct.

If 1 *pound Annuity forborn* 12 *years, give* 16.86994, *what shall an Annuity of* 20 *pound a year give, being forborn the same term of* 12 *years?* Answer, 337 *li.* 7 *s.* 11 *d.* 3 *q. fere*, as in the operation doth appear.

16.86994 — 20

20

337.39880

or

337 *l.* 7 *s.* 11 *d.* 3 *q. fere*

The

The Third TABLE sheweth, what one pound being forborn any number of Years under 31, is worth in ready Money, rebating yearly, after the rate of *6 per cent.* Interest upon Interest.

Y.	*6 per cent.*	Y.	*6 per cent.*	Y.	*6 per cent.*
1	,943396	11	,526787	21	,294155
2	,809996	12	,496989	22	,277505
3	,839619	13	,468839	23	,261797
4	,792093	14	,442300	24	,246978
5	,747258	15	,417263	25	,232998
6	,704960	16	,393646	26	,219810
7	,665057	17	,370364	27	,207367
8	,627412	18	,351343	28	,195630
9	,591898	19	,330512	29	,184536
10	,558394	20	,311804	30	,174110

The making of the Table

As——106——100——,943396——889996.

and again,

As 106——100——.889996——839619

Et sic ad infinitum.

If 356 li. *be payable at the end of* 7 *years, what is it worth in ready money, discounting or rebating after the rate of* 6 *per cent. interest upon interest.*

Look in the Table for 7 years, and against it you shall find .665057, being the ready money which 1 *li.* is worth payable at 7 years end, wherefore say by the Rule of Three,

If 1 li. *in* 7 *years rebate or decrease to* .665057, *to what will* 356 li. *rebate or decrease in the same time?* Answer, *to* 170 *li.* 5 *s.* 1 *d.* as by the operation doth appear.

1 ——— .665057 ———
256

3990342
3325285
1330114

170.254592
or
170 *l.* 5 *s.* 1 *d.*

The

The Fourth TABLE ſheweth the preſent worth of one pound Annuity, to continue any number of Years under 31, and payable yearly after the rate of *6 per cent.* Intereſt upon Intereſt.

Y.	*6 per cent.*	Y.	*6 per cent.*	Y.	*6 per cent.*
1	0,94337	11	7,88687	21	11,76305
2	1,83339	12	8,38384	22	12,04158
3	2,67301	13	8,85268	23	12,30337
4	3,46510	14	9,29498	24	12,55035
5	4,21236	15	9,71224	25	12,78335
6	4,91732	16	10,10589	26	13,00316
7	5,58238	17	10,47725	27	13,01053
8	6,20979	18	10,82760	28	13,40616
9	6,80169	19	11,15811	29	13,59071
10	7,36008	20	11,46902	30	13,76482

The uſe of this Table.

What is the preſent Rent or Annuity of 25 *pound* per annum *worth payable yearly, for* 22 *years, accounting Intereſt upon Intereſt at* 6 per centum.

Look

Look in the Table for 21 years, and right against it you shall find 11.76407, which is the present worth of one pound Annuity for 21 years, wherefore say by the Rule of Three.

If an Annuity of 1 li. per annum *for* 21 *years be worth* 11.76407 *ready money, what is an Annuity of* 25 li. per annum *worth in ready money for the same time?* Answer 294 *li.* 2 *s.* 0 *d.* 1 *q.* as by the operation following doth appear.

1——11.76407——25

25

5882035
2352814

294.1075
or
294 *l.* 2 *s.* 0 *d.* 1 *q.*

The

The Fifth TABLE sheweth what Annuity payable Yearly, one pound will purchase for any number of Years under 31, after the Rate of *6 per cent.* compound Interest.

Y.	*6 per cent.*	*Y.*	*6 per cent.*	*Y.*	*6 per cent.*
1	1.06000	11	.12679	21	.08500
2	.54363	12	.11926	22	.08304
3	.37411	13	.10297	23	.08127
4	.28859	14	.10758	24	.07967
5	.23739	15	.10296	25	.07822
6	.20336	16	.09895	26	.07690
7	.17913	17	.09544	27	.07569
8	.16103	18	.09235	28	.07459
9	.14702	19	.08962	29	.07357
10	.13586	20	.08718	30	.07264

The use of this Table.

What Annuity to begin presently, and to continue 28 *years, payable at yearly payments, will* 640 li. *purchase, accounting compound interest after the rate of* 6 per cent.

Look

Look in your Table for 28 years, and right against it in the next column you ſhall find .07459, which ſhews that one pound ready money will purchaſe an Annuity worth .07459, and to continue 28 years, wherefore ſay by the Rule of Three.

If one pound ready money will purchaſe an Annuity worth 07459 *to continue* 28 *years, what Annuity ſhall I purchaſe for the ſame time, paying* 640 li. *ready money?* Anſwer, 47 *li.* 14 *s.* 9 *d.* as by the operation doth appear.

1 —— .07459 —— 640

640

29836
44754

47.73760
Or
47 *l.* 14 *s.* 9 *d.*

FINIS.

INSTRUMENTAL ARITHMETICK.

The THIRD PART.

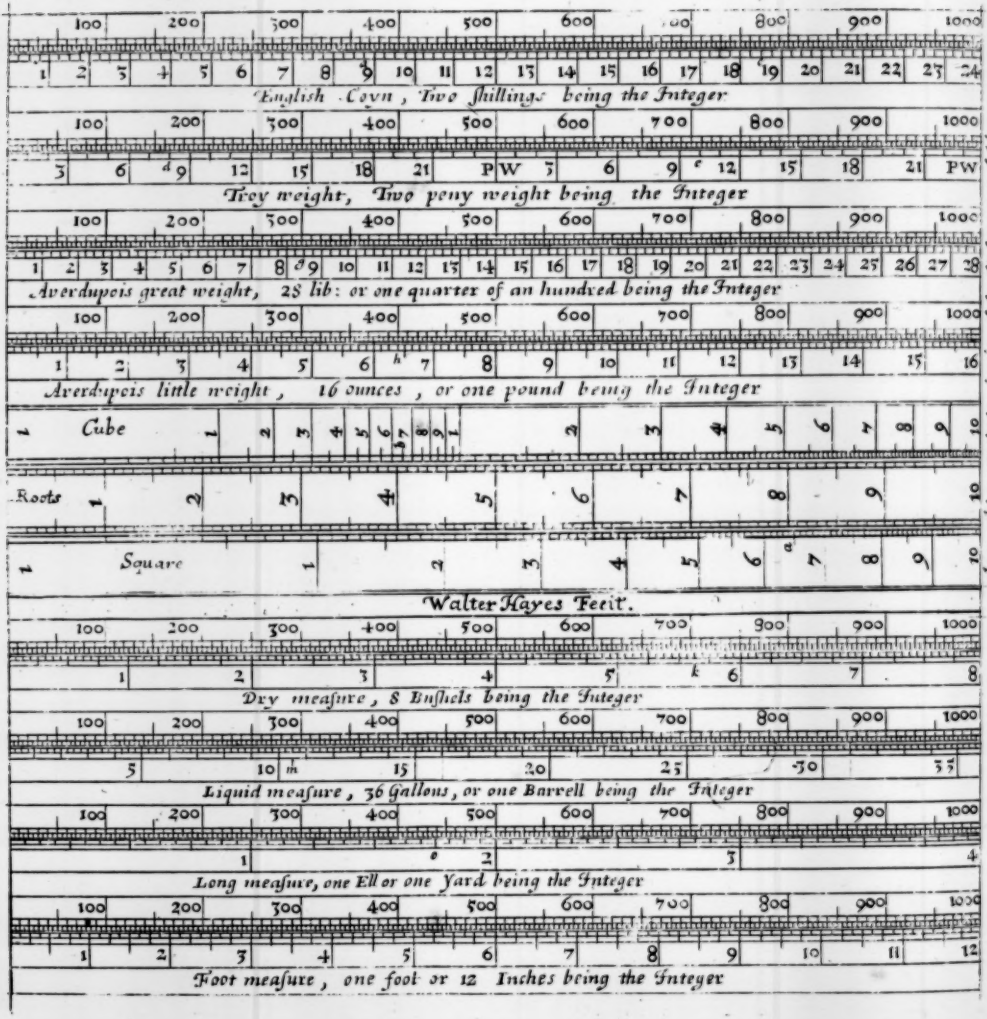
TEACHING,

By a new Artifice (not heretofore Publiſhed, to my Knowledge, in any Language) The manner how to ſet down any *Decimal Fraction* required: Or a *Decimal Fraction* being given, to find the value thereof in *Engliſh Money*, *Weight* or *Meaſure*; by inſpection only. By certain SCALES contrived, ſuitable to the *Coins*, *Weights* and *Meaſures* now uſed in *England*.

By *Will. Leybourn.*

LONDON,

Printed *Anno Dom.* 1678.


English Coyn, Two Shillings being the Integer
Troy weight, Two peny weight being the Integer
Averdupois great weight, 28 lib: or one quarter of an hundred being the Integer
Averdupois little weight, 16 ounces, or one pound being the Integer
Cube
Roots
Square
Walter Hayes Fecit.
Dry measure, 8 Bushels being the Integer
Liquid measure, 36 Gallons, or one Barrell being the Integer
Long measure, one Ell or one Yard being the Integer
Foot measure, one foot or 12 Inches being the Integer
place this at the beginning of the third part to fold out.

Instrumental ARITHMETICK.

The Third Part.

THe *Arithmetick* of which we now come to treat, and which I call *Instrumental Arithmetick*, is not any new kind of *Arithmetick*, but is indeed the same with *Decimal Arithmetick* before taught; only, whereas in *Decimal Arithmetick* there were certain *Tables* made of *Money*, *Weight* and *Measure*, by help of which the Decimal of any Fraction of *Money*, *Weight* and *Measure*, might be set down (as it were) in whole numbers; here in this *Instrumental part*, we have contrived certain *Scales* of *Money*, *Weight* and *Measure*, equally divided into the several Denominations into which the *Weights* and *Measures* for which they are contrived, may be equally divided. Unto all which Scales there is joyned a Scale of 100, 1000, or 10000, equal parts, according to the length of the Scale, so that by inspection only you may readily and exactly without Addition (as in using the forementioned *Tables* you must necessarily do) set down the *Decimal*

Fraction of any part of *Money*, *Weight* or *Measure*, with great celerity and exactness, if the Scale be any thing well divided, and be but of a reasonable length.

Now the Scales which I have chiefly made choice of in this Work; as being of most use with English men; (though other Scales may be made for the *Coins*, *Weights* or *Measures* of any other Countrey as well, and upon the same ground) are chiefly these, *viz.*

1 Of Money.
2 Of Troy Weight.
3 Of Averdupois great Weight.
4 Of Averdupois little Weight.
5 Of Liquid Measures.
6 Of Dry Measures.
7 Of Long Measures.
8 Of Dozens.

Unto every of these Scales, is joyned another Scale of 100 or 1000 equal parts, these Scales are made to face one another, so that if you look upon any one Division in the one, you shall also discern plainly what Division or part of a Division answereth thereunto in the other.

These Scales being thus disposed, as they may easily be upon any *Ruler* of *Silver*, *Brass* or *Wood*; but best of all upon a *square Ruler*, made in form of a *Parallelepipedon*, will by inspection only give you any Decimal fraction required without Addition, or (on the contrary) reduce the fraction into the known parts of the Integer, by inspection also, without *Substraction*.

Let thus much suffice for a general description of what I mean by Scales, the particular description of them will more plainly appear, when we treat of Numeration upon the Scales, unto which we shall now proceed. But first take a view of the Scales as they are here

here diſpoſed, and as they may be ſet upon ſuch a Ruler as I have here mentioned.

Numeration upon the Scales.

THe Scales here to be deſcribed are in number eight, as hath been already ſhewed, and as by the figure of them appears. *Now* Numeration *upon a* Scale, *is to find upon what part of the* Scale *any number upon the ſame* Scale *will fall.*

We will begin with the firſt, and ſo proceed till we have given an Example in every one.

1 The firſt Scale is of *Engliſh Money*, and is divided into 24 equal parts, which repreſent 24 pence or 2 ſhillings, theſe parts are numbred with Arithmetical figures, from the beginning thereof, by 1, 2, 3, 4, 5, &c. to 24, each diviſion repreſenting one peny, and the whole 24 diviſions repreſenteth 24 pence, or 2 ſhillings; So that where the figure 1 ſtandeth, that part of the Scale repreſenteth one peny, where the figure 2 ſtandeth, it repreſenteth two pence, where the figure 18 ſtandeth, it repreſenteth 18 pence, or one ſhilling ſix pence, and ſo of any other figure of the ſame Scale.

Then becauſe there are four farthings contained in a peny, each of theſe pence (or diviſions) is ſub-divided into 4 other equal parts by ſhort lines, every one of theſe repreſenting one farthing, ſo is the whole Scale divided in all in *96* equal parts, which are the number of farthings contained in two ſhillings. Thus if you look into the Scale of Money for 8 pence 3 farthings, you ſhall find it at the letter *a*, which letter is here put onely for example ſake, Alſo if you would find in the Scale the place of 18 *d.* half peny, you ſhall find it at *c*,

and thus may you find the place of any number of pence and farthings under two shillings upon the Scale.

Unto this Scale of money (as to all the rest of the Scales) there is joyned another Scale, which I shall alwayes hereafter call the Scale of 1000, the use of which Scale is this. When you have found any Number of pence or farthings upon the Scale of Money, you shall find upon the Scale of 1000, what parts of a thousand is the Decimal of those pence and farthings: Thus when in the Scale of money you find at the letter *a* 8 pence 3 farthings, if you cast your eye directly cross to the Scale of 1000, you shall find 364 to stand directly against 8 *d.* 3 *q.* which 364 is the Decimal of 8 *d.* 3 *q.* Also, if you find upon the Scale of money 18 *d.* half peny, which is at the letter *c*, you shall find against it in the Scale of 1000, this number .771, which is the Decimal of 18 *d.* half peny. And in this manner may the Decimal of any number of pence or farthings under two shillings be most easily and exactly obtained.

Now on the contrary, suppose a Decimal Fraction were given, representing some part of *English Coin*, if you look in the Scale of 1000 for your Number given, right against it in the *Scale of Money* you shall find what number of pence and farthings is represented thereby. As for Example, Suppose .364 were a Decimal given, and it were required to find what part of Coin it doth represent. Look in the Scale of 1000 for the Number 364, and right against it you shall find 8 pence 3 farthings. Also if .771 were a Decimal given, if you look in the Scale of 1000 for .771, you shall find against that number 18 pence 2 farthings. And thus of any other.

By what hath been already said, it may be easily discerned of what exceeding expedition these Scales thus disposed are of, for I dare affirm, that I will set down a (if not 3) numbers, by the Scale, as soon as one by the

the Tables, and if the Scale be but of any reaſonable length, altogether with as much exactneſs, but if I ſhould vary an unite in the laſt place, in my eſtimation in the Scale of 1000, it is not any thing material.

I have been very tedious in ſhewing the uſe of theſe Scales to find the fraction-parts of *Money*; but the reaſon is, becauſe I intend to be the briefer in the reſt, for *Weight* and *Meaſure*. the manner of working (when the Diviſion of the Scale is known) being the ſame in all reſpects without the leaſt alteration.

2 The ſecond Scale is of *Troy Weight*, two peny weight being the Integer, which Scale is divided into 48 equal parts or diviſions, each of which diviſions contains one grain, and are numbers by Arithmetical figures at every three grains by 3, 6, 9, 12, &c. to 24, and at the place where 24 ſhould ſtand, there ſtandeth P W, which ſignifieth one peny weight, or 24 grains, this P W ſtandeth in the middle of the line. Then is the ſame Scale continued farther by Arithmetical figures. 3, 6, 9, 12, &c. as before to 24, and there is written P W again, repreſenting two peny weight, or 48 grains.

The Scale being thus divided, it is eaſie to find the place where any number of Grains under 48 ſhall be upon the Scale; As for Example, if it be required to find where 8 peny weight ſhall fall, look upon the Scale of Troy weight, from the beginning thereof, and count the figures 3 and 6, then count alſo two of the ſmaller Diviſions, and that makes 8 grains, which you ſhall find to ſtand at the letter *d*. which is the place of 8 grains; Alſo if upon the Scale you would find the place of one peny weight 10 grains, you ſhall find it at the letter *e*, and ſo of any other number of grains under 48, or two peny weight.

But if you had a Decimal given, and would know what number of grains it repreſenteth, if you ſeek your

Decimal given in the Scale of 1000, right againſt it in the Scale of Troy weight, you ſhall find the number of grains repreſented thereby.

Example. Let .167 be a Decimal fraction given, If you look in the Scale of 1000, for .167, right againſt it in the Scale of *Troy weight*. you ſhall find 8 peny weight.

Alſo if .708 were a Decimal given, if you ſeek 708 in the Scale of 1000, right againſt it you ſhall find 1 peny weight 10 grains.

3 The third ſcale is of *Averdupois great weight*, 28 pounds, or one quarter of an hundred, being the Integer, this Scale is numbred by 1, 2, 3, 4, &c. to 28, which 28 repreſenteth 28 *l.* or a quarter of a hundred, and each of theſe is ſubdivided into four ſmall parts, each repreſenting one quarter of a pound.

Now if you would know what is the Decimal of any number of pounds or quarters under 28, if you ſeek the number of pounds in the Scale of *Averdupois great weight*, right againſt it in the Scale of 1000, you ſhall find the Decimal thereof.

Thus if it were required to find the Decimal of 8 pound and an half, if you look upon the Scale for 8 pound and an half, you ſhall find it at the letter *g*, and right againſt it in the Scale of 1000 you ſhall find .304, which is the Decimal of 8 pounds and an half.

On the contrary, ſuppoſe .304 were a Decimal given, and it were required to find what part of *Averdupois great weight* were repreſented thereby, if you look in the Scale of 1000 for .304, right againſt it in the Scale of *Averdupois great weight*, you ſhall find 8 pound and an half.

4 The fourth Scale is of *Averdupois little weight*, 16 ounces or one pound being the Integer; This Scale is firſt divided into 16 equal parts, and numbred by 1, 2, 2. 4. &c. to 16, each Diviſion repreſenting one Ounce.

Then

Then again, each of theſe ounces is ſub-divided into 8 other ſmaller parts or diviſions, each of which diviſions repreſenteth two Drams; but if your Scale be large enough, you may have each ounce divided into 16 equal parts or diviſions, each diviſion repreſenting one Dram.

Now to find the Decimal belonging to any number of Ounces and Drams, repair to the Scale of *Averdupois little weight*, and on it the quantity of ounces and drams required, and right againſt it in the Scale of 1000, you ſhall have the Decimal thereof.

Thus if it were required to find the Decimal of 6 Ounces and 6 Dram., if you look this in the Scale of *Averdupois little weight*, you ſhall find it at the letter *h*, and right againſt it in the Scale of 1000, you ſhall find 398, which is the Decimal of 6 ounces and 6 drams.

But if .398 were a Decimal given, and it were required to find the value thereof in *Averdupois little weight*, if yon look for .398 in the Scale of 1000, right againſt it in the other Scale you ſhall find 6 ounces 6 drams.

5 The fifth Scale is of *Dry Meaſures*, one *Quarter* or 8 *Buſhels* being the Integer; This Scale is firſt divided into 8 equal parts, and numbred by 1, 2, 3, &c. to 8, each of which diviſions repreſenteth a Buſhel, and each of thoſe parts is again ſub-divided, firſt into 4 equal parts or diviſions, each repreſenting one *Peck*, and then thoſe again ſub-divided into 4 other ſmaller parts, repreſenting *Quarters*, *Halves*, and *Three Quarters* of a Peck.

Now if you would know the Decimal belonging to any number of Buſhels (under 8 Buſhels or one Quarter) Pecks and parts of a Peck, if you ſeek the number of Buſhels, Pecks, and parts of a Peck in the Scale of *Dry Meaſures*, right againſt it in the Scale of 1000, you ſhall have the Decimal required.

As for Example, if it were required to find the Decimal belonging to 5 Buſhels 2 Pecks, and half a Peck, if you look into the Scale of Dry Meaſures you ſhall find 5 Buſhels, 2 Pecks, and an half to ſtand at the letter *k*, and right againſt it in the Scale of 1000, you ſhall find .702, which is the Decimal anſwering to 5 buſhels, 2 pecks and an half.

But if .702 were a decimal given, and it were required to find what number of buſhels, pecks, and parts of a peck it repreſenteth, if you look in the Scale of 1000 for .702, you ſhall find againſt it in the Scale of *Dry Meaſures*, 5 buſhels, 2 pecks and an half.

6 The ſixth Scale is of *Liquid Meaſures*, the Integer being 36 Gallons or one Barrel, this Scale is divided firſt into 36 equal parts or diviſions, and numbred by 5, 10, 15, &c. to 36, then every of theſe diviſions is again ſub-divided into 4 other ſmall diviſions, each repreſenting a quart, but (if the Scale be large enough) you may ſub-divide each Gallon into 8 parts, ſo will every part repreſent one pint.

Now to find the Decimal belonging to any number of *Gallons* (under 36 Gallons or one Barrel) quarts or pints, repair to the Scale of *Liquid Meaſures*, and ſeek there upon the Scale, the number of gallons, quarts or pints, and againſt it in the Scale of 1000, you ſhall find the decimal thereunto belonging.

So if it were required to find a Decimal repreſenting 10 gallons and two quarts, or 4 pints, which is all one, if you ſeek in the Scale of Liquid meaſures for 10 gallons, 2 quarts, you ſhall find it at the letter *m*, againſt which in the Scale of 1000, you ſhall find .292, which is the decimal of 10 gallons 4 pints.

Likewiſe if .292 were a decimal given, and it were required to find what number of gallons, quarts or pints were repreſented thereby, if you look in the Scale of 1000 for .292, right againſt it in the Scale of

Liquid

Liquid Meaſures, you ſhall find 10 gallons, 2 quarts, or 4 pints.

7 The ſeventh Scale is of *Long Meaſures*, the Integer being *Yards* or *Ells*, this *Scale* is divided into 4 equal parts, and numbred by 1, 2, 3, 4, repreſenting 1 quarter, 2 quarters, 3 quarters or 4 quarters of a Yard or Ell, theſe are again ſub-divided, firſt into 4 other equal parts, repreſenting *Nailes*, and thoſe may be again ſub-divided at pleaſure if need be.

Now if you would know what decimal belongeth to any number of *quarters or nailes of a yard or Ell*, it you ſeek the number of quarters and nailes in the *Scale* of *Long Meaſure*, the *Scale* of 1000 will give you the decimal thereof.

Thus if it be required to find the decimal belonging to 1 quarter and 3 nailes, if you ſeek this in the *Scale* of *Long Meaſure*, you ſhall find it to ſtand at the letter *o*, againſt which in the *Scale* of 1000 you ſhall find .437, which is the Decimal anſwering to 1 quarter and 3 nailes of a *Yard* or *Ell*.

Alſo if .437 were a decimal given, and it were required to find what quantity of *yards* or *ells* were repreſented thereby, if you look in the *Scale* of 1000 for .437, you ſhall in the *Scale* of *Long Meaſure* find againſt it one quarter and 3 nailes.

8 The eighth and laſt *Scale* is of *repreſentative inches*, the whole *Scale* being divided into 12 equal parts, and numbred by 1, 2, 3, &c. to 12, and thoſe parts are again ſub-divided into halves, quarters, and half quarters, as Carpenters-Rules are uſually divided.

Unto this *Scale* (as unto all the other) there is joyned a *Scale* of 1000, this *Scale* will readily diſcover what is the Decimal belonging to any number of Inches, halves or quarters, and the uſe is the ſame with the *Scales* before mentioned.

Thus I have given you a brief deſcription of theſe

Scales

Scales, and the uſes of them, and do now ſuppoſe my Reader to be perfectly acquainted with the way of numbring or counting upon them; wherefore I intend onely to give you a queſtion or two in the moſt uſual Rules of Arithmetick, and ſo conclude; for Decimal Arithmetick being already ſufficiently explained, I ſhall not need to repeat the Rules (or the manner of working them) again, but give you one Example, by which the exactneſs and expedition of theſe *Scales* may the more evidently appear, for when we work by Scales, it is ſuppoſed that we do not uſe *Vulgar*, but *Decimal Arithmetick* and Addition, Subſtraction, Multiplication, Diviſion, the Rule of Three, and indeed, all the other Rules of Arithmetick, are to be performed, as is before taught, the Scale ſerving only to avoid Reduction.

ADDITION.

WHat *Addition* is, and the manner of working of it hath been already taught, both in the firſt and ſecond Parts, we will now come to an Example, which let be in *Addition of Engliſh Coin*, and let the ſums to be added be 36 *l.* 8 *s.* 8 *d.* 29 *l.* 0 *s.* 2 *d.* 31 *l.* 16 *s.* 9 *d.* and 6 *l.* 2 *s.* 5 *d.*

Firſt, ſet down 36 *l.* 29 *l.* 31 *l.* and 6 *l.* one under another, in ſuch order as you ſee here in the margine, drawing a line by the ſide of them as you ſee here done, and alſo a line under them.

36	
29	
31	
6	
—	——

This done, ſeeing that your firſt number to be ſet down to 36 *l.* is 8 *s.* 8 *d.* you muſt for the 8 *s.* (becauſe two ſhillings

lings, which we called a Decade, or the tenth part of a pound, is made the Integer, in the Scale of Money) ſet down 4, which is done by memory, and after it make a Comma; Then your next number to be ſet by 29 *l.* being 0 *s.* 2 *d.* for the 0 *s.* ſet down a Cypher; Thirdly, for your number to be ſet by 31 *l.* being 16 *s.* 9 *d.* for the 16 *s.* ſet down 8 decades, with a comma after it, and laſtly, the number to be ſet by 6 *l.* being 2 *s.* 5 *d.* for the 2 *s.* I ſet down 1 decade with a comma after it, and then will your work ſtand, as here you ſee.

36	4,
29	0,
31	8,
6	1,
—	—

Then take your Scale in hand, and ſeeing your firſt number of pence are 8 *d.* look in your ſcale of money for 8 *d.* and againſt it in the ſcale of 1000, you ſhall find 333, which ſet 36 *l.* 8 *s.* behind the comma, then your next number of pence being 2 *d.* look in your ſcale for 2 *d.* and againſt it in the ſcale of 1000, you ſhall find 083, which ſet to 29 *l.* 0 *s.* behind the comma. Then your third number of pence being 9 *d.* look in your ſcale for 9 *d.* and againſt it in the ſcale of 1000, you ſhall find 376, which ſet to 31 *l.* 16 *s.* and laſtly, your laſt number of pence being 5 *d.* look in your ſcale for 5 *d.* and againſt it you ſhall find 208, which ſet to 6 *l.* 2 *s.* and then will your whole work ſtand, as here you ſee.

36	4,333
29	0,083
31	8,376
6	1,208
—	—

Your ſums being thus ſet down, which is done with more facility than you can imagine, tell you make trial and be ſomething perfect therein, you muſt then add all the numbers together, as in Addition of Decimals, and you ſhall find the ſum of them to be 103|4,000, Now to know this in money, is as eaſie as it was to ſet ſeveral ſums down, for the figures 103, which ſtand behind

behind the down right line, are 103 *l.* and the figure 4 which ſtands between the down right line and comma, are 4 decades or 8 *s.* and being the reſt to the right hand are all Cyphers, they ſignifie neither pence nor farthings, ſo is the total of this Addition 103 *l.* 8 *s.* 0 *d.*

li.	
36	4,333
29	0,083
31	8,376
6	1,208
103	4,000

That the manner of working may appear more plain, I will give you another ſhort Example as difficult as I can invent, which I performed by a Scale of Wood but of 8 inches long. Let the ſums to be added together be theſe following:

li.	*s.*	*d.*	*q.*
332	17	4	1
159	6	8	1
217	5	3	3
709	9	4	1

First ſet down your ſeveral ſums of pounds one under another as before, and draw a line by the ſide of them, and another under them. So will they ſtand as here you ſee.

332
159
217

1 Your ſums of pounds being thus orderly placed and lines drawn, repair to your Scale, and ſeeing your firſt number of ſhillings, pence and farthings is 17 *s.* 4 *d.* 1 *q.* for your 17 *s.* ſet down 8 Decades which is 16 *s.* with a comma after it, then will there reſt to be ſet down 1 *s.* 4 *d.* 1 *q.* or 16 *d.* 1 *q.* which if you ſeek in your ſcale of money, you ſhall find to ſtand againſt it in the ſcale of 1000 this number .677, which is the Decimal of 1 *s.* 4 *d.* 1 *q.*

2 Your ſecond number of ſhillings, pence and farthings is 6 *s.* 8 *d.* 1 *q.* for your 6 *s.* ſet down 3 decades, which

which is 6 *s.* and then there will remain 8 *d.* 1 *q.* which if you ſeek in your ſcale of money, you ſhall find to ſtand againſt it in the ſcale of 1000, this number .344, which is the decimal of 8 *d.* 1 *q.*

3 Your third number of ſhillings, pence, and farthings, is 5 *s.* 3 *d.* 3 *q.* for your 5 *s.* ſet down 2 decades, which is 4 *s.* with a comma after it, then will there reſt to be ſet down 1 *s.* 3 *d.* 3 *q.* or 15 *d.* 3 *q.* which if you ſeek in your ſcale of money, you ſhall find to ſtand againſt it in the ſcale of 1000, this number .656, which is the Decimal of 15 *d.* 3 *q.* or 1 *s.* 3 *d.* 3 *q.* and the three ſums to be added together will ſtand as here you ſee.

	l.	
	332	8,677
	159	3,344
	217	2,656
	709	4,677

These ſums being added together according to the Rule for Addition of Decimals, you ſhall find the ſum of them to be 709|4,677, now to know what this is in money, take notice that the 709 which ſtands to the left hand of the down right line are 709 pounds, and the figure 4, which ſtands between the down right line and the comma, are 4 decades or 8 *s.* but (becauſe the firſt figure next after the comma is above 5, *viz.* 6) you muſt add 1 *s.* to the 4 decades, making them 9 *s.*; then will there remain 177, wherefore if you look in the ſcale of 1000 for 177, you ſhall find againſt it in the ſcale of Money 4 *d.* 1 *q.* So is the whole ſum of this Addition 709 *l.* 9 *s.* 4 *d.* 1 *q.* as by the preceeding work doth appear.

¶ Here note, that when you had ſet down your 709 *l.* 4 decades or 8 *s.* there remained beyond the comma 677, which if you had ſought in your ſcale of 1000, you ſhould have found againſt it in the ſcale of Money 15 *d.* 1 *q.* or 1 *s.* 3 *d.* 1 *q.* (which is all one) as before, for it appeareth plainly by the Scale, that 500 in the line of 1000 is equal to one ſhilling.

I might proceed farther in giving you Examples in *Weight* and *Meaſure* anſwerable to the *Scales*, but that would only make the Reader ſpend his time to little purpoſe; for being before acquainted with *Decimal Arithmetick*, and (as by this time I ſuppoſe he is) with *Numeration* upon the *Scales*, he cannot be deficient in the applying of the other *Scales* of *Weight* and *Meaſure* to the ſame purpoſe for which they were contrived, I having ſo largely exemplified the uſe of the *Scale* of *Money*.

SUBSTRACTION.

Subſtraction (as hath been before ſaid) is the taking one or more ſmaller ſums out of one greater; I ſhall only give you an Example or two, as I have taken the numbers from a Scale.

Example.

Delivered to a Gold-ſmith of old Plate 297 *ounces*, 13 *peny-weight*, 19 *grains.*

Received of the ſame Gold-ſmith firſt 165 *ounces*, 11 *peny-weight and* 7 *grains, and after that received of the ſame Gold-ſmith* 32 *ounces* 19 *peny-weight and* 32 *grains, what Plate remaines in the Gold-ſmiths hands?*

Take your Numbers out of the Scale of Troy-weight, and ſet them down as you here ſee.

ounces

	ounces
Delivered	297|6,896
Received firſt	1655,646
Received more	329,979
Received in all	198|5,625
Reſts in the Gold-ſmiths hands	99|1,271

or

ounces	*peny-w.*	*gr.*
99	2	13

Then add the ſeveral weights of Plate received, together, and they make 198|5,625, or 198 ounces 11 peny-weight, 6 grains, which if you ſubſtract from 297|6,896, or 257 ounces, 13 peny-weight, 19 grains, which was the quantity of Plate delivered, there will remain 99|1,271, or 99 ounces, 2 peny-weight, 13 grains, and ſo much Plate is ſtill in the *Gold-ſmiths* hand. And let thus much ſuffice for *Subſtraction.*

Now we ſhould proceed to *Multiplication* and *Diviſion*, but when the numbers are taken from the Scale and ſet down, the manner of working doth not at all differ from *Multiplication* and *Diviſion* of *Decimals* before taught. But before I teach you how to Multiply and Divide Inſtrumentally, I ſhall ſhew you

The

The farther uſe of the Decimal Scales, *and how by them to find the* Square *or* Cube Root *of any* Number. *And*

2. *Any* Root *being given, to find the* Square *or the* Cube number *of that Root.*

And both theſe by inſpection only, without the help of either *Pen, Compaſſes*, or any other *Motion.*

For the effecting hereof, there is now inſerted, among the forementioned *Decimal Scales* of *Money, Weight, Meaſure*, &c. namely, between the Scales of *Averdupois little weight*, and that of *Dry Meaſures*, Two other *Scales*, one having written at the beginning thereof the word *Square*, and to the other there is added the word *Cube*, and between them, there is a third line, which hath written upon it, the word *Root*. And by theſe three *Scales* thus united, the *Square* and *Cube Roots* of any number may be extracted by inſpection only. For,

If you find any number whoſe *Square Root* you require, in the *Line* or *Scale* of *Squares*, right againſt it, in the *Line* of *Roots*, you ſhall have the *Square Root* of that number. Thus,

If the number 64 were given, and it were required to find the Square Root *thereof.*

Find the given number 64, upon the *Line* or *Scale* of *Squares* (which you may do at the letter *a*) and right againſt it, in the *Scale* of *Roots* ſtands the figure 8, which ſhews, that 8 is the *Square Root* of 64. And in the ſame manner you may find the *Square Root* of any other number.

For

For, againſt	81	in the Line of *Squares*, you ſhall find	9	in the *Scale* of *Roots*, which is the *Square Root* thereof.
	64		8	
	49		7	
	36		6	
	25		5	
	16		4	
	9		3	
	4		2	

In like manner,

If the number 64 *were given, and it were required to find the* Cube Root *thereof.*

Find the given number 64 in the *Scale* of *Cubes*, (which you may do, by counting the ſame number between the ſecond and third figures of 1 upon the Scale, at the letter *b*) and right againſt it, in the *Line* or *Scale* of *Roots*, ſtands the figure 4, which ſhews, that 4 is the *Cube Root* of 64. And in the ſame manner you may find the *Cube Root* of any other number.

For, againſt	729	in the Scale of Cubes, you ſhall find	9	in the Scale of Roots, which is the Cube Root thereof.
	512		8	
	343		7	
	216		6	
	125		5	
	64		4	
	27		3	
	8		2	

And by this *Artifice*, not onely the *Roots* of direct *Square* and *Cube* numbers may be found, but in numbers that be not directly *Square* or *Cube*, the *Fraction* part of the *Root* is nearly diſcovered alſo.

I have hitherto given you Examples in ſuch *Square* and *Cube* numbers, as are common and familiar, and that any man may compute almoſt by memory; but by

theſe the *demonſtration* of the Artifice is diſcovered, the *Lines* of *Squares* and *Cubes* being only *Square* and *Cube* numbers transferred to Lines. And now let us proceed to greater Numbers. And

1. *For the* SQUARE ROOT.

In the *Extraction* of a *Square Root*, it is uſual to ſet a *Prick* under the firſt figure, the third, the fifth, the ſeventh, and ſo forwards, beginning from the right hand towards the left. And as many pricks as fall under the *Square number* given, of ſo many figures will the *Root* of that number conſiſt: So that if the number given be leſs than 100, the *Root* ſhall be onely of one figure, if leſs than 10000, it ſhall be but two figures, if leſs than 1000000, it ſhall be three figures, and ſo forward.

Hence it is, That the *Line* or *Scale* of *Squares* is divided firſt into 100 parts, and if the number given be greater than 100, the firſt diviſion (which is the place where the firſt figure of 1 ſtandeth, and which before did ſignifie *One*,) muſt now ſignifie 100, and the whole line ſhall be 10000; If farther, the number be greater than 10000, you muſt count or eſteem the firſt figure of 1 to be 10000, and then will the whole line be 1000000 parts; and if this be too little to expreſs the number given, you may proceed farther, and call the firſt 1· 1000000, and ſo increaſe the Line 100 times more; but this is ſufficient.

Thus when any Square number is given, if you ſet it down, and prick it, (or imagine it ſo to be) If the laſt prick to the left hand ſhall fall under the laſt figure, (which will always be when the figures in the given number be odd) you muſt find all ſuch numbers upon the line, between the two figures of 1.——But if the

the laſt prick ſhall fall under the laſt figure but one of the given number, (which it will always do, when the figures of the number given are even) then you muſt find the number given in the line of *Squares*, between the ſecond figure of 1 and 10 at the end of the line.

This being conſidered, find the number given, whoſe Square Root is required, in its proper place upon the Line of Squares, and againſt it in the Scale of Roots you ſhall have its Square Root deſired. Thus,

The *Square*	36	will be found	6
Root of	3600	to be	60
	360000		600.

And in this manner may the neareſt Root of any number not exactly Square be obtained. For

The neareſt	725	will be found to be about	27
Root of	72500		269
	7250000		2691

And thus on the contrary, a Number may be *Squared*, as may partly appear by what hath been before delivered; for if you find the *Root* in the *Scale* of Roots, you have its *Square* in the line of *Squares*, and ſo,

Againſt	27	in the Scale of *Roots*, you ſhall find	725
	85		7250
	269		72500
	851		725000

the Square thereof.

Thus much for the Square Root. Now

2. *For the* CUBE ROOT.

In the *Extraction* of the *Cube Root*, it is usual to set pricks under the first figure, the fourth, the seventh, the tenth, and so on, pricking alwayes the 3 figure from the right hand towards the left. And as many pricks as fall under the *Cubick* number given, so many figures shall be in the *Root*. So that if the number given be less than 1000, the Root shall consist only of one figure; If less than 1000000, it shall be only two figures; if it be above 1000000, and less then 1000000000, it will be only three figures.

Hence it is, That the *Line of Cubes* is divided first into 1000 parts; And if the number given be greater than 1000, the first figures of 1 (which before did signifie only *One*) must now signifie 1000, and the second figures of 1, must now signifie 10000, and the third 1, must signifie 100000, and the whole line must be esteemed to be 1000000. Farther, If the number given be greater than 1000000, the first 1, must signifie 1000000, the second 10000000, the third 100000000, and the whole line 1000000000 parts. And if these be yet too little, you may proceed farther; but let these suffice.

Thus when any Cube number is given, if you set it down, and prick it; If the last prick to the left hand shall fall under the last figure, the number shall be reckoned between the first and second figures of 1, and the first figure of the Root shall be alwayes either 1 or 2——If the last prick fall under the last figure but one, the number given must be rekoned between the second and third figures of 1, and the first figures of the Root shall alwayes be either 2, 3, or 4.—— But if the

the laſt prick ſhall fall under the laſt figure but two, then the number given muſt be reckoned between the third figure of 1, and 10 at the end of the Line.

This being conſidered, find the number given, whoſe *Cube Root* is deſired, in the proper Section upon the *Line* or *Scale* of *Cubes*, and right againſt it in the *Scale* of *Roots*, you ſhall have its *Cube Root* deſired. Thus

849	The Cube Root of	8490000	will be found to be about	204
849		84900000		439
849		849000000		947

And the like of any other.

On the Contrary, a number may be *Cubed*; for if you find the number in the line of *Roots*, you ſhall have the *Cube* thereof right againſt it in the *Scale* of *Cubes*, giving the true denomination to the Cube, according as the part of the Line againſt which the *Root* ſtandeth doth require.

Thus have you by this Inſtrumental way of working, theſe things, which in the ordinary courſe are moſt hard and intricate, rendred very familiar and eaſie; And although at all times you do not make uſe of them, yet they are ready helps to confirm you in your working without the tedious way of proving by Reverſe working. And here by the way take this

Advertiſement.

The forementioned *Decimal Scales*, and theſe *Lines* of the *Square* and *Cube Roots*, as alſo *Nepairs Bones*, whoſe uſe followeth, are made in Silver, Braſs or Wood, by Mr. *Walter Hayes* at the *Croſs-Daggers* in *Moor-Fields*, who alſo maketh all ſorts of *Mathematical Inſtruments*.

SECTION I.

Concerning the

FABRICK *and* INSCRIPTION

Of these

RODS.

IN the foregoing Argument I told you, That the Author and Inventer of this kind of Instrument, of which I intend to shew the Use, called it *RABDOLOGIA*, and the Word he thus defines: *RABDOLOGIA est Ars Computandi per Virgulas numeratrices.* That is, *RABDOLOGIA* is the Art of Counting by Numbering Rods.

I. *Of the Fabrick of these Rods, according to the Inventors Description of them.*

These Rods may be made either of *Silver, Brass, Box, Ebony, or Ivory*, of which last substance I suppose they were at first made, for that they are (for the most part) by all that know or use them, called *NEPAIRS-BONES.*

But let the matter of which they are made be what it will, their form (according to this description) is exactly a square Parallelepipedon, the length being about three Inches, and the breadth of them about One tenth

tenth part of the length. But the length of theſe Rods are not confined to three Inches, but let the length be what it will, the breadth muſt be a tenth part thereof, but that may be accounted a competent breadth that is capable of receiving of two numerical Figures, for there is never upon one Rod required more to be ſet on the breadth thereof.

The breadth of theſe Rods being exactly one tenth part of the length thereof, when 10 of theſe are laid together they do exactly make a Geometrical ſquare, and if 20 of them be tabulated or laid together, they will make a right-angled Parallelogram, whoſe length is double to its breadth. If 30 be tabulated, the Figure will be ſtill a Parallelogram, whoſe length will be three times the breadth, and ſo if 40, four times the length, *&c.* 650.

The Rods being thus prepared of exact length and breadth, let each of them be divided into 10 equal parts, with this *Proviſo*, that Nine of the Ten parts ſtand in the middle of each Rod, and the other tenth part muſt be divided into two parts, half whereof muſt be ſet at the one end, and the other half at the other end of the ſame Rod. Then from ſide to ſide draw right Lines from diviſion to diviſion, ſo is your Rod divided into Squares on every ſide thereof. Laſtly, from corner to corner of every of theſe Squares draw a Diagonal Line, and that will divide every Square into two Triangles. The Rods being thus prepared and lined, firſt into Squares, and then into Triangles, they are then fit to be numbred.

The Figure 1. at the beginning of the Book ſhews the Form of one of theſe Rods lined as it ought to be.

SECT. II.

How these Rods are to be Numbred.

IN the two half Squares which are at the ends of each Rod on every side, there are set one single Figure, on each side of every Rod one, in the division at the end thereof, so every Rod containing four sides, Ten Rods will contain 40 sides, and so consequently will have 40 single Figures at the ends of every of them; that is, there will be upon the Ten Rods amongst them, four Figures of each kind, that is, four Ones, 1111. four twos, 2222. four threes, 3333. four fours, 4444. four fives, 5555. four sixes, 6666. four sevens, 7777. four eights, 8888. four nines, 9999. four Cyphers, 0000.

And here it is to be noted, That what Figure soever it be that standeth at the top of the Rod alone, the Figure that standeth alone on the other side of the same Rod, maketh that figure up the number 9. As for Example, If 1 stand on one side, 8 will stand on the other side, so 2 and 7, &c. As in this Table, where,

	1		8	
	2		7	
	3	stands alone	6	standeth on
	4	at the top of	5	the other
If	5	any side of	4	side of the
	6	any of the	3	same Rod.
	7	Rods, then	2	
	8		1	
	9		0	

This

This alſo is to be obſerved in the figuring of every Rod, that what figure ſoever ſtandeth alone at the top or ſuperiour part of the Rod, the figure or figures that ſtand in the two Triangles next underneath it, is double to the figure which ſtandeth at the top. And the figures which ſtand in the next two Triangles below, that is three times as much as the figure above. And that in the fourth place, or Triangles, is four times as much as the figure above, *&c.* till you come to the loweſt Triangles in that Rod, and then the figure or figures that ſtand in thoſe Triangles are nine times as much as the figure which ſtandeth at the top of the Rod.

So if a Rod have 4 at the top thereof, in the two Triangles which are juſt and next under it, hath 8 in them, which is double to 4: in the next two Triangles below there is 12, which is treble to 4; in the two Triangles below them is 1 and 6, which together make 16, which is four times as much as the 4 at the top; the next Triangles have in them 20, which is five times as much; the next 24, which is ſix times as much; the ſixth hath 28, which is ſeven times as much; the next Triangles have in them 32, which is eight times 4; the laſt hath 36, which is nine times as much. All which is viſible by the Figure 2 at the beginning of the Book.

And is evident enough by this little Table following, which is the Table of Multiplication, commonly called *Pythagoras* his Table.

Figures

Figures at the top of each Rod.

	0	1	2	3	4	5	6	7	8	9
The Figures in the First	0	1	2	3	4	5	6	7	8	9
Second	0	2	4	6	8	10	12	14	16	18
Third	0	3	6	9	12	15	18	21	24	27
Fourth	0	4	8	12	16	20	24	28	32	36
Fifth	0	5	10	15	20	25	30	35	40	45
Sixth	0	6	12	18	24	30	36	42	48	54
Seventh	0	7	14	21	28	35	42	49	56	63
Eighth	0	8	16	24	32	40	48	56	64	72
Ninth	0	9	18	27	36	45	54	63	72	81
Square, which are	1	2	3	4	5	6	7	8	9	
Times as much as the Figure at the top.	1	2	3	4	5	6	7	8	9	

Thus

have

Fig: 1.

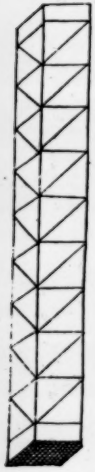


Fig: 2.

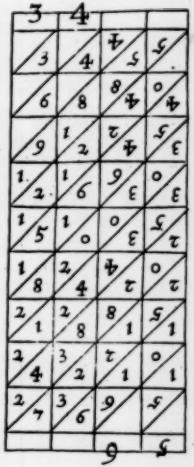
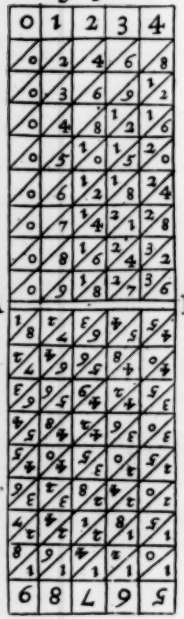


Fig: 3.



A B

Place this Against. 121. part third to fold out.

Fig: 4.

1	3	4	9	6	3 4 9 6
2	6	8	1	2	6 9 9 2
3	9	2	7	8	1 0 4 8 8
4	2	6	3	4	1 3 9 8 4
5	5	0	5	0	1 7 4 8 0
6	8	4	5	6	2 0 9 7 6
7	1	8	3	2	2 4 4 7 2
8	4	2	7	8	2 7 9 6 8
9	7	6	1	4	3 1 4 6 4

The Tabulat with Rods on it

Square.

1	2	1
4	4	2
9	6	3
16	8	4
25	10	5
36	12	6
49	14	7
64	16	8
81	18	9

Cube.

1	1	1
8	4	2
27	6	3
64	8	4
125	10	5
216	12	6
343	14	7
512	16	8
729	18	9

Thus have you the *Fabrick, Inſcription and Numbering* of theſe Rods, according to the Inventors contrivance of them: He makes mention of Ten of them, and hath in his Book ſet the figure of the ſaid Ten, of one of which Ten I have given you a Scheme at the beginning of the Book, which is *Figure* 2. I will now proceed to give you the deſcription of theſe Rods in another more commodious form.

SECT. III.

A Deſcription of theſe Rods according to their beſt and lateſt Contrivance.

THe Deſcription which I ſhall here give of theſe Rods, varies not at all from that before delivered in the matter of which they are made, for theſe may be made either in Silver, Braſs, Wood, Ivory, *&c.* Neither do they differ in their dividing, nor yet in their numbering: Only, whereas my Lord *Nepair* maketh them ſquare, each Rod to contain four ſides, theſe are made flat, conſiſting each Rod but of two ſides, and contain in length about $2\frac{3}{10}$ Inches.
and in breadth $\frac{1}{4}$ of an Inch.
and in thickneſs $\frac{1}{12}$ of an Inch.

One ſet of theſe Rods conſiſteth of five pieces, and therefore hath but ten Faces or ſides, whereas thoſe of the Lord *Nepairs* conſiſted of 40 Plaines or ſides.

Upon one of theſe five pieces (a Figure whereof is at the beginning of the Book, noted with *Figure* 3) you have a Cypher at the head of the firſt piece, and 9 at the bottom thereof. Upon the ſecond of them you have

have 1 at the head, and 8 at the bottom: upon the third you have 2 at the head, and 7 at the bottom; upon the fourth 3 at top, and 6 at bottom; and upon the fifth you have 4 at the top, and 5 at the bottom. Every of the two Figures at the top and bottom together make 9; as 0 and 9 is 9, 1 and 8, 2 and 7, 3 and 6, 4 and 5. And here observe, that the Figures 9 8 7 6 5, which stand at the bottom of the Scheme, stand with their heels upwards, in this manner, 6 8 L 9 S, and so do all the other figures under them, till you come to the double Line which is in the middle of the Scheme, noted with *A* and *B*, at which Line, if the Scheme were cut into two pieces, and folded or pasted on the back-side of the other half, so that the 9 at the bottom were placed upon the Cypher at the top, and so 8 upon 1, 7 upon 2, 6 upon 3, and 5 upon 4, and then the Scheme cut again into five little slippets by the down-right Lines; these five slippets would exactly represent one set of these Rods, for upon one side of one of these pieces, you should have a Cypher upon one side, and 9 on the other: Upon the next 1 and 8, upon another 2 and 7, on another 3 and 6, and on the other 5 and 4; both the Figures on either side making 9, as before was described.

These five slippets do now contain the whole Table of *Pythagoras* before mentioned, but so few are not of sufficient use, neither are the Ten before mentioned of the Lord *Nepair*'s order; for there can be but four Figures of one kind, which in all cases is not sufficient.

Therefore, as these Rods are made now a days, they do commonly make six sets of them, that is, 30 pieces, which contain 60 faces, and these will be of good use, and there will seldom be found a want, which in those of the Inventor's there will often be, except you have a great quantity, which will be far more cumbersom than

than these here described, for there is required as much Metal or Wood in one of his, as in four of these, and then for his Four sides we have here Eight.

Concerning a Case *for these Rods.*

For the orderly keeping and ready finding of these Rods, I have often (for my self and others) had a Box made of Walnut-tree or Pear-tree, with five partitions in it, each partition to hold five or six sets of these Rods, or more if more Rods were required. Every of these partitions being figured on the side thereof next the Eye, with such figures as the Rods in such a partition had figures at the top, so that the party that was to use them, could take them as readily out of his partition, as a Printer can take his Letters out of his respective Boxes to make any Word.

In this Box there is also convenient Room made for one other Rod, double in breadth to these here described, but of the same length and thickness; upon the one side whereof there is a Table or Plate useful in the Extracting of the *Square Root*, and on the other side another for the Extracting of the *Cube Root*, the Figure whereof is at the beginning of the Book, noted with *Square*, *Cube*: But I shall forbear to say any thing of them, till I come to shew you how to Extract the *Square* and *Cube Roots* by the help of them and the *Rods*

Of

Of a Board with a Frame, upon which to lay your Rods, when any Operation is to be wrought by them, known by the name of a TABELLET.

In the using of these Rods, care is to be had first of the orderly laying of them, and then secondly, for the keeping of them in that position till your work be ended. For the effecting whereof, both neatly and certainly; there is a little Table or Frame contrived, containing in breadth $\frac{1}{10}$ of an Inch more than the length of the Rods, and in length at pleasure, but it may well be about once and a half the length of the breadth.

It ought to be made of a thin piece of Pear or Walnut-tree, or of such matter as your Box or Case is made of, and it may very commodiously be contrived to be put into the Box as ever I had them made to do, for that I found it convenient to carry loose.

Upon the Superficies of this Board, close to one of the edges thereof, must be glewed or otherwise fastened with Pins, a small piece of the same matter, and also of the same length, breadth, and thickness of one of your Rods, which must be divided into 9 equal parts, and Lines drawn cross the piece, so will there be 9 Squares, in which you must Grave or Stamp the nine Digits, beginning with 1 at the top, and so descending by 2 3 4 to 9 at the bottom thereof: And it were necessary that these Figures (as also those which are at the head of every of your Rods) were Graven or Stamped of something a bigger Figure than the other figures of your Rods are.

Under the end of this ledge beginning at the Figures, and so continuing the whole length of the Board, must another ledge of the same matter and thickness, be glewed, or pinned, and then is your *Tabellet* finished. A Figure whereof you have at the beginning of the

Book.

Book, noted with *Figure* 4, it is called a *Tabellet*, for that, when the Rods are laid thereon, for any Operation to be wrought by them, we uſually ſay, the Rods are Tabellated.

Being thus prepared with Rods and *Tabellet*, you are ready for the work intended by them, and for which chiefly they were invented.

Thus much for the *Fabrick*, *Inſcription*, *and Numbering* of theſe Rods; let us now come to ſhew the Uſes of them.

SECT. IV

To what Uſe theſe Rods generally ſerve.

THe chief Uſes which theſe ſmall Rods ſerve unto, are all manner of Multiplications and Diviſions, as alſo of the Extraction of both the Roots, either Square or Cube; all which are ſo facilly and expeditiouſly performed, and that by the help of Addition and Subſtraction only, that it is inconceivable; for here is no charge at all required of the Memory, and you ſhall aſſuredly take your Quotient-Figure in Diviſion always certain; neither too great nor too little, an inconvenience ſo prejudicial, that I leave it to the cenſure of ſuch as have found it, to their great loſs of time, and other vexation which it hath put them to. But ceaſing to ſay more of their properties, I will now come to ſhew their Uſe.

SECT.

SECT. V.

How to apply or lay down any Numbers by the Rods.

PROP. I.

Any Number being given, how to Tabulate or lay down the same by the Rods.

LEt it be required to Tabulate or lay down this Number 3496.

First, From among your Sets of Rods, (or out of your Case) take four of them, of which let one of them have the Figure 3 at the top thereof, and lay it upon your Tabellet close to the edge thereof, then,

Secondly, Take another Rod from your Case, which hath the Figure 4 at the top of it, and lay that also upon your Tabellet close by the side of the other.

Thirdly, Take another Rod which hath the Figure 9 at the top of it, and lay that upon your Tabellet close by the other two.

And lastly, take a fourth Rod, having the figure 6 at the head thereof, and lay that also upon your Tabellet close by the rest.

These four Rods thus taken, and laid upon the Tabellet, you shall see in the uppermost Row (which standeth against the Figure 1 on the side of your Tabellet) these four Figures, 3 4 9 6, that is 3496, equal to your given Number. In the second Row (against the figure 2 of your Tabellet) you shall find the double thereof:

thereof. In the third (againſt the figure 3) you ſhall find the triple thereof. In the fourth the Quadruple thereof. In the fifth the Quintuple; and ſo on the ninth and laſt, in which you ſhall find the Nonuple of the Number given.

PROP. II.

How theſe Rods will appear when Tabulated, and being Tabulated, how to read the Multiplication of that Number ſo Tabulated, by any of the Nine Digits.

The Four Rods being Tabullated according to the Precepts delivered in the preceding *Propoſition*, they will appear exactly as they are repreſented in *Figure* 4 at the beginning of the Book, which Figure lively repreſents the four Rods lying upon the Tabellet, which mind well, for upon the true Tabulating, and right reading of the Rods ſo tabulated, depends the whole Work.

The Rods thus Tabulated, and as you ſee them in the *Figure* 4, do to the Eye appear in the form of a Glaſs-window, every Pane thereof repreſenting a Rhomboyades or Diamond-form: In the reading of the Figures which are in theſe ſeveral Rhomboyades or Diamond-form, obſerve theſe few Directions following, which will fully illuſtrate the whole buſineſs intended, and therefore eſpecially to be minded.

Note,

1. *That the Figures upon the Rods are to be read beginning at the right hand and reading towards the left*; which is contrary to our common courſe of reading and writing, which is from the left hand towards the right:

II. *That in every Rhomboyades or Diamond, there are either One Figure, or Two Figures, but never more than Two.*

III. *If there be but one Figure in a Rhombus, then that Figure is the Figure to be set down alone* (be it either a Figure or a Cypher) *but if there be two Figures in a Rhomboyades* (as for the most part there is) *then add them two Figures together, and set down their sum in one Figure.*

IV. *But if the sum of the two Figures in one Rhomboyades or Diamond do exceed Ten, then you must set down the overplus above Ten, and keep One in mind, which One you must carry to the next Rhomboyades.*

V. *Note that the first towards your right hand, and the last towards your left hand are but half Rhomboyades or Diamonds, and never have in them more than one Figure only, but all between them are whole ones, and for the most part have two Figures in them.*

VI. *If in either Rhomboyades or half Rhomboyades, you find no Figures but Cyphers, you must not neglect but set them down as if they were Figures.*

¶ These Rules being rightly understood, all that follows will be familiar and easie, and these I shall explain by Example following.

Example.

For the illustration of the preceding Rules, we will make use of those Rods which were before tabulated, therefore have recourse to *Figure* 4 at the beginning of the Book, where this Number 3496 is tabulated.

The

The Figures at the top of the Four Rods are theſe: 3,4,9,6. which ſignifie the former given number 3496, and this number ſtands againſt the figure 1 on the ſide of the Tabellet. Then I ſay, that the figures in the next row ſtanding againſt the figure 2 of the Tabellet are double thereunto, which I thus prove.

Repair to the Rods as they lie upon the Tabellet, and in that row which lieth againſt the figure 2, you ſhall find in the firſt half Rhomboyades towards your right hand (where *by Rule* 1 you muſt begin) the figure 2, wherefore ſet down with your Pen upon Paper the figure 2. In the next Rhomboyades, in the ſame row you ſhall find 8 and 1, which added make 9, ſet down 9 on the left hand of 2: In the next Rhombus you ſhall find 8 and 1 again, which is 9 alſo, ſet down 9 on the left hand of the other, and in the laſt Rhomboyades you ſhall find only 6, wherefore ſet down 6 on the left hand of 9, ſo have you in all 6992, which is double to 3496.

Again, the figures in the row which ſtands againſt the figure 3 in the Tabellet, are triple to 3496; for in the firſt half Rhomboyades towards your right hand, you have 8, ſet down 8.— In the next Rhom. you have 7 and 1, which is 8, ſet down 8 again.— In the next you have 2 and 2, which is 4, ſet down 4.— In the next Rhom. you have 9 and 1, which makes 10, ſet down 0 and carry 1, but it is the laſt Rhom. and becauſe there is never another to carry the 1 unto, you muſt therefore ſet it down, ſo have you this number 10488, which is triple to 3496.

Again, the figures ſtanding againſt 4 in the Tabellet, are Quadruple to 3496,—for in the half Rhom. you have 4, ſet it down: in the next 6 and 2, which is 8, ſet that down: In the next 6 and 3, which is 9, ſet that down: In the next 2 and 1, which is 3, ſet that down: and in the laſt half Rhom. you have 1;

which also set down: so have you 13984, which is Quadruple to 3496.

Also, the figures against 5 in the Tabellet: the first is a Cypher, therefore put down 0; the next is 5 and 3, which is 8, set down 8; the next is 0 and 4, set down 4; the next is 5 and 2, that is 7, set down 7; and the last is 1, therefore set down 1, so have you in all 17480, which is Quintuple to 3496.

Against 6 in the Tabellet, you have in the first place 6, set it down; then in the next 4 and 3, that is 7, set that down; in the next 4 and 5, that is 9, set 9 down; in the next you have 8 and 2, that is 10, set down 0 and carry 1 to the next Rhom. where you find only 1, to which add the 1, which you carried from the Rhom. before, and it makes 2, set down 2: so have you 20976, which is six times 3496.

Against 7 in the Tabellet, you have first 2, set it down; then 3 and 4, which is 7, set 7 down; in the next 8 and 6, which is 14, which being above 10, set down 4, and carry 1 to the next Rhom. where you have 2 and 1, which is 3, and 1 which you carried makes 4, set down 4; then in the last place you have only 2, which set down, so have you in all 24472, which is Septuple to 3496, or seven times as much.

Against 8 in the Tabellet, you have first 8, which set down; then 2 and 4, which is 6, set 6 down; then 2 and 7, which is 9, set 9 down; then 4 and 3, which is 7, set 7 down; and lastly 2, set that down; so have you 27968, which is Octuple to 3496, or eight times as much.

Lastly, against 9 in the Tabellet, you have in the first place 4, set that down; in the next you have 1 and 5, which is 6, set 6 down; in the next place you have 6 and 8, which is 14, set down 4, and carry 1 to the next Rhom. where you find 7 and 3, that is 10, which with 1 which you carried makes 11, set down

down 1, and carry 1 to the next Rhom. where you find only 2, and the 1 carried makes 3, therefore set down 3, and so you have 31464, which is Noncuple to 3496, or nine times as much as the tabulated number.

Thus have I given you Examples, in shewing you how the Numbers upon the Rods are to be read and written down; and in the delivery of this Example, I have made the whole work which is to follow, so plain and easie, that the meanest capacity (I think) if he can but tell his figures, and add any two figures together, he may by this here delivered, read or write down any number that can be tabulated; and that you may throughly understand this Chapter before you proceed further, I will give you the Products of 7009078 multiplied by all the nine Digits, which I would have your self to tabulate, and see if you find your working by your Rods to agree with those which are here written, which numbers if they do, you need not scruple at the most difficult that can be proposed to you, therefore study it, and try it.

7009078 being multiplied by

	Produceth
2	14018156
3	21027234
4	28036312
5	35045390
6	42054468
7	49063546
8	56072624
9	63081702

Thus have I sufficiently described these Rods, and the manner of Numbring upon them; and now I think it time to apply them to that use for which they were intended, namely,

namely, the more difficult parts of Arithmetick, as Multiplication, Division, and Extraction of Roots: But first let me give you

An Admonition concerning Addition and Subſtraction.

Whereas it was the difficult operations of *Arithmetick*, which by the benefit of theſe Rods, the Inventor chiefly aimed at (of which kind he eſteemed *Multiplication*, *Diviſion*, and *Extraction of the Square and Cube Roots*) he omitted to ſay any thing concerning *Addition* and *Subſtraction*, as things obvious to every *Tyro*, he therefore omitting them, begins to ſhew the uſe of his Rods in *Multiplication*, whoſe Method I ſhall here follow.

SECT. VI.

Multiplication by the Rods.

IN Multiplying by the Rods, you are to conſider (as in vulgar Arithmetick) three Terms, Things, or Numbers, *viz.*

1. The *Multiplicand*, which is the Number to be multiplied.

2. The *Multiplier*, which is the Number by which the *Multiplicand* is multiplied.

3. The *Product*, which is the ſum produced by the multiplying of the two former together.

And here note, that the *Product* doth contain the *Multiplicand*, ſo many times as there be *Unites* in the *Multiplier*.

Thus for the definition of *Multiplication*, now for the working thereof by the Rods, for which this is

THE

THE RULE.

Firſt, Set down upon your Paper the Multiplicand, and orderly under it the Multiplier. It matters not greatly which of the two given Numbers be made *Multiplicand* or *Multiplier*, but it is uſual and beſt to make the greateſt Number *Multiplicand*, and the leſſer *Multiplier*. *Then draw a Line with your Pen under them, and having Tabulated your Multiplicand (or greater Number) look what Numbers in your Rods ſtand againſt the firſt Figure towards your right hand, and that Number which you ſhall find upon your Rods ſtanding againſt that firſt Figure found in your Tabellet, ſet down under your Line which you formerly drew under your Multiplicand and Multiplier: And having ſo done with the firſt Figure of your Multiplier, do ſo with the reſt, ſetting them down one under another, removing every Figure one place more towards the left hand, then that which went before it, as is done in common Multiplication, and as you ſee in the following Example.*

Example 1. *Let it be required to multiply* 3496, by 489. As if it were required to know how much 489 times 3496 would amount unto.

Firſt, Set down your given Numbers 3496, and 489, one under another, and draw your Line under them, as here you ſee done.

Secondly, 3496 your Multiplicand being Tabulated, and 9 being the firſt Figure to the right hand in your Multiplier, look upon your Rods, what ſum there ſtands againſt 9 in the ſide of your Tabellet, and you ſhall find (as by the Rules in the ſecond Prop. of the Fifth Chap. you were directed)

3496 Multiplicand,
489 Multiplier,

31464
27968
13984

1709544 Product.
Sec.

31464, which is the Product of 3496 multiplied by 9, wherefore set down this Number 31464 under your Line, as you see in the Example.

Thirdly, Look what sum upon the Rods stands against 8, which is the second Figure of your Multiplier, and you shall find 27968; set this Number under the former, moving it one place forward towards the left hand.

Fourthly, Look what sum upon the Rods stands against 4, which is the third Figure in your Multiplier, and you shall find 13984, which set down under the other, one place more to the left hand.

Lastly, Under these three sums draw a Line, and add the three sums together, and they make 1709544, which is the Product of 3496 multiplied by 489, and this 1709544 the Product, contains 3496 the Multiplicand, 489 times.

Practise well this first Example, and compare it with the Rods as they are Tabulated in Figure 4 at the beginning of the Book, as also with the Rules in the Fifth Chapter, and you may perform any Multiplication. However I will give you one or two more Examples, and some other ways of *Multiplication*.

Example 2. *Let it be required to multiply the same sum* 3496, by 261.

3496	Set the Numbers down as here is done, then
261	look upon the Rods for the Product of 3496
———	by 1, and you shall find it to be the same,
3496	wherefore set down 3496 under the Line—
20976	then look upon the Rods for the Product of
6992	3496 by 6, and you shall find it to be 20976,
———	which set down under the other Number, one
912456	place more towards the left hand.——A-
	gain, look in the Rods for the Product of
	3496

3496 multiplied by 2, and you ſhall find it to be 6992, which ſet down under the other two.

Laſtly, Draw a Line under them, and add the three Numbers together in order as they ſtand, and the ſum of them will be 912456, which is the Product of 3496 multiplied by 261.

Example 3. *Let it be required to multiply the ſame Number* 3496 by 520.

Set down your Numbers as here you ſee done——

3496	Then becauſe the firſt Figure of your Mul-
520	tiplier towards your right hand is a Cypher,
———	wholly omit it, and multiply 3496 by 52
6992	only, ſo ſhall you find the Product of 3496
17480	by 2, to be 6992, which ſet down: Alſo
———	the Product by 5 will be 17480, which
1817920	ſet down under the other, one place fur-

ther; Then draw a Line——and add theſe two ſums together, and they make 181792, to the which if you add a Cypher for the Cypher which you omitted in your Multiplier, the ſum will be 1817920, which is the Product of 3496 by 520.

Example 4. *Let it be required to multiply the ſame* 3496 by 7003——

Set down your Numbers as before, and as you ſee here done; Then having Tabulated 3496,

3496	ſee what the Product thereof is upon the
7003	Rods, being multiplied by 3 the firſt Figure
———	in your Multiplier, and you ſhall find it
10488	to be 10488, which ſet down under the
24472	Line——Then the two next places of
———	your Multiplier being Cyphers, make two
24482488	pricks under the former Number, one un-

der 8, the other under 4, as you ſee in the Example; or inſtead of 2 pricks you may make

two

two Cyphers,—Then look in the Rods for the Product of 3496 by 7, and you ſhall find it to be 24472, which ſet down under the other ſum, beginning your number at the fourth place, or beyond the two Pricks or Cyphers. Laſtly, draw a Line and add theſe two ſums together, and their ſum is 24482488, which is the Product of 3496 multiplied by 7003.

Thus have you four Examples in *Multiplication*, in which are included all the Varieties that may at any time happen in that Rule, *viz.* Two where the Multiplier conſiſted all of Figures, as in the firſt and ſecond Example they did.—Another where the latter place of the Multiplier conſiſted of a Cypher——And this laſt Example where Cyphers were intermixed among the Figures.

And thus much for this kind of Multiplication, but before I leave, I will ſhew you

Another Form of

MULTIPLICATION.

Whereas in the foregoing Form of Multiplication, which is the beſt and moſt uſual, (only I inſert this following for variety.) You began (your Rods being Tabullated) with that Figure of your Multiplier which ſtands next your right hand, but there is no neceſſity for that, for you may begin with that Figure which ſtandeth next to your left hand, and by ſo doing, and placing your ſeveral Products one place more to the right hand, as you did before place them to the left hand, thoſe Products added together in the Form they then ſtand, ſhall produce a Product equal to the former.

Example, For our Example we will take the firſt Example before-going at the beginning of this Section,

on, where it was required to Multiply 3496 by 489. Set the numbers down as before in that firſt Example, and as you ſee here done.

Then 3496 being Tabulated, look upon your Rods, for the Product thereof multiplied by 4, (which is the firſt Figure of your Multiplier towards your left hand) and you ſhall find the Product thereof to be 13984, which ſet down.—— Secondly, look the Product of 3496 by 8 (your ſecond Figure) and you ſhall find it to be 27968, which muſt not be ſet down as in the other firſt Example but as you ſee it in this, 8 the firſt Figure thereof muſt be ſet one place forwards towards the right hand, as in the other it was ſet a place backward towards the left.——Laſtly, ſeek in your Rods for the Product of 3496 by 9 your laſt Figure, and you ſhall find it to be 31464, which ſet under the other two Numbers, yet one place more to the right hand.——So a Line being drawn under, and theſe three Numbers added together produce 1709544 equal to that in the firſt Example: And that you may the better ſee the difference of the work, I have ſet them one by the other.

3496	3496
489	489
31464	13984
27968	27968
13984	31464
1709544	1709544

One

One Example more in Multiplication, which shall be for Advertisement and direction, I will give, and so conclude Multiplication.

I said in the gencarl Rule for working of *Multiplication* (at the beginning of this Section) that it mattered not which of your Numbers were made the Multiplicand, or which the Multiplier, of which I will here give you a President where the lesser Number shall be Tabulated, and the greater Number only set down; and I will work it here according to this last way of Multiplication, and the Example shall be as followeth.

Example, *Let it be required to multiply* 868437 by 3496, and let 3496 (the lesser Number) be Tabulated

Let the Numbers be set as you here see, then 3496 being Tabulated, begin with the first Figure towards the left hand of your Multiplier, which here is 8, and upon your Rods find the Product of 3496 multiplied by 8, which is 27968, set that down under the Line——then find the Product of 3496 by 6, the second Figure of your Multipler, and you shall find that to be 20976, set this number under the former, one place more towards the right hand ——Again the third Figure of your Product is, 8 whose Product is 27968, as before, set that under the other, still one place more to the right hand.——In this manner do with the other Figures of the Multiplier, as 4 the next Figure, whose Product is 13984, which also set down a place forward.——So also the Product of 3 which is 10488, which set down.—And lastly,

```
  3496
868437
------
 27968
  20976
   27968
    13984
     10488
      24472
-------------
3036055752
```

laſtly, of 7, which is 24472.——All theſe Products being ſet down in the order as you ſee them in the Margent, if you add them together, the ſum of them will be 3036055752, which is the Product of 3496 multiplied by 868437, the leſſer number being Tabulated.

Other ways of Multiplication I could have added, but theſe I eſteem ſufficient.

SECT. VII.

DIVISION *by the Rods.*

AS in Multiplication, ſo in Diviſion there are three Numbers, Terms, or Things required, *viz.*

1. The *Dividend*, or number to be divided.
2. The *Diviſor*, or Number by which the Dividend is divided, and
3. The *Quotient*, which is the Number iſſuing from the Dividend's being divided by the Diviſor; And this *Quotient* doth always conſiſt of ſo many *Unites* as the *Diviſor* is times contained in the *Dividend.*

Thus much for the *Definition* of *Diviſion*, now let us come to the *Practice* of it by the *Rods*, to perform which, this is

THE RULE.

Tabulate the Diviſor, (which is always the leſſer Number of the two given) and ſet down the Dividend, and ſet the Diviſor on the left hand, and draw a crooked Line on the right hand for your Quotient, as in common Arithmetick.

tick. *Then look upon your Tabulated Rods (always) for the Number leſs than the Number in the firſt Figures of your Dividend, and what Figure ſtands againſt that Number on the edge of your Tabellet muſt be the Figure you muſt put in your Quotient, and that Number you muſt always ſubſtract from the Figures of your Dividend, and to the remainder add another Figure, ſo proceeding from Figure to Figure till your Diviſion be wholly ended.*

Example, *Let it be required to divide* 1709544, by 3496. Having Tabulated 3496, ſet down your Dividend, your Diviſor on the left hand thereof, and a crooked Line for the Quotient on the right hand thereof, as by the Rule preceding you were directed, and as you ſee done in the Exanple adjoyning.

And becauſe at your firſt ſetting down of your Diviſor 3496, it would reach (if it were ſet under your Dividend 1709544) as far as the Figure 5, therefore under the figure 5 make a prick, to intimate how far you are gone on in your work, and under this prick draw a Line quite under your Dividend; then is your Sum ſet down ready for work, and will appear as here you ſee;

3496) 1709544(

Your Sum thus prepared, ask, how often can you have 3496 in 17095, look in your Tabulated Rods for 17095, which you cannot there find, but the neareſt number thereunto amongſt the Rods, which is leſs than 17095 (for you muſt always take a leſs number) is 13984, which number ſtands againſt the Figure 4 in the Tabellet, wherefore ſet 4 in your Quotient, and 13984 under the Line, and ſubſtract 13984 from 17095, and there will remain 3111, ſo is the firſt part of

of your Division ended, and your work will stand thus;

```
        3111
3496 ) 1709544 ( 4
        .
      ---------
       13984
```

Then make another Prick under 4, the next Figure of your Dividend, so will the remaining number be 31114,——Then look among your Rods for the number 31114 (or the nearest less than it) and the nearest less you shall find to be 27968, which stands against 8 in your Tabellet; put 8 in your Quotient, and set 27-968 under 31114, and substract 27968 from 31114, so will there remain 3146, which set over head, so is the second part of your Division ended, and your work will appear thus;

```
        3146
        3111
3496) 1709544 ( 48
        ..
      ---------
       13984
        27968
```

Lastly, Make another Prick under the next Figure of your Dividend, which is 4 also, making the remaining number to be 31464, seek among your Tabulated Rods for this number (or the nearest less) but looking you shall find the very number, against which stands on your Tabellet the Figure 9; set 9 in the Quotient, and the number 31464 under the Line, and substract it from 31464 the remainder which stands above

bove the Line, and nothing remains, and being there is never another Figure in your Dividend, your Division is ended, and your work will stand thus, and 3496 is contained in 1709544, 489 times.

```
                 00000
                 3146
                 3111
Divisor,  ) 1709544 ( Quotient
  3496 )      ...   ( 489
        ----------
                13984
                 27968
                  31464
```

Another Example, and by another way of Division.

Let it be required to divide 912456 by 3496, set down your Dividend and Divisor, draw a crooked Line for your Quotient, and also make a Prick under the fourth Figure of your Dividend, and draw a Line under your Dividend, so is your Sum prepared to be divided, and will stand thus;

```
3496 ) 912456 (
       .
       ------
```

Then your Divisor 3496 being Tabulated, look amongst your Rods for the nearest number to 9124 which is less, and you shall find it to be 6992, against which, stands on your Tabellet the Figure 2, set 2 in the Quotient, and this Number under the Line, and substract it from 9124, and there will remain 2132, to which number add the next Figure of your Dividend, namely 5, and it makes 21325, under which number draw

draw a Line, then will your Sum stand thus,

3496) 912456 (2
..

6992
21325

Then among your Rods seek the neareſt number to 21325 and you ſhall find 20976 to be the neareſt number leſs, againſt which, in your Tabellet ſtands 6, ſet 6 in the Quotient, and 20976 under the Line, ſubſtracting it from 21325, which when you have done, there will remain 349, to 349 add the next Figure in your Dividend, which is 6, your laſt Figure, and it makes 3496, under which draw a Line, and your work will ſtand as here you ſee,

3496) 912456 (26
...

6992
21325
20976
3496

This done, look amongſt your Rods for the neareſt number to 3496, and you ſhall find the exact number at the top of the Rods, againſt which ſtands the Figure 1 on the Tabellet, ſet 1 in the Quotient, and ſubſtract 3496 from 3496, the remainder is nothing, and ſo is your Diviſion ended, the work ſtanding thus, and 3496 the Diviſor is contained in 912456 the Dividend, 261 times.

```
3496 ) 912456 ( 361
          ...
       ------
        6992
        21325
       ------
        20976
          3496
       ------
          3496
          0000
```

A third Example ready wrought by the last and best way of Division. I will only set it down ready wrought, leaving the practice of it to your self.

Let it be required to divide 73020506 by 3496.

```
3496 ) 73020506 ( 20886 3050/3496
          .....
       ---------
        6992
         31005
       ---------
         27968
          30370
       ---------
          27968
           24026
       ---------
           20976
            3050
```

This Sum thus divided, produceth in the Quotient 20886, and 3050 remaining, so that the Quotient with Fraction and all is, 20886

20886 $\frac{3050}{3496}$ which shews

that 3496 the Divisor is contained in 73020506 the Dividend, 20886 times, and 3050 remaining.

This Example well practised, together with them beforegoing, are sufficient instruction for any Student whatever, and he that can perform these, need not despair of the most difficult that can be proposed. And so I conclude with Division.

SECT. IX.

Of the Extraction of ROOTS.

THe Extraction of *Roots*, which is the difficultest part of Multiplication and Division, is expeditiously and certainly performed by the Rods, for the easie and expedite performance of which, there are two Rods on purpose, one for the Square, the other for the Cube Root, of which I will speak; first, Of their Fabrick: secondly, of their Use.

Of the Fabrick of the Rods for Extracting of Roots.

Of the same matter, and of the same length and thickness of your other Rods, let there be made another Rod, but three times the breadth of the former, the Inscription on one side serving to extract the Square, and that on the other side for the Cube Root, each of which are divided into three Rows or Columes.

That which serveth for the Square Root, hath in

the top or uppermost Square between the Diagonal thereof, these Figures 0-1, in the second 0-4, in the third 0-9, in the fourth 1-6, in the fifth 2-5, in the sixth 3-6, in the seventh 4-9, in the eighth 6-4, and in the ninth or lowermost 8-1, which are the Square Numbers belonging to the nine Digits.

In the second Column of the same Rod, in the first Square is inscribed 2, in the second 4, in the third 6, in the fourth 8, in the fifth 10, in the sixth 12, in the seventh 14, in the eighth 16, and in the ninth 18.

In the last or third Column, there are the nine Digits orderly descending, namely, 1,2,3,4,5,6,7,8,9. This Rod thus made is fitted for the Square Root.

That which serveth for the Cube Root, hath in the top or uppermost Square of the first Column towards the left hand, between the Diagonal thereof, these Figures, 0-01, in the second 0-08, in the third 0-27, in the fourth 0-64, in the fifth 1-25, in the sixth 2-16, in the seventh 3-43, in the eighth 5-12, and in the ninth 7-29 which are Cube Numbers orderly descending.——The second Column of this Rod contains these Square Numbers, 1, 4, 9, 16, 25, 36, 49, 64, 81, orderly descending.——The third and last Column of this Rod hath in it the nine Digits, 1,2,3,4,5,6,7,8,9, orderly descending also.

This Rod thus prepared and inscribed, is fit for extracting of the Square and Cube Roots, a Figure of either side whereof you have at the beginning of the Book: That which serveth for the Square Root having the word *Square* written by the side, that for the Cube Root, hath *Cube* written by the side.

Thus having given you the Fabrick and Inscription of these Rods, I will now shew you their Use; And first,

SECT.

SECT. IX.

The Extraction of the Square Root.

TO extract the Square Root of any number, you must first prepare it, that is, set down the number on a paper, then under the first & lowest figure next the right hand, make a point with your pen, and under the third figure make another, under the fifth another point, and so forth, under every second figure of the number make a point, always leaving between each point one figure unpointed, according to the ordinary Rule by the pen; this being done, you shall see how many figures will be in the Root, for so many points as you have, so many figures shall you bring into the quotient for the Square Root of the number given; next draw a quotient-line, as in Division, and your number is ready prepared for Operation, and will stand as in the Example following, where the number given is 119025, and the Root square thereof is required, this number being set, and pointed, as afore is shewed, you may perceive that the Root thereof will be of three figures, because there be three points under the number given, the two figures belonging to the highest point next the left hand are 11. the two figures belonging to the second point are 90, & the two figures belonging to the third point are 25. and the figures for the Root answerable to those several points, are to be found by the Rods, as followeth.

```
119025 (
 . . .
```

Take the Frame, the Rods, and Lamina, and lay them

them before you; and first place the Lamina in the Frame next to the left hand ledge, with that side upwards, whereupon are the inscriptions belonging to the Square Root, and marked at the top with the Letter S. then consider what is the greatest square number in 11. the figures belonging to the first point; the Lamina presently sheweth you that the greatest square number in 11 is 9, and his Root is 3, for 3 times 3 is 9; therefore put 3 in the quotient for the first figure of the Root, then set 9 under the 11 and substract it therefrom, and there will remain 2, this 2 set over 11, and cancell the 11 as you use to do in Division, and so you have gained the first figure of the Root, and the work will stand as in this Example.

2	
11 9025	(3
9	

Having the first figure of the Root; to get the second, and so all the rest in order, you must proceed in this manner; duple the Root found, which duple place, or Tabulate upon the Rods between the Lamina and the ledge of the Frame; As in this example, the duple of 3, the Root found is 6, therefore place a Rod that hath in his top or upper square 6, between the ledge and the Lamina; Then look upon your number given, what figures, or number it is that belongeth unto the second point, which you shall see will be 290 in this our Example; Then turn your eye to the Lamina and Rod now Tabulated, and search thereupon what number will (being less yet) come nearest unto 290, the number out of which the second figure of the Root is to be found, And there you may soon see it is the number 256, which of any number upon the Rods, less than 290 cometh nearest thereunto, for the next greater number upon the Rods, above 256, is 325, which is greater than 290, and therefore cannot be

be taken out of it; but 256 is the only number to work withal, against which, on the ledge and Lamina, is this figure 4, this 4 must you put in the quotient for the second figure of the Root, and then subtract 256 out of that 290, and there will rest 34, this 34 set over the 290 in its due place, and cancell the 290, and the work will stand as in the Example. If you please, you may write your numbers to be subtracted, under the number out of which Subtraction is to be made, as I have done here in this Example, for instruction sake, or you may omit that if you please, being you have them before you upon the Rods.

234
119025 (34
• • •
9
256

And now for the third figure of the Root, look upon the number given, and there you shall see that the Remainder 34, with the two figures 25 still uncancelled, belonging to the third point, being all joyned together, make 3425, out of which the third figure of the Root is to be extracted; To find out what this third figure shall be, duple the Root found already, which is thus done very readily; Take forth a Rod that on his top Square hath 8, the duple of 4, the figure last found, and this Rod put into the Frame between the Lamina and the Rod that is already Tabulated; This being done, you have no more to do, but to look over the Rods and Lamina for such a number as will come neerest unto the number 3425, that belongeth to the third point, and is less than it, and the figure that you see against that number so found, put

in the quotient for the third figure of the Root; Thus looking upon the Rods you shall at the first sight find the very number it self 3425 that belongeth to the third point, in the fifth line of Squares, against the figure 5, therefore put 5 in the quotient for the third figure of the Root, and if you substract 3425, the number now found from 3425, that number belonging to the third point, there will be no remainder, so is the work done, and the number given, 119025 is a perfect square number, and the square Root thereof is 345, which was the thing required to be found.

	234
	119025 (345
	. . .
	9
	256
	3425

Now if you multiply this 345 by it self, it produceth 119025, the first given number, which proveth the work to be truly wrought; for note this evermore, that for proof of the Extracting the square Root, you must multiply the Root found, by it self, (to the product adding the remain, if any be) and the total will produce the first number given, if the work be truly wrought, otherwaies not.

For a second Example, let there be given this number 117716237694, and the Root thereof required: Now to perform the work, first write down the number, and then prepare it with points and a quotient-line, as you were instructed before in the former Example: This being done, the number will appear as here you see in this Example; Now in 11. the figures belonging to the highest point, the greatest square number is 9, whose Root is 3, and 2 remaineth to be set over head; Now for the second figure of the Root, Tabulate the duple of the Root found

	221
	117716237694. (34
	. . . . .
	9
	256

found, and that is no more to do, but to place a Rod that hath 6, the duple of 3, the Root found, in his top square, between the Lamina and the Ledge, and on the left hand of the Lamina, the said Lamina being Tabulated with that face upward, that is, for the square Root; then seeing that the number belonging to the second point, out of which the second figure is to be extracted, is in this Example 277, therefore search upon your Tabulated Rod and Lamina for such a number, as will come neareſt to that 277, which you shall quickly find to be 256, and right against it, on the right hand Column of the Lamina is the figure 4, therefore put 4 in the quotient for the second figure of the Root, and substract 256 from 227 and set the remainder 21 over the head of it, and also cancell the 227, and so have you done with your second point.

And now for the third figure of the Root, observe, that the 21 remaining with the other 16, the two figures uncancelled belonging to the third point, being joyned together, make 2116, out of which the same third figure is to be extracted; to perform which work, take a Rod that carrieth in his upper square the figure 8, the duple of 4, the figure last found, and put that Rod into the Frame between the Lamina and the other Rod before Tabulated,

then look for the number upon those two Rods, and Lamina, that will neareſt take away 2116, the number belonging to that third point, and at first sight you shall find that the next number lesser than 2116 is 2049, and his figure for the quotient, 3, therefore put 3 in the quotient, for the third figure of the Root, and substract 2049, from his respective number 2116,

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22167
117716237694 (3430
. . . . . .
9
256
 2049
```

and

and there remaineth 67 to set over head, this being done, and the 2116 being cancelled, you have done with the third point, and third figure, and are to proceed to the fourth.

Where first you see that the 67 last remaining, with the 23, make 6723, the figures belonging to the fourth point, whereout you must extract the fourth figure of the Root. Therefore go on as before, taking such a Rod, as in its top Square hath 6, the duple of 3, the figure last found, and Tabulate it between the Lamina and the other Rods, and then seek what number there can be found, that will be less than 6723, but at the very first sight you shall see no number upon the Rods so small as that 6723, for the very first number against 1, is 6861, which is greater than 6723, and therefore cannot be taken out of it, so that here you can put no figure in the quotient, but must supply the place with a Cypher, therefore put a 0 in the quotient for the fourth figure of the Root, all the rest of the number standing as it did; Now to the next point, which is the fifth, you have this number belonging 672376, whereout the fifth figure of the Root is to be found. Now here, in regard your last figure of the Root found is a 0, you must ever in such a case (in stead of putting a Rod, that hath the duple of the figure found in his top square,) you must take such a Rod as upon one of his faces carrieth only Cyphers, and Tabulate it between the Lamina and the other Rods already Tabulated; This being done, question the Rods, what number you must take from that

```
      5
  221674895
117716237694 (34309
 . . . . . .
 9
 256
  2049
   617481
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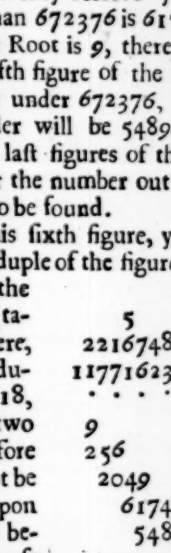
that number 672376, or what number it is there, that being leſs than 672376 yet cometh neareſt unto it; and alſo what the figure ſhall be that you muſt put in the quotient for the reſpective figure of the Root; The Rods will ſuddenly reſolve you, that their greateſt number leſs than 672376 is 617481, and its reſpective figure for the Root is 9, therefore put 9 in the quotient for the fifth figure of the Root, then if you will write 617481 under 672376, and make ſubſtraction, your remainder will be 54895, which joyned to the 94, the two laſt figures of the number given, make 5489594, for the number out of which the ſixth and laſt figure is to be found.

To find this ſixth figure, you muſt Tabulate upon the Rods the duple of the figure laſt found between the Lamina, and the Rods already tabulated; but here, becauſe the duple of 9 is 18, conſiſting of two figures, therefore this 18 cannot be Tabulated upon one Rod, as before we did uſe to do, when the duple was contained of but one figure, now in this caſe, (and ſo of all the like) Firſt, Tabulate a Rod bearing 8 in its top ſquare, between the Rods formerly Tabulated and the Lamina, and next to the Lamina, then for the unite 1, being the higheſt figure of the 18, you muſt Tabulate that upon the laſt and loweſt Rod, formerly Tabulated, which is done by encreaſing the figure in its top ſquare, one unite more than it was before; ſo here the laſt Rod before Tabulated, carrying only 0; either turn it, or elſe



else take it away, and place such a face of that Rod, or of some other, that hath in his upper square the unite 1 in room and stead of that face, with Cyphers, so is your number 18 the duple of 9 Tabulated, this done, look over the Rods, for a number that will come nearest unto 5489594, in the Eighth line is this number 5489504, and its respective figure for the Root, 8; this 8 put into the quotient, and Substraction being made as is used to be done, there will remain 90, and so is your whole work ended, and the square Root of 11771623769 4 is 343098, if you multiply this Root by it self, and to that product add the 90 that remaineth, you shall produce again the first number given, which argueth that the work is truly wrought.

But now when any thing remaineth, the Extraction being ended, as here it doth, make a Fraction of that remainder as you do in Division, in this manner; Set the number so remaining after the Extraction is ended, over a line for Numerator, and for the Denominator, set the duple of the whole Root found, with one added thereunto, as here in this example 90 remaineth; this 90 put over a line for the Numerator of the fraction, then double 343098 the Root, and it is 686196, to which add one unite, and then it will be 686197, this set down under the line for Denominator to the fraction, and then the true Root square of the number given will be $343098\frac{90}{686197}$ and will stand as in the Example. This is the vulgar, and ordinary way to make a fraction of the remainder.

But the best and most certain way to attain unto the true value of the fraction remaining, and that too by the Rods, very easie and speedy, is to add two, four, or six, Cyphers to the remainder, and continue the work of Extraction, and then your fraction will be in primes, seconds, thirds, &c. that is in 10 parts, 100 parts, and 1000 parts, in the same manner as in Division; for note,

note, that for every two Cyphers that be added or adjoyned to the number given, you shall have one fractional figure in the quotient, which will represent the fraction in Decimal parts of an unite, we will add 6 Cyphers to the remainder in this our last Example, and it will then be 90000000, and then we will continue the work, therefore Tabulate 16, the duple of 8, the figure last found; which to do, put a Rod that carrieth 6, the lowest figure of 16 next the Lamina, between it & the Rods afore Tabulated, & then instead of that Rod last in place next the Lamina, put another Rod that hath in his upper square one unite more than that, as here, change the Rod from 8 to 9, and the Rods are Tabulated, and you are now to look out a number that will neareſt take away 9000, the number belonging to the first fraction-point, but the Rods give you none so small, therefore put a 0 in the quotient for the first figure of the fraction, and because there is no more to do about this first figure, you are next to Tabulate a Rod with Cyphers next the Lamina, and then see for a number that will come neareſt unto 900000, the number belonging to the second point of the fraction, but yet you shall have none upon the Rods so small, therefore put another 0 in the quotient for the second fraction figure: again, Tabulate a Rod with 0 next the Lamina, and you shall yet again find no number on the Rods so small as 90000000, the number belonging to the third point, therefore put a third Cypher in the quotient for the third figure thereof. Thus have you done with your three points of Cyphers, which you first added, but because you are resolved to get at least on fraction-Figure, therefore add two Cyphers more to the remainder, making it 9000000000, and also Tabulate a Rod with 0 next to the Lamina, and then see if you can find a number little enough upon the Rods. And now here at last you shall find,

one

one significative figure in the quotient of your Fraction, for the number now belonging to the fourth Fraction point is 9000000000, consisting of 10 places of Figures, and the number Tabulated is also now become to be of 10 places, and withal the highest figures less than the highest figures of that number which belongeth to that fourth point, therefore seek upon the Rods for a number less than that 9000000000, and that you shall have in the second line upon the Rods, and it is 6861960004, whose respective figure for the root is 2; now substraction being made, there will remaine 2138039996. Thus have you gotten one figure into the quotient of your fraction, and that in the fourth place Descending, and may be thus expressed fraction-wise $\frac{0002}{10000}$, or thus, $\frac{2}{10000}$ signifying 2 parts of 10000 of an Unite; for note, that so many fractional points as you bring into the quotient to produce a new Numerator, the Denominator is always an Unite, with as many Cyphers as you have made fractional figures. This new found fraction joyned to the whole parts of the Root found, will stand as here in the Example; Or else without a Denominator, thus, which is all one with that other; Or else thus, according to *Simon Stevens*, and is thus to be read 343098, 0 primes, 0 seconds, 0 thirds, 2 fourths. These Examples might be sufficient to shew the excellent use of the Rods in Extracting the square Root of any

2138039996
9000000000 (0002
• • • •
6861960004

343098 $\frac{2}{10000}$

343098.0002

1 2 3 4
343098. 0. 0. 0. 2.

any number; but yet to ſhew the more variety of works, take one Example more; and if in any thing I be thought too tedious, know, that it is out of a deſire to plainneſs, even of ſuch a plainneſs as is anſwerable to that of the Rods.

Let there be given this number, 97419256 and the ſquare Root thereof required, Write down the number, and prepare it with points under each ſecond figure, and a quotient Line, and then proceed as before, and firſt look upon the Lamina, what is the greateſt ſquare number there, that can be had out of 97, the two figures belonging to the firſt and higheſt point; the Lamina ſheweth that it is 81, whoſe Root is 9, put this 9 in the quotient for the firſt figure thereof, and then ſubſtract 81 from 97, and that 97 cancel, the remaine is 16 for the ſecond figure, Tabulate 18, the Duple of 9, upon two Rods, between the Lamina and the Ledge, and then upon thoſe two Rods and Lamina ſeek out the number that comes neareſt to 1641, the number to the ſecond point belonging, and that you ſhall find to be the number 1504, and his reſpective figure for the Root 8, which being put into the quotient, and ſubſtraction made, according to the inſtructions afore-delivered, the number remaining to the Third point will be 13792, and to find out his proper figure for the Root, Tabulate 16 the duple of 8 laſt found, in this manner, place a Rod that carrieth in his top ſquare the figure 6 betwixt the Lamina and the former Rods, and increaſe the former lowermoſt Rod one unite, by changing it from 8 to 9, then ſhall you ſee the number upon the Rods neareſt un-

to

to 13792 is 13769 1
in the seventh line, 163723
and after Substraction made, there will rest 23, making the number for the least point to be 2356, now to find the respective figure of that fourth point, Tabulate 14, the duple of 7 as before you were instructed, and then you shall see at the very first, that no number upon the Rods is so small as that 2356, the number belonging to the last point, therefore put 0 into the quotient for the last figure of the Root, and so have you ended the work, for the whole part of the Root sought for, which in this Example appeareth to be 9870, and the Remainder 2356. But now to make a Fraction of this Remainder, as you were before shewed, set the same 2356 over a line, at the end of the whole part of the Root found 9870, and then duple the Root found, and to that duple add one unite, and the total will be 19741, which set under the line for Denominator, and then the whole work is finished, and the true Root found answering the demand is $9870\frac{2356}{19741}$, and standeth as in the example it appeareth.

97419256 ($9870\frac{2356}{19741}$
· · · ·
81
1504
13769

But if you desire to be yet more exact, and would have the truest value and estimate of your fraction; then turn it into a Decimal, and proceed as before; first add to the remainder so many times two Cyphers as you desire to have figures for your Numerator of your new fraction, that is to say, two Cyphers if you would have but only one figure, four Cyphers for two figures, six for three figures, and so forth, for as many as you would have, and untill you think your fraction is small enough: In this Example we add to the remainder three points of Cyphers, that is, six Cyphers, because

because we would have three Decimal figures in our fraction-Root; then Tabulate the duple of that figure of the Root last found, but because that is a 0 which doth neither increase nor diminish, therefore Tabulate a 0 next the Lamina, between it and the other Rods; next see upon the Rods what number there will come nearest to 235600, the number that now belongeth to the highest point of the fraction-points, and that is only the very first, *viz.* 197401, that will do it, therefore put 1 in the quotient, for the first fraction-figure, and then make Substraction; and there will remain for the second fraction-point 3819900.

To find the second fraction-figure, Tabulate 2, the duple of 1, and seek the number that commeth nearest to that remainder 3819900, and that is again only the first, therefore put 1 in the quotient, and make Substraction, and then to your third point will belong 184587900, and to find its respective figure for the Root, Tabulate 2, the duple of 1, last found, and inquire what number upon the Rods will come nearest to that 184587900, and that you shall find in the ninth line of squares to be 177662061, therefore put 9 in the quotient, and when Substraction is made, you will have remaining 6925839; thus have you three figures in the quotient, which are enough to give the fractions value in any ordinary question; if you please to continue the work the fourth figure will be 3, and now is your fraction turned into a Decimal fracti-

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          1
         600
       1849238
       381585867
      235699793951
97419256.00000000 ( 1193
          . . . .
      197401
       1974021
        177662061
         512207149
```

 on

on, whose Numerator is 1193, and his Denominator 10000, and being set fraction-ways, will stand thus $\frac{1193}{10000}$, Or it may very well be expressed without the Denominator, only with a line, or point of distinction, thus, 9870,1193, and so the value of this fraction is 1193 parts of 10000. Or according to *Simon Stevens*, 1 prime, 1 second, 9 thirds, 3 fourths, and more briefly may be thus expressed 1^{1}, 1^{2}, 9^{3}, 3^{4}, or thus, 1^{I}, 1^{II}, 9^{III}, 3^{IIII}. If you will, you may continue the Extraction unto 9 or ten degrees, but 3 or 4 for ordinary works is enough. If you multiply this Root found 9870. 1193 by it self, the Product will be 97-419254. 99623249, from whence, by a point or line cut off 8 places of figures, according to the rules of Multiplication in Decimal Arithmetick, and the number remaining towards the left hand, will want (the fraction considered,) but a very little more than one unite of the number first given, but if you take the paines but to continue the work to one place lower, it will not want an unite, and so the lower you work the fraction, the nearer still you come to the exact truth.

SECT. X.

The Extraction of the Cube Root by the Rods.

TO Extract the Cube Root of any number, you are first to write down the number given, whose Root is required, and make a point under the loweſt figure next to the right hand, and another point under the fourth figure, and so under every third figure, omitting between every two points two figures unpointed, and then so many points as you have

have under your number, ſo many figures ſhall you have in your Root; next draw a quotient-line, as in the Extraction of the Square Root, this being done, the number is prepared for Extraction; then to go to the work; firſt, ſeek what is the greateſt Cube number in the number ſtanding above, or belonging to the higheſt point next the left hand, which the Lamina will ſhew upon his left ſide of that face for the Cube Root, and in the Column, upon the right ſide of the ſame face is the Root thereof, and when either by your memory or Lamina, you have found the greateſt Cube Root in that number belonging to that firſt point, then (as in Extraction of the ſquare Root) Subſtract the greateſt Cube number, to that firſt point belonging, or that in the number, to the ſaid firſt point belonging, can be found from the ſaid number, and then cancell it, and ſet the remainder over head thereof, as before in the Extraction of the ſquare Root, and put the reſpective Digit number found in the quotient, for the firſt figure of the Root: Now to find the ſecond figure of the Root, (and ſo all the reſt, how many ſoever they be) you muſt always Triple the Root found, and that Triple multiply again by the Root found, and that laſt product Tabulate upon the Rods on the left hand of the Lamina, as before in the Extraction of the ſquare Root, you did the duple of the Root found, then look upon thoſe Tabulated Rods and Lamina together, what number you can find upon them will come neareſt to the number belonging to the number next following, and leſs than it, which number is the Diviſor, and the figure on the right hand of the Lamina is the figure for the Root anſwerable to that point, and that figure put in the Quotient for the reſpective figure of the Root, belonging and anſwerable to that point; but note always that you take the Diviſor ſo often, and no oftener, but that you may

yet also take another number from the number belonging to that point, which other number is square of the Digit new found, multiplied into the former Triple, the product add to the Divisor, with this proviso, that you place this new product one place higher towards the left hand than is the Divisor, that is to say, set the lowest place of that new product under the second place of the Divisor, the total of this Addition Substract from the number belonging to the point in action, cancelling the said number, and the remain set over head thereof, as you use to do in Division and Extraction of the square Root; and so proceed to the next point, if you have any more: But to make all plain, we will illustrate this by variety of Examples in all the kinds and differences of works.

First, let 110592 be a number given, and the Cubick Root thereof required; this Cubick Root is thus found, first prepare your number, that is to say, write it down, and make a point under 2, the lowest figure thereof next the right hand, and one other point under 0, the fourth figure thereof, leaving two figures between unpointed, then draw the quotient-line, and then the number will stand ready prepared as in the example, with two points, whereby it appeareth that the Root will consist of two figures, which are to be found out according to the former directions; and first observe that 110 is the number belonging to the first point, and upon the Lamina you may also observe, that the greatest Cube number in that 110 is 64, and his Cube Root 4, therefore put 4 in the quotient, for the first figure of the Root, and then Substract 64 from 110, and there will remain 46 this 46 set over the 110, and cancell 110, and the

46	
110592	(4
64	

wor

work of the first point is done, and here you may now observe, that the 46 remaining, with the other figures 592, make 46592, which is the number belonging to the second point, and where out the second figure is to be found.

To obtain this second figure, proceed in this manner, triple 4, the Root found, and it is 12, and that triple multiply by the Root found 4, and the Product is 48, this Product 48 Tabulate upon the Rods on the left hand of the Lamina, between it and the Ledge, then view over these Rods and Lamina thus Tabulated, what number there will come neareſt unto 46592, the number belonging to the second point, and be less than it, you shall see the number that cometh neareſt to it, is that in the ninth line, 43929, and his respective figure for the Root is 9; now square this 9, and it is 81, this 81 multiply by the former Triple 12, and it yieldeth 972, this 972 add unto 43929, the number found on the Rods (being set in Addition, one place higher than is ordinary, as was before shewed) and the total will be 53949, which if you compare with 46592, the number belonging to the point in action, you shall see it is too great to be taken out of it: Whereby it appeareth that you must not take 9 for your Root, for by your general rule you must not take the Divisor no oftner, but that you may take also the Product made by the square of that new Digit number multiplied into the first triple, out of that number belonging to that second point, therefore take a less number upon the Rods, as the number 38912 in the eighth line, which will surely serve the turn, wherefore put 8 in the quotient for the second figure of the Root sought for, now write (if you will) this number 38912, just under 46592, the number belonging to the said second point; then according to the Rule, Square this Digit 8 new found, and it giveth 64, this

64 multiply by the former triple 12 produceth 768, write down under the former number 38912, ſetting the loweſt figure 8 of this new product directly under 1, the ſecond figure of 38912, and its ſecond figure 6 under the third figure 9, and ſo of the other in Order, then add theſe two numbers together, and the total will be 46592, which is equal to that number above, therefore if Subſtraction be made, there will nothing remain, and ſo the work is ended, whereby you may conclude that 110592, the number given, is a right Cubick number, and that 48 is the Cube Root thereof, which was the thing required, to be found.

	46
110592	(48
	64
	38912
	768
	46592

Now if you would at any time prove your work, whether you have wrought truly or not; multiply the Root found Cublickly, and add the remainder, when any is, to that product, and if the total be the firſt number given, then the work is truly wrought, or elſe not, as here in this Example; if you multiply the Root found 48 by 48, it is 2304, and this 2304, multiply again by 48, produceth 110592, the number firſt given, and therefore conclude that the work is truly wrought.

For a ſecond Example, we will take this number, 41063625, and ſeek the Cube Root thereof, firſt, prepare the number, by writing it down, and making a point under the loweſt figure 5, and another under 3, the fourth figure, and another under 1, the ſeventh figure, and draw the quotient-line, theſe three points do declare that the Root will conſiſt of three figures: Now to fall to work, to find the firſt of them, conſider what

what the greateſt Cube number in 41 is, which appeareth on the Lamina to be 27, and its Root 3, therefore put 3 in the quotient for the firſt figure of the Root, and then ſubſtract 27 from 41, and ſet the remainder 14 over head, and the work of the firſt figure is ended, and the number that belongeth to the ſecond point is 14063, out of which the ſecond figure is to be found, behold the Example: to get this ſecond figure, triple 3, the Root found, makes 9,

this triple 9 multiply by 3,	14
the Root found giveth 27,	41063625 (3
this product 27 tabulate upon	. . .
the left hand of the Lamina,	27

and then the number that will come neareſt unto that 14063 belonging to the ſecond point now in action, remembring the former Caution, that it be taken no oftener, but that withal there may be taken from thence alſo the product, produced by multiplying the ſquare of the reſpective figure into the former triple, as here the diviſor may be had five times, but by reaſon of that other number, that muſt alſo be taken from thence, cannot be taken alſo, therefore it can be had but only 4 times, wherefore put 4 in the quotient for the ſecond figure of the Root, and ſet the number 10864 found in that fourth rank of ſquares under the number 14063, the number belonging to the ſecond point, then multiply 16 the ſquare of 4 the digit now found by 9, the product is 144, this 144 ſet under the former 10864, but according to the former proviſal, one place higher towards the left hand,

as set its lower figure 4 under 6, the second figure of that 10864, the next 4 under 8, and the uppermost being 1 undea 0, as you see it stand in the Example, then add these two numbers together, and they make 12304, this take from 14063 leaveth 1759, thus is the work of the second point at an end; behold the Example.

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  1
 14759
41063625 (34
 .  .  .
27
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10864
 144
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12304
```

Now for the third figure of the Root, you are first to observe, that the number belonging to the third and last point is 1759625, from whence the third and last figure of the Root is to be extracted, which figure to find out, triple the Root found 34, & it is 102, & that multiply again by 34, the Root found yieldeth 3468, this product 3468 is the divisor, and this Tabulate upon the Rods as before, and joyn the Lamina close to them, then seek upon those Rods and Lamina what number will come nearest unto that 1759625, the number belonging to the third point now in action, (remembering the former Caution, but here is no need of that in the work of this point,) you shall find the number for the purpose, to be the number 1734125 standing in the fifth line, and its respective figure for the Root 5, therefore put 5 in the quotient for the third figure of the Root, now transcribe the number 1734125 from off the Rods into the paper, just under the 1759625, then to find the other number to be hereunto

```
  1
 14759
41063625 (345
 .  .  .
27
10864
 144
 -----
12304
 -----
 1734125
   2550
 -------
 1759625
```

added

added; square 5 the figure last found, and it makes 25, this square 25 multiply by the former triple 102, and the product is 2550, this set under the other number 1734125, according to the former proviso, as you see it stand in the Example, and add these two numbers together, and their total will be 1759625, and is equal to that above, belonging to that third point, so that if substraction should be made, there would nothing remain which declareth the number given to be a perfect Cubick number, and the Cubick Root thereof to be 345 which was the thing required to be done; if you will multiply this Root 345 Cubickly, it produceth the number first given 41063625, which proveth the work to be truly wrought, the like is to be observed in all other works of this nature whatsoever.

For a third Example, we will take at an adventure this great number 859271650667, and seek the Cube Root thereof. This number being prepared with points, and a quotient-line, sheweth by his four points, that his Cube Root will consist of four figures, and by the former directions, the first figure will appear to be 9, for the greatest Cube number in 859, the number belonging to the first point, is 729, which taken from that 859, leaveth 130, which with the 271 between that 130 and the next point, make 130271, out of which the second figure of the Root is to be extracted; this second figure by the former rules, will be found to be 5, after the work of this second point is ended, there will be remaining to the third point 1896650, and the Root

```
  1
130896
859271650667 (95
 .  .  .  .
729
------
121625
  675
------
128375
```

found

found is 95, this tripled, and also multiplied again by the Root, produceth 27075 for the divisor, this divisor tabulated, there is no number to be found upon the Rods so small as is 1896650, the number belonging to that third point, therefore put a 0 in the quottent for the third figure of the Root, and so have you done with that third point.

Now to find the fourth figure answerable to the fourth point, you need here do no more, but Tabulate two 0 between the Lamina, and the other Rods, without any altering of the Rods already Tabulated, and then seek the fourth figure as before, the reason is, because 0 doth neither multiply nor divide but only raiseth the places of figures higher towards the left hand, for here if you triple 950, the Root found, it yieldeth 2850, and this multiplied again by 950, the said Root found, produceth 2707500, which is the same with that former tabulated number, saving only the two Cyphers, and therefore it is, that when there is a 0 in the quotient, there needs no more to be done, but to Tabulate two Cyphers between the Lamina and those Rods before Tabulated, when you sought for that last figure, which happeneth to be a 0. Now upon those Rods thus Tabulated, seek what number upon those Rods and Lamina will come nearest unto that number, which belongeth to the fourth point, which is here 1896650667. By viewing the Rods, you shall find that number in the seventh rank of

```
   1
100896  3842
859271650667 ( 9507
 .   .   .   .
729
----
121625
  675
------
128375
------
  1895250343
     139650
-----------
  1896646843
```

of ſquares to be the number that will ſerve the turn, *viz.* 1895250343, and its reſpective figure to be put into the quotient 7. To this number add 139650, the number made by multiplying 49 the ſquare of 7, the Digit new found by 2850, the Triple of the Root afore found, and the total is 1896646843. This ſubſtracted from that number to the laſt point belonging, leaveth remaining over head 3824.

Thus have you finiſhed all your points, and have found the Cube Root of your number given to be 9507. And being there is a remainder left, it appeareth that the number firſt given is not a perfect Cube number; but the greateſt Cube number therein is 859271646-843, and his Cubick Root is 5907. The truth of this work you may examine by multiplying the Root found Cubickly, which if you do, and add the remainder to the product, you ſhall produce the firſt number given.

Now for the remainder, to make a fraction thereof in ſuch ſort, that it may aptly expreſs the neareſt Cube Root, I could ſhew ſeveral ways delivered by ſeveral Authors, how to bring it neereſt to the truth. But for a brief and eaſie Rule, and which in my conceit is the moſt exact, is, that which that good Artiſt, my old acquaintance Mr. *John Tap* delivereth in his Book of Arithmetick, and which he ſaith, is the Method uſed by *Goſſelin* upon *Tartaglia*. And the Rule in his own words is this. The Root in whole numbers being extracted, ſet the remainder over a line for numerator, and then for the denominator, triple the Root found, that triple multiply again by the Root found, and to the product add the former triple, the total ſet under the line for denominator. According to this Rule, if you triple the Root found 9507, it giveth 28521, and that multiplied again by 9507, the Root found

found 9507, it giveth 28521, and that multiplied again by 9507, the Root found produceth 271149147, to this product add the former triple 28521, and it giveth this total sum 271177668 to be set under the line for a Denominator to 3824 the remainder; And so the neerest Cube Root of the number given, is found to be, as here it standeth in this Example. This Rule, of a brief Rule, is the best I know.

9507 $\frac{3824}{271177668}$

But the most absolute and best rule to get the Cube Root of any number not Cubick, is this: Add to the remainder so many several points, or Ternaries of Cyphers, as you desire to come nearer to the true Root, (in the same manner as you did in the extraction of the square Root) and then continue on the work for extraction, as you do in whole numbers, and the fraction will be turned into a Decimal fraction, the numerator whereof will not be the remainder, as before it was, but other figures new found, and then so many as you add Ternaries of Cyphers, and the Denominator will have an Unite, with as many Cyphers, as you added Ternaries, or points of Cyphers to the remainder; a short Example will serve to explain this Rule.

Let 2112 be a number given, whose Cube Root is required, This number I can perceive is no Cubick number, therefore I set it down, and to it add three Ternaries of Cyphers, and then prepare it with a quotient-line, and points, as you use to do, where by the points you may see that you shall have two figures for the whole part of the Root, and three that you would have in the fraction. Now to go to work, the greatest Cube number in 2 is 1, and 1 is the first figure of the Root, and by the former Rules, the second figure of the Root is found to be 2, so that the whole part of the Root of that number given 2112 is 12, and the remainder

mainder is 348, whereof a fraction is to be made. Thus having done with the whole part, go on now to the fraction, as you use to do with whole numbers, for indeed it is no other but a whole number.

Therefore triple 12, the Root found, and it makes 36, that multiply again by the Root found 432, this product Tabulate, and joyn the Lamina to the Rods, and you shall find that you may have this Divisor 8 times out of that number 384000, the number belong-

```
      1 340
     12 315000
   1384 848513856
   2112,000000000 ( 12.134
   .  .   .  .  .
    608
    12
   ----
    728
   ----
    346112
     2304
   ------
    369152
   --------
    12501 627
       31 86
   ---------
    12533487
   ---------
     1975306864
        20528
   ----------
     1975512144
```

ing to the first fraction-point, put 8 therefore in the quotient

quotient for the firſt fraction-figure; when you have finiſhed this work, as you have been before inſtructed, you will find remaining to the next point 14848000, whoſe reſpective figure for the Root will be found to be 3, and to the third point remaineth 2315513000, whoſe reſpective figure for the Root is found to be 4, thus have you gained three figures into the quotient, for the Numerator of your new fraction, whoſe Denominator will be one Unite, with three Cyphers, becauſe three Ternaries of Cyphers were added, and ſo the Root found is $12\frac{834}{1000}$ repreſenting 12, and 834 ſuch parts whereof 1000 make one Unite, or $8^{1}\ 3^{2}\ 4^{3}$, that is to ſay, 12 Unites, 8 Primes, 3 Seconds, 4 Thirds.

If you ſeek a Denominator for this remainder 384 by that other Rule, you ſhall find the Root to be $12\frac{384}{477}$, but this laſt Rule by addition of Cyphers, though not ſo eaſie as that other, yet is the moſt exact, as by tryal you ſhall find.

Thus much of the Extraction of Roots.

Example

Example in the Rule of Three Direct.

IF 37 *Ells and a half of Linnen Cloth, coſt* 24 l. 7 s. 9 d. *what ſhall* 283 *Ells and an half coſt?*

Firſt, ſet down your 37 Ells, then if you look in your Scale for your half Ell, you ſhall find it to ſtand againſt 500 in the Scale of 1000, which 500 may be called 5 only, for the two Cyphers may be omitted. Then ſet down your 24 *l.* and for your 7 ſet down 3 decads, which is *6 s.* and look in your Scale for the Decimal of *1 s. 9 d.* which you ſhall find to be 875; Laſtly, ſet down your 183 Ells, and for your half Ell ſet down 5 as before, ſo will your numbers ſtand thus,

Ells		*li.*		*Ells*
If 37.5	coſt	24.3875,	what	183.5

183.5

1219375
731625
1951000
243875

4475.10625

37.5)

37·5) 4475.10625 (119.3361
or

375 (119 *li.* 6 *s.* 8 *d.* 2 *q.* $\frac{36}{100}$
725
375
501
3375
1260
1125
1356
1125
2312
2250
625
375
250

Your numbers thus taken out of your Scale, and placed as here you see, if you multiply the second and third together, you shall find the product of that multiplication to be 4475. 10625, which divided by the first number 37. 5, giveth in the quotient 119.3361, which is 119 pounds, 3 decads, or 6 shillings, and 361, which reduced by the Scale, giveth 8 pence, 2 farthings, and something more.

Example

Example in the Rule of Three Reverſe.

IF when the price of a Quarter of Wheat is 1 li. 5 s. 6 d. *the peny white loaf ſhall weigh* 12 *ounces* 16 *peny weight; I demand what the peny white loaf ſhall weigh, when the price of the Quarter of Wheat is* 7 s. 6 d.

If you place your numbers according to the tenor of the Queſtion, they will ſtand as followeth.

li.	*s.*	*d.*		*ou.*	*pw.*		*s.*	*d.*
1	5	6	——	12	16	——	7	6

But being taken out of the Scales of Money and Troy Weight, they will ſtand thus,

li.		*ounces*		*s.*
1.275	——	12.8	——	.375

12.8

10200
2550
1275

16.3200

.375) 16.3200 (435
1500
1320
1125
1950
1875
75

or

ounces. pw.
43—10 $\frac{75}{100}$

Here if you multiply 1.275, which is the Decimal of 1 *li.* 5 *s.* 6 *d.* by 12.8, which is the Decimal of 12 ounces 16 peny weight, you shall find the product of that multiplication to be 16.3200, which being divided by .375, which is the Decimal of 7 *s.* 6 *d.* the quotient will be 43.5, which is the Decimal of 43 ounces, 10 peny weight 3 grains; and so much ought the peny white loaf to weigh, when the quarter of Wheat is sold for 7 *s.* 6 *d.*

Example in the Double Rule of Three.

IF 24 *yards of stuff of three quarters broad, cost* 4 l. 14 s. *what shall* 328 *yards of the same stuff cost being* 5 *quarters broad.*

If you place your numbers according to the directions of this Rule, they will stand thus,

yards	*quarters*	*li. s.*	*yards*	*q.*
If 24	of 3	cost 4 14,	what shall 328	cost of 5

But if you take your fraction-numbers out of their proper Scales, they will stand thus,

yards	*quarters*	*li.*	*yards*	*quarters*
24	3	4.7	328	5
3			4.7	
72			2296	
			1312	
			15416	
			5	
			7708.00	

72

```
72 ) 7708.000 ( 105055
     .....
     ---------
     72                 or
      508          li.   s.   d.
     ---------     107—1—-1
      504
        400
     ---------
        360
         400
     ---------
         360
          40
```

First, multiply the two first numbers, as 24 and 3 together, they make 72 for Divisor, then multiply 4.7, which is the Decimal of 4 *l.* 14 *s.* by 328, and the Product is 15416, which again multiplied by 5, the last number giveth 77080; unto this Product, (that there may be a competent number of figures in the quotient,) I add two Cyphers, making it 7708000, which I divide by 72, and the quotient is 107.055, which is 107 *l.* 1 *s.* 1 *d.* and so much is 328 yards of stuff worth, being 5 quarters broad.

Example in Barter.

TWo Merchants having two ſeveral Commodities, are willing to Barter, or Exchange the one with the other. The one hath Indigo, which he will ſell at 4 s. *the pound for ready money, but in Barter he will have* 4 s. 9 d. *the pound; the other Merchant hath Kerſies, which for ready money he will ſell for* 3 s 6 d. *the yard. Now the queſtion is, at what price he muſt rate his Kerſies in Barter, to equalize the* 9 d. *advanced upon the pound of Indigo.*

The tenor of the Queſtion is this.

If 4 s. *in Barter require* 9 d. *what ſhall* 3 s. 6 d. *require?*

Your numbers placed will ſtand thus,

s.	*d.*	*s.*	*d.*
4	9	3	6

But being taken out of your Scale, they will ſtand thus,

Decads

Decads	*d.*	*s.*
2	·375	1.75

1.75

1875
2625
375

2) .656|25 (.328

65
4

16

Say then by the Rule of Three Direct, If 2 Decads or 4 *s.* in Barter require .375, which is the Decimal of 9 *d.* what shall 1 75 require? which is the Decimal of 3 *s.* 6 *d.*

First, multiply .375 by 1.75, the product is .65625, but being it is a fraction, I cut off the two last figures, because we require only three figures in the quotient, which divided by 2, giveth in the quotient .328, which is the Decimal of 7 *d.* 3 *q.* this 7 *d.* 3 *q.* added to this 3 *s.* 6 *d.* maketh 4 shillings 1 peny 3 farthings, and so much ought he to rate his Kersies at by the yard in Barter, to save himself harmless.

Example in Fellowſhip.

THree *Perſons* A, B *and* C. *bought* 4000 *Sheep*, *which coſt* 483 li. 6 s. 8 d. *of which money* A. *paid* 203 li. B. *paid* 165 li. 6 s. 8 d. *and* C. *paid* 114 li. 11 s.

Firſt, ſay by the Rule of Three Direct.

1. If 483 *li.* 6 *s.* 8 *d.* buy 4000 ſheep, how many ſheep ſhall 203 *l.* (which is A's ſhare) buy? *Anſwer*, 1680.

2. Say, If 483 *li.* 6 *s.* 6 *d.* buy 4000 ſheep, how many ſheep ſhall 165 *li.* 15 *s.* 8 *d.* (which is B's ſhare) buy? *Anſwer* 1372.

3. Say again, if 483 *li.* 6 *s.* 8 *d.* buy 4000 ſheep, how many ſheep ſhall 114 *li.* 11 *s.* (which is C's ſhare) buy? *Anſwer* 918.

Your numbers being taken out of your Scale, proceed as followeth.

Firſt for A.

ſheep	*li.*
If 483.3333 *buy* 4000, *what*	103
	4000
	812000

483.

483.3333) 8120000000 (1680

4833333
32866670
28999998
38666720
38666064
00000560

Secondly for B.

li.	*sheep*	*li.*
483.3333	4000	165.7833

4000

66313330000

483.3333) 6531332000 (1372

4833333
17979990
14499999
34800010
33833331
9666790
9666666
124

Thirdly for C.

li. *sheep* *li.*
483.3333 —— 4000 —— 114.55

4000

458200.00

483.3333) 4582000000 (948
...

43499997
23200030

19333332
38666980

38666664
316

The manner of Work.

For *A.* multiply 203 *l.* (which is *A*'s share) by 4000 (which is the number of sheep bought) and the product is 812000, which number should be divided by 483.3333, but being it is greater than 81200, I therefore add four Cyphers thereto, that I may have four figures in the quotient, and it makes 8120000000, which divided by 483.3333, giveth in the quotient 1680, and so many sheep belong to A.

2. For *B*, multiply 165.7833 (which is the Decimal of *B*'s share) by 4000, (the number of sheep bought) and it produceth 663133200o, which divided by 483 3333, giveth in the quotient 1372, and so many sheep belong to B.

3. For

3. For *C.* multiply 114,55, (which is the Decimal of *C*'s ſhare) by 4000, (the number of ſheep bought) it produceth 45820000, which number ſhould be divided by 483.3333, but being it is not large enough to give figures enough in the quotient, I therefore add two Cyphers, making it 4582000000, which divided by 483.3333, giveth in the quotient 948, and ſo many ſheep ought *C.* to have.

Now for proof, if you add the number of ſheep that *A*, *B* and *C.* ſhould ſeverally have, you ſhall find them in all to make 400, which demonſtrates the Work to be true.

A	1680
B	1372
C	948
	4008

Examples in Loſs and Gain.

IF one Yard of Stuff coſt 6 s. 8 d. *and I ſell the ſame again for* 8 s. 6 d. *what ſhall I gain in laying out* 100 li. *upon ſuch a Commodity?*

Take the difference between the price that your Commodity coſt, and the price for which you ſell it, that is, in this Example, the difference between *6 s.* 8 *d.* and 8 *s.* 6 *d.* which is 1 *s.* 10 *d.* then ſay by the Rule of Three Direct,

If *6 s.* 8 *d.* gain 1 *s.* 10 *d.* what will 100 *li.* gain?

If you place your numbers according to the Rule of Three Direct, as they are here given, they will ſtand thus,

6 s. 8 *d.*

s.	*d.*		*s.*	*d.*		*li.*
6	8	———	1	10	———	100

But being taken out of your Scale and placed, they will ſtand as followeth.

s.		*s.*		*li.*
.3333	———	.425	———	100

100

.3333)42500|00 (127.5

*****

3333
9170
6666
25040
23331
17090
16665
425

Your numbers being placed, multiply .425, which is the Decimal of 1 *s.* 10 *d.* by 100 *li.* and the Product is 42500, to which I add two Cyphers (that I may have a competent number of figures in the quotient) and it makes 42500|00, which divided by .3333, the decimal of 6 *s.* 8 *d.* giveth in the quotient 127.5, which is 127 *li.* 5 Livers or 10 *s.* ſo there is 17 *li.* 10 *s.* gained in laying out of 100 *li.*

I will here prove this queſtion by the converſe.

If by one yard of Stuff which is sold for 8 s. 6 d. *there was gained* 27 li. 10 s. *in laying out of* 100 li. *I demand what the said stuff cost a yard at the first hand?*

Add 100 *l.* and 27 *l.* 10 *s.* together, and they make 127 *li.* 10 *s.* Then say by the Rule of Three Direct,

If 127 *li.* 10 *s.* give 100 *l.* what shall 8 *s.* 6 *d.* give?

Take your numbers out of your Scale, and place them as here you see.

li. *li.* *s.*
If 127.5 give 100, what .425

127.5) 42500|0 (3333
...
3825
4250
3825
4250

Here if you multiply 425, which is the Decimal of 8 *s.* 6 *d.* by 100, you shall have 42500, to which if you add a Cypher, you make it 42500.0, this number being divided by 127,5, which is the Decimal of 127 *li.* 10 *s.* giveth in the quotient 3333, and if you had added more Cyphers to the Dividend, you should have had more threes in the quotient, and no other figures, but these four threes are enough, and are a Decimal fraction representing 6 *s.* 8 *d.* and so much did the yard of Stuff cost at the first hand.

Examples

Examples in Loſs and Gain upon Time, wrought by the Double Rule of Three.

IF one Ell of Lockeram coſt me 2 s. 8 d. *ready money, and I ſell the ſame again for* 2 s. 10 d. *the Ell, to be paid at the expiration of three months; I demand what I ſhall gain in* 12 *months, laying out* 100 li. *upon that Commodity?*

This and ſuch like queſtions, although they may be wrought by the Rule of Three Direct, at two Operations, yet they are beſt performed by the Double Rule of Three compounded of five numbers, wherefore the queſtion may be thus ſtated.

If 2 *s.* 8 *d.* in three months, gain 2 *d.* what ſhall 100 *li.* gain in 11 months?

If you take your numbers out of your Scale, and place them according as was directed in the firſt part of this Book, you ſhall find them to ſtand thus,

If

s. *mo.* *d.* *li.* *mon.*
If 1.333 *in* 3 *gain* 83, *what ſhall* 100 *gain in* 12

3 100

3.999 8300
12

16600
8300

3.999) 99600 (25 *fere*
..

7998
19620

19995
375

Your numbers being placed according to the tenor of the queſtion, if you multiply 1.333, which is the Decimal of 2 *s.* 8 *d.* by 3 months, the product will be 3.999, which muſt be your Diviſor, then multiply.83, which is the Decimal of 2 *d.* by 100 *li.* and it makes .8300, that again multiplied by 12 months, giveth for the product 99600 for your Dividend, wherefore if you divide 99600 by 3999, it will give you in the quotient 25 almoſt, which is 25 *li.* for the Decimal fraction remaining is ſo ſmall, that it wanteth not near a farthing of 25 *li.* and therefore we call it 25 *li.* and ſo it is exactly, as you may try, if you reduce all the numbers to their leaſt denominations, and work as is before taught in *Vulgar Arithmetick.*

I will prove this queſtion by the converſe.

If

If one Ell of Lockeram cost me 2 s. 8 d. *ready money, for what price shall I sell the same again to be paid at the end of three months. So that I may gain* 25 li. *in* 100 li. *for* 12 *months?*

Say by the Rule of Three Direct

If 100 li. *in* 12 *months gain* 25 li. *what shall* 2 s. 8 d. *gain in* 3 *months?*

If you take your numbers out of your Scale, and place them according to the Double Rule of Three, they will stand as followeth,

```
 li.    m.    li.    s.        mon.
 100——12——25——1.333——3
  12                 25
 ———                ———
  200               6665
 100                2666
 ———                ———
 1200               33325
                        3
                   ———
              1200) 99975 (.83
                    •
                   ———
                    9600
                     3975
                   ———
                     3600
                      375.
```

Your numbers being thus placed, if you multiply 100 *li.* by 12 months, you shall find the product to be 1200, which is your Divisor. Then multiply 25 *li.*

b

by 1.333, which is the Decimal of 2 *s.* 8 *d.* and the product thereof will be 33325, which multiply again by 3, & the product will be *99975* for your Dividend, this *99975* divided by 1200, giveth in the quotient .83, which is the Decimal of 2 *d.* which 2 *d.* added to 2 *s.* 8 *d.* the price which the Ell of Lockeram cost, giveth 2 *s.* 10 *d.* and at that price must you sell the same at 3 months time, so that you may gain 25 *li.* in the 100 *li.* in 12 months.

I might further proceed to shew you Examples in divers other Rules, As in *Alligation*, *Position*, &c. but those Rules being already handled in the First Part of this Book, it will be easie to apply them to the Scales. And (as I intimated at the beginning of this Third Part,) that although I have only made choice of the eight *Scales* there mentioned and described, yet it was easie to contrive *Scales* for the *Coins*, *Weights* and *Measures* of other *Countries*, and not only so, but when the value of the *Money*, *Weight* or *Measure* of one *Country*, and the *Money*, *Weight* or *Measure* of another Country is known, it is easie to contrive two *Scales*, which facing one another, shall immediately tell you how many *Pounds* in one place shall be equal to so many *Crowns*, or other *Coin* in another place; but this I do only intimate, that such as are desirous, may fit themselves with Scales answerable to their most necessary occasions. And thus shall I conclude this Third Part, referring some things necessary to such a Work, to the Appendix following.

The End of the Third Part.

AN ABRIDGEMENT OF THE PRECEPTS OF ALGEBRA.

The Fourth Part.

VVritten in French
BY
JAMES de BILLY.

And now Translated into Engliſh.
VVith divers Queſtions added,
which were not in the Original.

Publiſhed by Will. Leybourn.

LONDON,
Printed by *Tho. James* for *George Sawbridge*,
MDCLXXVIII.

ALGEBRA BREVIS.

The Fourth Part.

I Have been always of opinion, that the practice of *Algebra* should not be entangled with great number of precepts: This Science is of it self dark enough, without adding unto it new obscurity, by the confusion of many different operations. You have here an Abridgement which hath pleased many of good Judgments, and I hope, such as will with attention read it, shall from thence receive both satisfaction and profit.

I shall in the first place set forth a Table of three Ranks: In the first of which there is a Progression natural, of which the terms are disposed in that order, that immediately under them you have the Cossick Characters, of which they are Exponents, and in the loweſt Cell, a Progression Geometrical, which beginning with an Unite, may be doubled, trebled, quadrupled, *&c.* We have for the greater ease only doubled them. Observe then that R is a Cossick Character signifying a Root, and that the Exponent thereof is marked above in the uppermost Rank, and C is the Cossick Character of a Cube, whose Exponent

is 3, and so of the rest. I call those terms, which are in the upper Cell, Exponents, because they expound the Cossick Characters, and the Numbers of the Geometrical Progression which are below. Mark well this manner of speaking, consider diligently this Table, and for the present content your self with this.

Exponents	0	1	2	3	4	5	6	7	8	&c.
Characters Cossick	N Numbers absolute	R Root	Q Square	C Cube	QQ Biquadrate.	S Surfolide.	QC Square Cube	SQ Solide second	QQQ Square Square Square	&c.
Progr. Geometr.	1	2	4	8	16	32	64	128	256	&c.

You may continue this Table, if you please, infinitely in this manner. Take two numbers, which if you multiply, they will produce some Exponent, you shall presently see what a Character is to be put under that Exponent. For Example, If you would know the Character of the Exponent 6, take the numbers 2 and 3 (because these multiplied together make 6) after that add their Characters which are Q and C, and you shall have QC for the Character of the Exponent 6. In like manner the Character of the Expo-

nent

nent 8 is QQQ, becauſe under the Exponents 2 and 4, which multiplied together produce 8, are contained the Characters Q and QQ. Alſo the Characters of the Exponent 12 will be QQC, becauſe 12 is a number produced by the Multiplication of 2 and 6, or of 3 and 4.

But if the Exponent be a firſt number (that is to ſay, a Number not produced by the Multiplication of any two other) mark in what order it is after the Exponent 5, and call it Surſolide Second, Third, or Fourth, *&c.* according to its Rank. The Character of 5 is ſolid, of 7 ſolid ſecond, of 11 ſolid 3; and ſo conſequently under ſuch Exponents as are firſt numbers, under which only are found ſuch ſolids.

CHAP. I.

The Algorithm of Coſſick Numbers, *ſimple, compounded, or diminiſhed.*

BY the word *Algorithm*, I mean all the Operations comprehended under theſe four kinds, *Addition, Subſtraction, Multiplication*, and *Diviſion*. By the word *Coſſick Simple*, I underſtand ſuch as have not this + (which ſignifies *Plus*, nor this − ſignifying *Minus*, expreſſed before them. On the other ſide, by *Numbers Compoſed*, are meant ſuch as have the ſign +, and by diminiſhed, ſuch as have the ſign −. Note ſuch Numbers as have no ſign expreſſed, are ſuppoſed to have this of +.

Sect. 1. Addition *of Simple Coſſick Numbers.*

All Simple Coſſick Numbers of the ſame Denomination (that is to ſay, have the ſame Character) or

of different: If they be of the ſame Denomination, the Addition is as in Common Arithmetick. *Example*, 5 Q added with 3 Q, makes 8 Q.

If they be of different Denominations, they muſt be added by the interpoſition of the Figure +, as 6 R added to 4 Q, makes 6 R + 4 Q in like manner 3 added to 4 R makes 3 + 4 R.

Sect. 2. Subſtraction of *Coſſick Numbers.*

Either they are of the ſame, or different Denominations: if of the ſame, you muſt ſubſtract as in ordinary Arithmetick; for Example, 3 Q ſubſtracted from 8 Q, there reſts 5 Q.

If of different, you muſt ſubſtract by the interpoſition of the ſign — as 6 R ſubſtracted from 4 Q, there reſts 4 Q — 6 R, ſo 3 C ſubſtracted from 66 there remains 66 — 3 C.

Sect. 3. Multiplication *of Simple Coſſick Numbers.*

You muſt here have regard both to the obſolute Number, and to the Coſſick Characters: If therefore a Coſſick Number be to be multiplied by an abſolute, you muſt multiply the abſolute Numbers, and unto the Product give the ſame Character, as 5 R, multiplied by 12 produce 60 R.

But if you multiply Coſſick Numbers by Coſſick, you muſt multiply the abſolute Numbers together, and to the Product give the Character of that Exponent, which is made by the Addition of the Exponents belonging unto the aforeſaid Coſſick Characters. For Example, 2 R multiplied by 3 Q make 6 C, becauſe the Exponent of Q is 2, added to 1, the Exponent R, makes 3 the Exponent of C, which in this reſpect ought to be given to the Product. In like manner 5 R mul-

multiplied by 4 C it makes 20 Q Q for the reaſon above given.

Sect. 4. Diviſion *of ſimple Coſſick Numbers.*

The ſpeculation of this is marvellous. But the practice of it is by putting the Diviſor under the Number to be divided, drawing between them a little ſmall line in manner of common Fractions. For example, 13 Q divided by 7 R, the quotient is $\frac{13}{7}$ and 6 Q Q divided by 5 C, the quotient is $\frac{6}{5}$ C.

Sect. 5. Addition *of Numbers compoſed and diminiſhed.*

Some order is to be obſerved in this, in which the Numbers are to be diſpoſed in ſuch manner, that thoſe that are of the ſame denomination muſt be put right under one another. After you have done this, if they have the ſame ſign, they are added as in common Arithmetick, and to the product give the ſame ſign. As for example, 7 Q — 4 C added to 3 Q — 2 C, the Sum is 10 Q — 6 C.

But if the Numbers be of different ſigns, the leſſer muſt be ſubſtracted from the greater, and to the reſidue you muſt give the ſign of the greater number as 6 Q + 7 R added with 7 Q — 12 R, give for the Sum total 13 Q — 5 R.

6 Q + 07 R	6 Q — 07 R
7 Q — 12 R	7 Q + 12 R
13 Q — 05 R	13 Q + 05 R

Sect. 6. Subſtraction *of numbers compoſed and diminiſhed.*

There is nothing more intricate to beginners than

the precepts commonly given for Subſtraction. You have here an order plain, ſure, and very eaſie to practice.

Change the ſign of the particulars of that Number you deſire to ſubſtract, and after this change add them with the Number from which the Subſtraction is to be made, and you ſhall have the reſidue: as if from 6 Q — 10 R you would ſubſtract 18 Q — 15 R by the 5 Sect. the reſidue will be 5 R — 12 Q. So likewiſe if you would ſubſtract — 8 R — 9 Q from 16 R + 6 Q the reſidue will be 24 R + 15 Q.

6 Q — 10 R — 18 Q + 15 R	Add	16 R + 6 Q 8 R + 9 Q	Add
— 12 Q + 5 R Reſid.		24 R + 15 Q Reſidue.	

Sect. 7. Multiplication *of Numbers compoſed and diminiſhed.*

Mark what I ſaid of Coſſick ſimple in the 3 Sect. and remember that the ſame ſigns have always the ſign + in the product and different — and there is no difficulty in Multiplication, ſo as you multiply every particular of the Multiplicand by every particular of the Multiplicator, as in common Arithmetick, as if you multiply 3 Q — 2 R by 8 R, 2 R by 8 R, it makes 18 Q and becauſe the Multiplicator and Multiplicand have different ſigns, the product muſt have the ſign of —: and therefore that ſhall be 16 Q, farther 3 Q by 8 R make 24 C, to which you muſt give the ſign +, becauſe the Multiplicator and the Multiplicand have the ſame ſign, ſo that the product of this Multiplication will be 24 C — 16 Q. So 2 R + 4 Q multiplied by 3 Q — 5 the product is 6 C + 12 QQ — 10 R — 20 Q.

3 Q —

3 Q — 2 R Multiplicand	2 R + 4 Q multipl.
8 R Multiplicator	3 Q — 5 multipl.
24 C — 16 Q Product.	— 10 R — 20 Q
	6 C + 12 Q Q
	6 C + 12 Q Q — 10 R — 20 Q.

Sect. 8. Division *of Numbers compoſed and diminiſhed.*

There is no great difficulty in this, only put a line between the Dividend and the Diviſor, and you have the Quotient, as 4 C — 3 Q + 2 R divided by 5 R — 4 C make for this quotient.

$$\frac{4\,C - 3\,Q + 2\,R}{5\,R - 4\,C}$$

Sect. 9. Algorithm *of Fractions.*

I ſhall not give any particular precepts, becauſe if a man underſtand the Fractions of common Arithmetick, and practice according to what is before ſaid, there will be no need of them.

CHAP. II.

The Rule of Algebra, *with the explication thereof.*

IT was meerly neceſſary by the precedent rules, to trace out inſenſibly the way to *Algebra*, which cannot be practiced without *Addition, Subſtraction, Multiplication,*

plication, and *Diviſion*. Having therefore made plain theſe difficulties, we will proceed in the *propoſition* of the Rule of *Algebra*, and the explication of every part of it briefly and plainly.

Sect. 1. *The Rule of* Algebra.

You muſt firſt for the Number unknown put 1 R, and after examine this root according to the tenour of the queſtion, until you come to an Equation. Secondly, this Equation is to be reduced, if need require. Thirdly, you muſt divide every part of the Equation by the Number of the greateſt Coſſick Character. After which, either the quotient, or ſome root of the quotient, will give the root unknown. This is the Rule of *Algebra*, let us now explain it.

Sect. 2. *How the* Equation *muſt be found.*

The Rule ſaith, this is done by examining the queſtion propounded, according to the tenour of the ſame. That is to ſay, you muſt well obſerve all the conditions of the queſtion propounded, to the end you may fully accompliſh it. For after you have gone thorow it, you ſhall find an Equation between two Numbers. As if I ſearch a Number, which added with its ſquare ſhall make 20, I ſuppoſe this Number unknown to be 1 R, the ſquare thereof is 1 Q, (becauſe every Number multiplied by its ſelf, makes its ſquare) then 1 Q + 1 R is equal to 20. See thus an Equation found between 1 Q + 1 R and 20.

Sect. 3. *How your* Equation *muſt be reduced.*

Your Equation muſt be found, it is reduced by adding the ſame Number to both the terms of the Equation,

quation, or by ſubſtracting from them the ſame number. So is it performed by multiplying, or dividing both the terms by the ſame number. For by this means your Equation ſhall remain the ſame after theſe things done. As for example, 1 R + 1 Q = 20, adding throughout 2 C, you ſhall have alſo an Equation between 1 R + 1 Q + 2 C and 20 + 2 C. So ſubſtracting 1 R from the terms, you have 1 Q = 20 — 1 R. Likewiſe multiplying, or dividing both your terms by 3, you ſhall have by the multiplication 3 R + 3 Q equal to 60, and by the diviſion the Equation will be between on $\frac{1}{3}$ R + $\frac{1}{3}$ Q and $20\frac{2}{3}$.

Now to make your reduction judiciouſly and profitably, you muſt take care always that your greateſt character remain alone on one ſide of your Equation. As of all the reductions before made, there is none uſeful but the ſecond, becauſe in that only you find on the one ſide alone 1 Q = 20 — 1 R which is the only end of your reduction.

I ſaid in the rule of *Algebra*, that your Equation muſt be reduced, if it be neceſſary; becauſe it ſometimes happens, that there is no need of it: as when your Equation falls out between two ſimple collaterals numbers, I call thoſe numbers collateral, whoſe exponents do not ſurpaſs one another by more than an unite.

Sect. 4. *When you muſt* extract *the* root.

When your coſſick numbers are ſimple and collateral, you muſt not extract any root; but if you divide by the number of the greateſt coſſick character, the quotient ſhall ſhew you the value of the root, which is all you ſeek for in *Algebra*. For example, If you find an Equation between 2 R and 28, dividing ſimply 28 by 2, the quotient ſhall be the value of 1 R.

But

But when the terms of your equation are not collateral, you must extract some root; either square cube, squared square, *&c.* according to the cossick character which remains after your Hypobibasm.

Now Hypobibasm is nothing else but an abatement, or depression of the character, and is done by substraction of the lesser exponent from the greater. As if you find an equation between 10 Q C and 90 Q Q, take notice of the exponent of Q C in the Table inserted at the beginning, the which exponent is 6; afterwards look to the exponent of Q Q which is 4, substract 4 out of 6 there rests 2, of which the cossick character is Q. From hence I conclude, that 10 Q are equal to 90, after dividing 90 by 10, and finding 9 in the quotient, I conclude that the square root of 9 must be extracted by reason of the character Q.

Sect. 5. *How to* extract *the* square root *of numbers, compound and diminished.*

No man hath yet perfectly found out the way to extract the root of numbers compound and diminished, unless the exponents of three terms of the equation keep between them in some situation or other, an Arithmetical proportion, that is to say, the same distance. As if the equation be between 1 Q and 20 — 1 R, you may now extract the root of 20 — 1 R, because the exponents of the three numbers which make the equation, are 2, 0, 1, which thus place, 0, 1, 2, keep the same distance.

The greatest cossick character left after the Hypobibasm, shews the root to be extracted; as in this example, before the square root is to be extracted, because the greatest characters is Q, the method to be followed in the extraction, see here propounded in general terms. Take

Take first the half of the number of roots. Secondly, to the square of this half, add or from it subduct your absolute number according to the sign of + or —. Thirdly, extract the square root of this sum, or of the residue. Fourthly, to this root add or from it substract half the number of roots, and this last sum or residue shews you the value of the root unknown. For example, I would find a number, the double whereof added to its square, should be equal to 24, I shall find an equation between 2 R + 1 Q and 24 by the second Section. Moreover, I shall reduce this equation after this manner 1 Q equal to 24 — 2 R by the third Section. Then if I divide 24 — 2 R by 1, the number of the greatest cossick character, there still remains 24—2R, because an unite doth neither multiply nor divide. Then in as much as the three terms of the equation do keep an Arithmetical proportion, I extract the square root of 24—2R in this manner. I take first half the number of roots, which is 1. Secondly, the square of 1 is 1, to which I add the absolute number, which is 24, because of the sign + before it, that makes 25. Thirdly, I extract the square root of 25, which is 5. Fourthly, from this root I extract the moiety of the number of roots which is 1, because of the sign — the residue will be 4; whence I conclude that to be the value of one root, and that the number sought is 4, whose double is 8, added to the square (of 4,) 16, makes 24.

Here note that numbers diminished, where the absolute number hath the sign —, have two roots. The greater is extracted, as before we have taught; the lesser is found out by substracting the square root of the residue from the half sum of the roots. As if I seek a number whose octuple diminished by 12, shall be equal to its square, you will find an equation between 1 Q and 8 R — 12. The greatest root is 6, the

lesser

leſſer is 2, here both the roots anſwer the queſtion. But this happens not often.

But if you be to extract the biquadrate root: Firſt, extract the ſquare root, as is taught, and again extract the ſquare root of this, and this ſhall be your biquadrate root. As if the equation be between 1 QQ and 2 Q + 8, you may find the ſquare root to be 4 by the method taught, firſt, taking half the number of the ſquares, *&c.* and afterward you muſt extract the ſquare root of 4, which is 2, this ſhall be the value of the root. In like manner, if your equation be between 1 QC and 2 C + 48, firſt, I extract the ſquare root of 2 C + 28, which is 8, of which extract the ſquare root 2, becauſe the root to be extracted is the ſquare cube, as the character QC, which is one of the terms of the equation, denoteth.

Sect. 6. *How to know if the* queſtion *be impoſſible, vain, or ill propounded.*

You may know the queſtion to be impoſſible, if you come to an equation impoſſible: As if following the conditions of the Probleme, you meet with an equation between 6 R and 24 R, or between 3 Q + 5 and 4 + 2 Q.

Secondly, the queſtion is vain, when the equation is between two equal numbers of the ſame denomination, as between 6 Q and 6 Q.

Thirdly, the queſtion is ill propounded, when without any difficulty many numbers will anſwer the probleme propounded.

CHAP.

CHAP. III.

Algorithm *of Second Roots, with their Uſe.*

ALgebraiſts ſometimes uſe more than one root to find out divers numbers propounded, and then to the end they may proceed with leſs confuſion, they uſually help themſelves with ſecond roots which they expreſs thus, 1 A, 1 B, *&c.*

Sect. 1. Addition *of Second Roots.*

If your Second roots be of the ſame denomination add the nmbers, and to the ſum give the ſame denomination, as 5 A added to 4 A make 9 A, if they be of different denomination, add them with the ſign of + as 5 A added to 6 B make 5 A + 6 B.

Sect. 2. Subſtraction *of Second Roots.*

If they be of the ſame Denomination, ſubſtract one Number from the other, and to the reſidue give the ſame Denomination, as 5 A taken from 9 A, there remains 4 A, if different, they are ſubſtracted with the ſign — as 6 B taken from 8 A, there reſts 8 A — 6 B.

Sect. 3. Multiplication of *Second Roots,*

If they be of the ſame Denomination, do as you do with the firſt Roots, as 4 A multiplied by 7 A make 28 AQ, if of different, both Denominations are retained in the Product, as 3 R multiplied by 5 A make 15R A.

Sect. 4.

Sect. 4. Division *of Second Roots.*

Division is only performed by the interposition of a little line, as is before taught, notwithstanding it is to be observed, that if for Example 3 AR be to be divided by the divisor 1 R, the quotient shall be A, because in such case there needs nothing else but to take from the Dividend the Character of the Divisor.

Sect. 5. *The Extraction and Use of Second Roots.*

After you have found and reduced your Opinion, according to the manner of working in second Roots, you must extract the Root after the manner taught in the precedent Chapter: As if 1 A be equal to 25, I say that 5 is the value of the second Root, and if 1 A Q be equal to 4 A + 12, you must take the moiety of the number of Roots, *&c.* As is said in the 5 Sect. of the precedent Chapter, and you shall find 6 to be the value of 1 A.

Now since the end of the second Roots is to be reduced to first, you must not forget after you have found the value to begin again your work, and to put in first Roots that which you have found to be the value of the second, as I shall shew you in some Examples in a Chapter following.

CHAP.

CHAP. IV.

The Algorithm *and Extraction of the Roots of Surd and Irrational Numbers.*

SUrd Roots are those that have a radical sign before them, and which in propriety of speech ought to be called absolute Numbers, notwithstanding they cannot be expressed by any common Number, neither whole, nor broken; we will hereafter express the Radical Sign by this Character R.

There are many sorts of Surd Roots, some are simple, as R Q 5, that is to say, the root square of 5, others are compound, as R Q 5 + R C 6, that is to say, the root square of 5, *Plus* the root cube of 6; some are universal, whose radical character, extends to all the particulars following it, and for that end are enclosed in a Parenthesis in this manner, R Q (14 + R Q 4) the root universal of 14, joyned with the root square of 4, all which number is 4 for 14 + the root square of 4 which is 2, maketh 16, whose root is 4.

Sect. 1. Reduction *of Surd Roots Simple to the same Denomination.*

First, you must put the radical signs under the numbers to which they belong. Secondly, you must multiply the numbers by the signs a cross, for to get new ones. Thirdly, you must add the signs together, which is done by multiplying their Exponents, and give the Character of the product common to the two new products; as if you would reduce to the same denomination R Q 5 and R C 4, you must first place them as followeth.

Secondly, you must multiply the numbers 4 and 5 by their signs across, that is to say, you must take the square of 4, and the cube of 5, which are 16, and 125. Thirdly, the exponents of the signs R Q and R C, which are 2 and 3, ought to be multiplied together, the product is 6. I look then in the Table what cossick character is under the exponent 6, and finding Q C, I take that for my common denominator, and instead of my two first surd roots, which were of different denomination, that is to say R Q 5, and R C 4. I have two new ones of the same denomination, that is to say, R Q C of 125, and R Q C of 16.

Sect. 2. Multiplication *and* Division *of Surd Simple Roots.*

If the roots be of the same denomination, you must only multiply and divide the numbers by themselves, and to the product and quotient give the same radical sign; as R Q 7 by R Q 2, give for the product R Q 14. In like manner R Q 36 divided by R Q 12, gives for the quotient R Q 3.

But if the Roots be of different denomination, you must reduce them to the same denominat ion by the precedent

precedent Paragraph, and after multiply and divide as ſhall be ſhewed; for example, R Q 3 multiplied by 2, the product is R Q 12, and R Q 12 divided by 2, the quotient is R Q 3.

Sect. 3. *How to know whether two Surd Roots be commenſurable or not.*

You muſt divide the greateſt root by the leſſer, if the quotient be rational, the two roots are commenſurable: if otherwiſe, they are not. As becauſe R Q 24 divided by R Q 6, gives for the quotient R Q 4, which is 2 a rational number; I conclude theſe two roots R Q 24 and R Q 6, to be commenſurable. In like manner, ſince root ſquare 24 divided by R Q 8, the quotient will be 3, a ſurd number and irrational, you may conclude thoſe two roots, R Q 24, and R Q 8, to be incommenſurable.

Sect. 4. Addition *of Simple Irrational Roots.*

If the roots be incommenſurable, you muſt add them only by the ſign + as R Q 24 added unto R Q 8 makes R Q 24 + R Q 8. But if they be commenſurable, you muſt add a unite to their quotient rationa', and you ſhall have a ſum, which being multiplied by the leſſer of the two roots to be added, will give a product which ſhall be the ſum ſought. As R Q 24 added with R Q 6 makes R Q 54, becauſe R Q 24 divided by R Q 6 gives 2 for the quotient rational, to which I add a unite, and it is 3, by which (always reducing them to the ſame denomination) I multiply R Q 6, which is the leſſer of my two roots, and I find for my ſum R Q 54.

Sect. 5. Subſtraction *of Simple Irrational roots.*

If they be incommenſurable, you muſt ſubſtract them

them by prefixing the sign — as R Q 8 subſtracted out of R Q 24, the reſidue ſhall be R Q 24 — R Q 8.

But if they be commenſurable, you muſt take away a unite out of the quotient rational, and you ſhall have the reſidue, the which being multiplied by the leſſer of the roots given, ſhall give a product which ſhall be the reſidue ſought; as if I be to ſubſtract R Q 6 from R Q 24, dividing the greater by the leſſer, the quotient is 2, from which if you take 1, there remains 1, by which (reducing them firſt to one denomination) if you multiply the leſſer root, that is to ſay, R Q 6, the reſidue will be R Q 6.

Sect. 6. Addition *and* Subſtraction *of Surd Numbers, compoſed and diminiſhed.*

I have here no new Precepts, only advertiſe you that you may remember what I have ſaid before of Coſſick Numbers, touching the ſigns of + and — in the fifth and ſixth Paragraph of the firſt Chapter, and what is delivered in the fourth and fifth of this Chapter, touching the Addition and Subſtraction of Simple Surd Numbers; and theſe will be no difficulty, as if you be to add 5 + R q. 24 with 3 + R q. 6, you will find 8 + R q. 54. In like manner, if you ſubduct 3 — R q 6, from 5 + R q 24, there reſts R q. 54 — 42.

Sect. 7. Multiplication *of Numbers ſurd, compoſed and diminiſhed.*

This Multiplication hath no great difficulty, nor needs new precepts. Remember only that the ſame ſigns have in the Product + and different — with this, that your Multiplication is not good, if the particulars to be multiplied be not firſt reduced to the ſame denomination. For example, 5 + R q. 24 by 3 — R q. 6

q. 6, 'tis to be done after this manner: -|- Rq. 24 by — Rq. 6, maketh — Rq. 144, or — 12 after -|- 5 by — Rq. 6, maketh — Rq. 150. Further, -|- Rq. 24 by 3 is -|- Rq. 216. Lastly, -|- 5 by 43 is -|- 15, then the whole Product will be 15 -|- Rq. 216, — Rq. 150. — Rq. 144, or 3 -|- Rq. 316 — Rq. 150, because Rq. 144 is a rational number, to wit 12, which being subducted out of 15, because of the sign — leaveth 3.

Example:

5 -|- Rq. 24
3 — Rq. 06

— Rq. 150 — Rq. 144
15 -|- Rq. 216

15 -|- Rq. 216. — R 150 — Rq. 144
3 -|- Rq. 216 — Rq. 150

25 9
6 5 24 3
X X
Rq. o Rq o
Rq. 150 Rq. 216

Sect. 8. Division *of Surd Numbers, compounded or diminished.*

If the Divisor be simple, the Division is made by the interposition of a line between the Divisor and the compound number to be divided; as if Rq. 2 -|- Rq. 5 be divided by 8, the quotient will be $\frac{\text{Rq. 2 -|- Rq. 5,}}{8}$ and so of others.

But because it sometimes may fall out (though

very ſeldome) if the Diviſor alſo will be a Binome, or compound number, that is to ſay, a ſurd number compounded of two particulars with the ſign ✠, or a Trinome that is compounded of three particulars, *&c.* See here the manner to divide in ſuch a caſe.

If the Diviſor be a Binome, you muſt multiply by his Apotome, as well the number to be divided, as the Diviſor (and if the Diviſor be an Apotome, you muſt divide by the Binome, as well the Dividend, as the Diviſor) by means of this Multiplication you ſhall have a new Dividend, and a new Diviſor. Now this new Diviſor will be always rational, and therefore needs only to be ſet under the Dividend with a line between As for example, R q. 6 — 2 by R q. 5 + R q. 3, I take the Apotome of my Diviſor (that is) R q 5 — R. q. 3, by which I multiply both my Dividend and my Diviſor, by one of the Multiplications is produced R q. 30 — R q. 20 — R q. 18 ✠ R q. 12, for my new Dividend, and by the other is produced 2 for my new Diviſor. So that the quotient of my Diviſion will be

$$\frac{\text{R q. 30} - \text{R q. 20} - \text{R q. 18} \dagger \text{R. q. 12}}{4}$$

becauſe 2 muſt be firſt ſquared, and then 4 put underneath the Dividend.

If your Diviſor be a Trinome, you muſt obſerve the ſame method, multiplying the Dividend and the Diviſor by the Apotome of the Diviſor, that is to ſay, by the ſame Diviſor onely, changing the ſign of the laſt particular. After this is done, you ſhall have a new Dividend and a new Diviſor, which ſhalll be a Binome. Then you muſt again ſeek a new Dividend and Diviſor, which now will be ſimple and rational.

Laſt of all, if you will not take this pains, the Diſion is good, if under the Dividend you ſubſcribe the Diviſor with a line between.

Sect. 9.

Sect. 9. Multiplication *of roots univerſal.*

You muſt reduce the root to be multiplied, and the Multiplicator to their ſquares or cubes, according to the radical ſign prefixed, and afterward perform your multiplication, as is taught in Sect. 7. of this Chapter. Afterward you muſt affix the ſign radical, and incloſe all in a Parentheſis.

This is better underſtood by an example, as if you multiply R q. (7 R q. 3) by 2, the ſquares of the one and other numbers are 7 + R q. 3 and 4, then the firſt being multiplied by the laſt, maketh 28 + R q. 48, and therefore if you cloſe this number within a Parentheſis, and put before it the ſame radical ſign, the product will be R q. (28 + R q. 48.)

In like manner, if you would multiply this number $\frac{7}{2} - \frac{1}{2}$ R + R q. $(\frac{49}{4} - \frac{1}{4}$ q. $- \frac{7}{2}$ R) by it ſelf, to get the ſquare of it, you muſt call to mind the fourth propoſition of the ſecond Book of *Euclide*, which ſheweth that a line being divided into two parts, the ſquare to the whole is equal to the ſquare of the parts, and to double their Rect-angles, you muſt therefore conceive this number, as divided into two parts, of which the firſt is $\frac{1}{2} - \frac{1}{3}$ R, and the laſt R q. $(\frac{21}{4} - \frac{1}{4}$ q. $- \frac{7}{2}$ R) take then the ſquares of the parts, which are $\frac{49}{4} + \frac{1}{4}$ q. $- \frac{7}{2}$ R, the double of the Rect-angle of the parts is R q. $(\frac{2604}{16} + \frac{220}{16}$ q. $- \frac{3744}{8}$ R + 7 C $- \frac{12}{16}$ q. q.) then the ſquare of the number propoſed is the ſum of theſe three numbers, that is to ſay, $\frac{49}{2} - \frac{1}{2}$ q. $- 7$ R + R q. $(\frac{2604}{16} + \frac{220}{16}$ q. $- \frac{3744}{8}$ R + 7 C $\frac{11}{16}$ q. q.) The ſquare of the Apotome is the ſame number, putting only the ſign — before the univerſal root, and the ſum of the two ſquares is 49 — 1q. — 14 R.

Sect. 10. Diviſion *of Roots univerſal.*

You muſt reduce the roots to be divided, and the

Diviſor to their Squares, Cubes, *&c.* And after divide them as is taught in the 8 Sect. and when this is done, encloſe all in a Parentheſis, with the ſame radical ſign which was before. As if you divide Rc q. (13 + Rc q. 17) by the root ſquare of 5, their ſquares are 13 + Rc q. 17 and 5, then the firſt being divided by the laſt, the product will be $2\frac{3}{5}$ + Rc q. $\frac{17}{25}$, then the quotient of the diviſion propoſed will be Rc q. ($2\frac{3}{5}$ + Rc q. $\frac{17}{25}$.)

Sect. 11. Addition *and* Subduction *of roots univerſal.*

Many trouble themſelves to give precepts intricate enough, the ſhort and moſt certain is to add them with the ſign +, and ſubduct them with the ſign —. As for example, Rc q. (3 + Rc q. 2) added with Rc q. (Rq. 5 + 6) will be Rq. (3 + Rq. 2) + Rq. (Rq. 5 + 6) and the ſame firſt root ſubducted from the laſt, the reſidue is Rq. (Æq. 5 + 6) — Rq. (3 + Rq. 2.)

Sect. 12. Extraction *of the roots of* Binomes *and* Apotomes.

1. Take the difference of the ſquares of the one and the other part of the Binome. 2. Add and ſubduct the ſquare root of this difference from the greateſt part of the Binome. 3. Conjoyn the ſquare root of the moiety of the ſum, with the ſquare root of the moiety of the reſidue, by the ſign +, if it be a Binome, and by the ſign —, if it be an Apotome; and thus the extraction is finiſhed. As if you would extract the ſquare root of this Binome $\frac{1}{2}$ + Rq. $\frac{1}{4}$, you ſhall firſt take the ſquare of the firſt part, which is, $\frac{1}{4}$, and the ſquare of the ſecond, which is $\frac{1}{4}$, the difference of theſe two is $\frac{0}{4}$, that is to ſay, 1. Secondly, you muſt extract the ſquare root of this difference, which is 1, you ſhall add and take it away from the firſt part of the Binome, by the addition you ſhall have $\frac{1}{2}$ for your ſum, and by ſubduction $\frac{1}{2}$ for your reſidue.

Thirdly,

Thirdly, joyn the root ſquare of the ſum, with the root ſquare of the reſidue by the ſign +- and you ſhall have R q. $\frac{1}{2}$ +- R q. $\frac{1}{2}$ for the ſquare root of your Binome propounded, and conſequently R q. $\frac{1}{2}$ — R q. $\frac{1}{2}$ ſhall be the root ſquare of the Apotome $\frac{1}{2}$ — R q. $\frac{1}{4}$.

CHAP. V.

The Uſe of Algebra.

'TIS much to have taken the pains to learn all that we have hitherto ſhewn or taught: but I dare boldly ſay, that thoſe that ſhall reſt here, do as yet know nothing to the purpoſe, although they may know all the precepts; it behoveth us then to make a ſtep further, to apply and bring thoſe precepts into uſe and exerciſe. 'Tis that which I deſire to demonſtrate in this Chapter, by ſome *Queſtions*, the ſolutions of which will give great light to the attaining of perfection in this Art. Wherefore I intreat thee (Reader) not to omit this Chapter, in which I pretend to yield thee ſome pleaſure and delight, as alſo an illuſtration of what hath been before treated of.

Sect. 1. *Queſtions reſolved by one ſimple* Equation.

Queſtion. I.

ALexander *one day told* Epheſtion, *That he was elder than him by two years*; *thereupon* Clitus *tells them, That he was as old as both of them (their ages added together) and four years over and above. The Philoſopher* Calliſthenes *being preſent at this diſcourſe (ſaith he) I well remember that my father, who was* 96 *years old, had the age of you three. It is demanded here, how old* Alexander *was when he held this diſcourſe,*

course, as also how old Clitus *and* Epheſtion *were?*

I put for the age of *Epheſtion* 1 R of years, whence it follows, that *Alexander* had 1 R + 2; therefore *Clitus* had 2 R + 6, and thoſe three together, according to the condition of the queſtion, ought to be equal to 96: therefore there is an equality between 4R + 8 (which is the ſum of the three ages) and 96, take away 8 from both parts of the equation, ſo there will remain on one ſide 4 R equal to 88, which divide by the number of the greateſt Coſſick Character, that is to ſay by 4, the quotient gives 22 for the value of one root, which was ſuppoſed for the age of *Epheſtion.* Therefore *Epheſtion* was at that time aged 22 years, *Alexander* 24, and *Clitus* 50, which altogether make 96 years.

Queſtion II.

A Hare is a 100 *Geometrical paces diſtant from a Dog that ſwiftly purſues her, and the Dog runneth two times and half faſter than the Hare. It is demanded how many Geometrical paces the Hare will have run when the Dog overtaketh her?*

I put for theſe Geometrical paces 1 R; therefore the Dog which runs 100 paces more than the Hare, will have run 100 + 1 R: and for that the Dog runs twice and an half ſwifter than the Hare, I take two numbers in like proportion to one another, that is to ſay 5 and 2, and conclude that there is the ſame proportion between 100 + 1 R to 1 R, as between 5 and 2; therefore the product of the firſt number 100 + 1 R multiplied by the laſt number 2 (which is 200 + 2 R) is equal to the product of the two means 1 R and 5 (which will be 5 R) therefore if you take from both parts 2 R, there will remain 200 by 3, which is the number of the greateſt Character, and find

find in the quotient 66 $\frac{2}{3}$, which will be the value of the root. I ſay therefore that the Hare will have run 66 Geometrical paces and $\frac{2}{3}$, when the Dog ſhall have overtaken her, and the Dog will have run 166 $\frac{2}{3}$ paces, which make twice and an half more than 66 $\frac{2}{3}$.

Queſtion III.

The Architect Vitruvius *in his* Ninth Book Chap. 3. *tells us, That* Archimides *found the quantity of ſilver that a Goldſmith had mixed in a Golden Crown which he had made for the King* Hiero *(who was obliged by Vow to preſent it to the gods) weighing* 100 *pounds. It is demanded by what means* Archimedes *could arrive to the knowledge of that ſecret?*

The common Opinion is, that he took two maſſes, the one of gold, the other of ſilver, which weighed as much as the Crown; afterward he filled a Veſſel up to the brim with water, which Veſſel was placed in ſome great Baſon, that the water that ſhould be forced out of the firſt Veſſel, might be preſerved and not loſt. Thirdly, he gently put in the two maſſes and the Crown, one after the other, into the prepared Veſſel, taking exact notice of the quantity of water that iſſued out of the Veſſel at each time, and concluding from thence, that the Goldſmith had mingled 6 pounds and $\frac{2}{3}$ parts of ſilver. We will ſuppoſe then, that the maſſe of gold weighing 100 pounds, did caſt out of the Veſſel 60 pounds of water, and that the maſſe of ſilver alſo weighing 100 pounds, caſt forth of the Veſſel 90 pounds of water, and that the Crown caſt forth 65 pounds. I put afterward for the ſilver mixed in the Crown 1 R, and conſtitute twice the Rule of Three after this manner.

If 100 pounds of gold give me 60 pounds of water, how much will 100—1 R? and I find $\frac{6000-60R}{100}$ for my fourth number.

Secondly,

Secondly, if 100 pounds of silver give me 90 pounds of water, how much shall 1 R? and I find $\frac{90R}{100}$. Now these pounds of water $\frac{6000-42R}{100}$ and $\frac{90R}{100}$ added together do make $\frac{6000+32R}{100}$ pounds of water cast out, which ought to be equal to 65 pounds of water, cast forth by the Crown, and therefore if we reduce them, we shall find 6000 -|- 30 R equal to 6500 (this reduction is made by multiplying the denominator 100 by 65, for seeing that this fraction $\frac{6000\times 32R}{100}$ is equal to 65, it shall be also equal to $\frac{65}{1}$, and therefore there will be the same proportion of the numerator 6000 -|- 30 R to the denominator 100, as of the second numerator 65 to 1. Therefore the product under the extreams 6000 -|- 30 R (is equal to the product of the means 6500) take away therefore from both parts 6000, and there will remain an equation between 500 and 30 roots, and therefore divide 500 by 30, the numbet of the greatest character, you shall have the value of the root $16\frac{2}{3}$ for the pounds of silver mingled by the Goldsmith in the Crown.

Mark well this kind of reduction once for all.

Sect. II. *Questions resolved by an* Equation *compounded.*

Question. I.

TO divide 8 *into two such numbers as their squares being added together, may make* 34?

I put for the first 1 R, therefore the second shall be 8 — 1 R, their squares are 1 Q, and 64 -|- 1 Q — 16 R, which added together, do give for their sum 64 -|- 2 Q — 16 R, the question imports that the sum of the squares is 34. Therefore there is an Equation between 64 -|- 2 Q — 16 R, and 34, which being reduced by addition and substraction, there will remain

also

alſo an Equation between 2 Q and 16 R — 30, and the whole divided by 2, which is the number of the greateſt Coſſick Character, there will yet remain an Equation between 1 Q and 8 R — 15, from which I extract the root, as hath been ſhewn in the fifth Section of the ſecond Chapter. The half of the root is 4, his ſquare is 16, from which take the abſolute number 15, reſt 1, whoſe ſquare root 1 added to the half of the number of roots, gives for its ſum 5, which is the value of the root; therefore the two numbers ſought ſhall be 5 and 3.

Queſtion II.

To find two numbers whoſe product may be 12, *and the difference of their ſquares* 32 ?

I put for the one of them 1 R; therefore ſeeing that the product is 12, the other number ſhall be $\frac{12}{1R}$ (for if the product of two numbers be divided by one of thoſe two numbers, the quotient ſhall be the other number) their ſquares are 1 Q and $\frac{144}{1Q}$, whoſe difference is $\frac{144}{1Q}$ — 1 Q equal to 32, as appears by the queſtion; therefore there will be an equality between $\frac{144}{1Q}$ and 32 + 1 Q, and therefore if we make the reduction, as in the third Queſtion of the firſt Section, we ſhall alſo find an Equation between 144 and 32 Q + 1 QQ, alſo between 144 — 3 Q and 1 QQ; it behoveth then to extract the ſquare root of the number 144 — 32 Q. The half of 32 is 16, whoſe ſquare is 256, to which add 144, makes 400, whoſe ſquare root is 20, from which take the half of 32, to wit 16, there remains 4. See here the ſquare root, but ſeeing that the ſquared ſquare root ought to be taken, I take again the root of 4, and I find 2 for the value of the root. Therefore ſeeing that the ſecond number hath been put $\frac{12}{1R}$ the ſame ſecond number ſhall be $\frac{12}{2}$, that is to ſay 6,

Queſt.

Question III.

Two Merchants joyn in Company, and together bring in the sum of 165 Crowns; but the first mans money hath been exposed twelve months intire, and the second mans money only eight months: it happens that they gain but 28 Crowns, which added to 165 make 193, which they distribute to one another in such sort, as the first takes 67 Crowns, as well for his principal money as for his profit; and the second takes 126 Crowns, the question is what each of those Merchans brought into Stock?

I put for the money of the first man 1 R, therefore seeing that the sum of both was 165 Crowns, the second money is 165 — 1 R. Now if you take away 1 R, which is the sum the first man brought in, from the sum he received, which was compounded of the principal and profit; you will find that the first mans profit will be 67 — 1 R, and by the same Argument you shall find, that the second mans profit will be 1 R — 39. Now you must find what one root gaineth in eight months, which will be done by the Rule of Three, thus, If in 12 moneths there be gained 67 — 1 R, how much will there be gained in eight moneths, and the fourth number shall be $\frac{134}{3} - \frac{2R}{3}$ for the first mans profit in eight moneths; after that I seek what the second man hath gained by another operation of the Rule of Three, saying, If 1 R gain $\frac{115}{3} - \frac{3R}{2}$ what will 165 — 1 R gain? and I find for my fourth number $\frac{7370 + \frac{2}{3}Q - \frac{464}{3}R.}{1R}$ which is equal to the second mans profit, which we have already found to be 1 R — 39, and therefore by Reduction there will be an equation between 1 Q — 39 R and 7370

$7370 + \frac{2}{3}Q - \frac{464}{3}R$, all which 39 R being added and $\frac{2}{3}Q$ taken away, there will be an equation between 7370 and $\frac{1}{3}Q + \frac{347}{3}R$, and consequently between $7370 - \frac{347}{3}R$ and $\frac{1}{3}Q$, and therefore multiplying all by $\frac{1}{3}$, which is the number of the greatest character, there will be yet an equation between 1 Q and $22110 - 347$ R, out of which the square root must be extracted. The half of the number of roots is $\frac{347}{2}$, whose square is $\frac{120409}{4}$, which added to 22110, make $\frac{208849}{4}$, whose square root is $\frac{457}{2}$, from which if you take half the number of roots, there will remain $\frac{110}{2}$, that is to say, 55 for the value of one root, and was the money the first man put in bank; and for that we have found in the pursuit of these operations, that the first mans profit was $67 - 1$ R, that same profit will be $67 - 55$, that is to say 12, by the same reason the second mans stock shall be 110, and his profit 16.

Sect. III. *Questions resolved by* Surd Numbers.

Question I.

To divide any given number (as for example 4) *according to mean and extream reason*, that is to say, *to divide* 4 *into two numbers, in such manner as that the whole* 4 *may bear the same proportion to its greater part, as the greatest part bears to the least?*

I put for the greatest part 1 R, therefore the least shall be $4 - R$. Therefore there is the same proportion of 4 to 1 R, as of 1 R to $4 - 1$ R, and therefore the square of the middle part 1 Q is equal to the product of the extreams $16 - 4$ R, from which I extract the root according to the rule before prescribed. The half of the number of roots is 3, whose square 4, add to 16, makes 20; out of which the square

root

root ought to be extracted according to the precept; but seeing it is no square number, I must content my self by putting the radical sign before it thus, R Q 20, from which I take the half of the number of roots, and I have for residue R Q 20 — 2, which is the value of the root; by which I shall find with facility, that the other part will be 6 — R Q 20. For the proof of this operation, it behoveth that these two parts added together make 4, and that the lesser 6 — R Q 20, being multiplied by 4, make the product equal to the square of the greater part R Q 20 — 2.

Question II.

To divide 8 *into two parts, between which* 2 *may be mean proportional.*

I put for the first part 1 R, therefore the lesser shall be 8 — 1 R, and seeing that 1 R and 2, and 8 — 1 R, ought to be proportional, it behoveth that the square of 2, which is 4, be equal to the product of the extreams, which is 8 R — 1 Q, and therefore after the reduction, it will be found that 1 Q is equal to 8 R — 4, whose square root is R Q 12 + 4. For the one part of 8, and for the other part 4 — R Q 12, both the one and the other root do resolve the question, as you will find, if you take the pains to examine it.

From this practice may be framed an universal Canon, which may serve for the resolution of an infinite number of Algebraical Problems, which may be conceived after this manner. The sum given, which containeth the two extreams, ought to be distributed into two equal parts, that the square of the half may be taken, from which the square of the mean proportional given, must be taken, and the square root of the residue added, and taken from the half of the given sum, will shew the two parts sought.

As

As for example, I take the half of 8, which is 4, whose ſquare is 16, from which I take 4, the ſquare of 2, which is the given mean, and there remains 12, whoſe ſquare root added to the ſame half, makes $4 + R Q 12$, and taken from the ſame half, makes $4 - R Q 12$.

Queſtion III.

To divide any given number (as for example 4) *into three numbers continually proportional, in ſuch ſort as that the ſquares of the extreams joyned together, may be triple the ſquares of the mean.*

I put 1 R for the number of the middle part, then ſeeing all three ought to make the ſame ſum of 4, we ſhall have for the ſum of the extreams $4 - 1$ R. Now ſeeing that of three numbers continually proportional, the ſquare of the ſum of the extreams is equal to the ſquares of the extreams, and to the double of the ſquare of the middle part; I take the ſquare of this ſum $4 - 1$ R, which is $16 + 1 Q - 8$ R, from which I take 2 Q, which is the double of the ſquare of the middle part, and there will remain $16 - 1 Q - 8$ R for the ſum of the ſquares of the extreams: Therefore ſeeing that the condition of the queſtion requireth a triple proportion, there will be an equation between $16 - 1 Q - 8$ R and 3 Q; add therefore 1 Q on both ſides of the equation, and you ſhall have 4 Q equal to $6 - 8$ R, and dividing the whole by 4, which is the number of the greateſt character, the equation will be 1 Q equal to $4 - 2$ R, whoſe ſquare root is $R Q 5 - 1$ for the middle number ſought, and the ſum of the extreams ſhall be $5 - R Q 5$, which being divided into two parts, by the Canon of the precedent queſtion, in ſuch ſort as $R Q 5 - 1$, be the middle proportional, you will find that the extreams are 2 and $3 - R Q 5$, therefore the three numbers are

2 and R Q 5 — 1, and 3 — R Q 5, which altogether make 4, and are in continual proportion, and the ſquares of the extreams are triple the ſquare of the mean.

Sect. IV. *Geometrical queſtions reſolved by* Algebra.

Queſtion I.

Before you attempt the reſolution of ſuch queſtions, you muſt draw their figures.

There is a piece of ground of a greater length than breadth, whoſe angles are right angles, and in a triple proportion, and their ſquares taken together, are quintuple their ſum. The ſides, the diameter, and the capacity or ſuperficies of that piece of ground is demanded.

I put for the leaſt ſide 1 R, therefore ſeeing they are in a triple proportion, the other ſide ſhall be 3 R, their ſquares ſhall be 1 Q and 9 Q, which added together make 10 Q, which ought to be quintuple, the ſum of the numbers. Now the ſum of the numbers is 4 R, and its quintuple 2 R, and by conſequence, ſee here an equation between 10 Q and 20 R, which are two collateral Characters, and therefore dividing 20 by 10, which is the number of the greateſt character, you ſhall find 2 for the leaſt ſide, therefore the greateſt ſide ſhall be 6. Therefore the ſuperficies ſhall be 12, and the diameter R Q 40.

Queſtion. II.

There is an equilateral triangle, whoſe ſuperficies is R Q 243. The ſide and perpendicular is demanded.

Suppoſing that the perpendicular of an equilateral triangle doth always divide the ſide into two equal parts, I put for the half of the ſide divided, 1 R; therefore the ſide ſhall be 2 R. Now ſeeing that in every

every equilateral triangle; the ſquare of the ſide is equal to the ſquare of the perpendicular joyned to the ſquare of the half of the ſide, which is 1 Q of the ſquare of the whole ſide, which is 4, I ſhall have 3 Q for the ſquare of the perpendicular, and ſo R Q 3 Q ſhall be perpendicular, which is multiplied by the half of the ſide, which is 1 R (reducing it firſt to its ſquare becauſe of the radical ſign, which is in the number multiplied) you ſhall have R Q 3 QQ for the ſuperficies of the triangle: therefore there will be an equation between R Q 243 and R Q 3 QQ, and therefore there will be alſo an equation between their ſquares, which are 243 and 3 QQ, and the whole being divided by 3, the number of the greateſt character, there will be yet an equation between 81 and 1 QQ. extract therefore the ſquared ſquare root of 81, and have 3 for the half of the ſide, 6 for the ſide, R Q 27 for the perpendicular, and R Q 243 for the ſuperficies of the triangle.

Queſtion III.

There is a Semicircle, whoſe Diameter is divided according to mean and extream reaſon; on which there is raiſed a perpendicular produced to the circumference, and the leſſer line which is drawn from the extremity of the diameter, to this point of the circumference, is R Q 20 — 2, the quantity of the Diameter, and of its parts, and of this perpendicular, is demanded.

To reſolve this queſtion, it is preſuppoſed that the greater part of the diameter ſhall be equal to the given line, as may with facility be Geometrically demonſtrated. That being done, I put for the leſſer part of the diameter 1 R; therefore ſeeing that the other part is given R Q 20 — 2, the whole diameter ſhall be R Q 20 — 2 + 1 R, which multiplied by 1 R, giveth

for the product R Q 20 — 2 R + 1 Q, equal to the ſquare of the given quantity, which is 24 — R Q 320, and by due tranſpoſition you ſhall find 1 Q equal to 24 — R Q 320 + 2 R — R Q 20, from which the ſquare root ought to be extracted, taking exact notice that the particles which have the Coſſick Characters may hold place, with the number of roots. I conſider therefore in this term of the equation, the number of roots which is 2 — R Q 20, whereof I take the half, which is 1 — R Q 5, to whoſe ſquare 6 R Q 20, the abſolute number 24 — R Q 320, ought to be added, and the ſum will be 30 — R Q 500, whoſe ſquare root ought to be extracted as from Apotomes, as hath been ſhewn in the laſt Section of the fourth Chapter, that root is 5 — R Q 5, which added to the half of the number of roots 1 — R Q 5 gives for the ſum 6 — R Q 20, which is the value of 1 root, that is to ſay, of the leſſer part of the diameter: and therefore if you add it to the greater part you ſhall have 4 for the quantity of the whole diameter, from whence the perpendicular is eaſily known, provided the rules of Geometry be in any reaſonable manner underſtood.

Sect. 5. *Queſtions reſolved by the* Second Roots.

Queſtion I.

THree men have amongſt them a ſum of money: The firſt ſaith to the ſecond, If you deliver me the half of your money, I ſhall have 100 *Crowns: The ſecond ſaith to the third, If you deliver me* $\frac{1}{3}$ *of your money, I ſhall have* 100 *Crowns: The third ſaith to the firſt, If you deliver me* $\frac{1}{4}$ *of your money, I ſhall have* 100 *Crowns. demand how much money each one hath.*

I put for the firſt mans money 1 R of Crowns, and ſo

for the second mans money 1 A, and for the third mans money 1 B: therefore the first which hath 1 R with $\frac{1}{2}$ of the second mans money shall have 1 R $+\frac{1}{2}$ A equal to 100, and by consequence $\frac{1}{2}$ A shall be equal to 100 — 1 R, and multiplying the whole by 2, 1 A shall be equal to 200 — 2 R. I begin again therefore the operation, and instead of 1 A, I put for my second number 200 — 2 R. Now the question requires that the second man (with $\frac{1}{3}$ of the third mans) shall have 100: therefore there will be an equation between 200 — 2 R $+\frac{1}{3}$ B, and between 100, add to both parts of the equation 2 R, and take away 200, there will yet remain an equation between $\frac{1}{3}$ B and 2 R — 100, and multiplying the whole by 3, you will have 1 B equal to 6 R — 300: That being found, I begin again the work, and instead of 1 B I put for the third number 6 R — 300, which added to $\frac{1}{4}$ of the first mans, make $\frac{25}{4}$ R — 300, which ought to be equal to 100, therefore if you add on both parts 300, there will be an equation between $\frac{25}{4}$ R and 400, therefore if divide 400 by $\frac{25}{4}$, you shall find 64 for one root; therefore the second, which had 200 — 2 R, shall have 200 — 128, that is to say 72, and the third shall have 84. Those three numbers do perfectly satisfie all the conditions of the question.

Question II.

Two men divide between themselves three hundred Crowns, in such sort, as that the second mans money divided by that of the first mans, makes $\frac{1}{2}$. *It is demanded how much each of them hath.*

I put for the first mans money 1 R, and that of the second 1 A, there is therefore an equation between 1 R $+$ 1 A, and 300, and therefore 1 A is equal to 300 — 1 R, therefore $\frac{300-1A}{1A}$ is equal to $\frac{1}{2}$, and by

conſequence in croſs multiplying theſe two fractions, I ſhall have 600 — 2 R equal to 5 R, and dividing 600 ſhall be alſo equal to 5 R, and dividing 600 by 5, I ſhall find 120 for the firſt mans money, ſo ſhall the other have 180.

Queſtion III.

Two find two numbers whoſe product may be 10, *and the ſum of their ſquares* 29.

I put for the firſt 1 R, and for the other 2 A, the product is 1 R A, equal to 10; therefore in dividing the whole by 1 R, there will be an equation between 1 A and $\frac{10}{1R}$, and therefore I begin again the operation, and put for the firſt 1 R, and for the ſecond $\frac{10}{1R}$, their ſquares are 1 Q $\frac{100}{1q}$ equal to 29, and after the reductions and extractions of the roots, I find 5 and 2 for my ſought numbers.

Sect. VI. *Queſtions* reſolved indefinitely.

THat queſtion is ſaid to be reſolved indefinitely, in which the numbers are demonſtrated in ſuch Algebraical terms, as do ſatisfie all the conditions of the queſtion propoſed.

Queſtion I.

To divide 12 *into four numbers Arithmetically and continual.*

I preſuppoſe that when there is four numbers in Arithmetical proportion, the ſum of the extreams is always equal to the ſum of the means: whence it follows, that in our queſtion the ſum of the extreams ſhall be 6, and the ſum of the means alſo 6. I put for the ſecond 1 R, therefore the third ſhall be 6 — 1 R, their

their difference is 2 R — 6, presupposing 1 R to be the greater number of the two; if therefore I add this difference to 1 R, I shall have 3 R — 6, and if I take it from 6 — 1 R, I shall have 12 — 3 R, and and therefore the four numbers in continued Arithmetical proportion, shall be 3 R — 6 | 1 R | 6 — 1 R | 12 — 3 R | and the question is indefinitely resolved, in pursuit of which you may take such a number as you please for the value of 1 R, provided nevertheless, that you do admit of fained numbers less than nothing: However, if you will have no other numbers than what are real, you ought to take the value of 1 R, beneath 4 and above 2, which will be easily understood by a little experience.

Take for Example $\frac{1}{2}$ for the value of 1 R, therefore the first number, which is 3 R — 6, shall be $1\frac{1}{2}$ — 6, that is to say $\frac{1}{2}$, the second shall be $\frac{1}{2}$, the third $\frac{2}{2}$, and the fourth $\frac{2}{2}$, which added together make 12, and are in continued Arithmetical proportion. And so you may take an infinite number of others.

Question II.

A Vintner hath three sorts of Wine, the first is worth 4, *the second* 6, *and the third* 10 *pence the pint; of these three sorts of Wine he desires to fill a Vessel which contains* 80 *pints, which may be worth* 8 *pence the pint; I demand how many pints he ought to take of each sort.*

You ought here to consider, that the number 80 must be divided into three such numbers, as the first multiplied by 4, the second by 6, and the third by 10, the sums of the three products added together, may make 640 (because that all the Wine which shall be in the Vessel to be filled will cost 640 pence, seeing that if one pint be worth 8 pence, 80 pints will be worth 640 pence) I put therefore for the third number 1 R,

which multiplied by 10, makes 10 R, which being taken from 640, doth leave for the residue 640 — 10 R, which is a number containing the first 4 times, and the second 6 times. On the contrary, seeing that the third hath been put 1 R, therefore 80 — 1 R, shall make the sum of the first and the second, which multiplied by 4, will give 320 — 4 R, which being substracted from 640 — 10 R, will leave 320 — 6 R double to the second number, and therefore the second shall be 160 — 3 R. In like manner the same sum 80 — 1 R multiplied by 6, will produce 480 — 6 R; therefore if you take away 640 — 10 R, there will remain 4 R — 160, double to the first, and therefore the first shall be 2 R — 80. See here the Question resolved indefinitely, the first number is 2 R — 80, the second 160 — 3 R, and the third 1 R. The terms between which you ought to take the value of 1 R are 53 $\frac{1}{3}$, and 40; if therefore you take 46 for the value of 1 R, you shall have 46 pints of Wine of 10 *d.* the pint, 22 of 6, and 12 of that of 4 pence the pint.

A special Note.

Here I intreat you to consider, That it is impossible perfectly to understand the Rule of Allegation, without the knowledge of *Algebra*: For if you propose this question to one skilful only in Arithmetick, he will give you for the 3 sought numbers, 40, 20, and 20. And if you tell him, that of the sort of Wine of 4 pence the pint, you have but 16 pints, he will remain astonished, whereas by your question resolved indefinitely by *Algebra*, you will be able to give satisfaction to this condition infinite kinds and manners.

Question III.

Two numbers are sought, which have 56 for the difference of their Cubes, and which added together make 6.

I

I put for the difference of those numbers 1 R, and if from the difference of those two Cubes you take the Cube of the difference of the sides, dividing this residue by the triple of the difference of the sides, you have for the quotient the product of the sides, it follows, that if from 56 you take 1 C, and that the residual $56 - 1\,C$ be divided by the difference of the sides, which is 1 R, the quotient $\frac{56}{1} - \frac{1\,C}{R}$ shall be triple the product of the sides; and therefore if you divide this quotient by 3, you will have for the product of the sides $\frac{56}{3} - \frac{1\,C}{R}$, which is equivalent to $\frac{56}{3R} - \frac{1}{3}q$, therefore if you divide 6 (which is the sum of your two sought numbers) into two parts, whose product may be $\frac{56}{3R} - \frac{1}{3}q$, you will have the question resolved indefinitely. Now to attain this, I have given you a Canon in the second Question of the third Section, the half of the sum is 3, whose square is 9, from which if you take the product found, there will remain $9 + \frac{1}{3}q - \frac{56}{3R}$, whose square root added, and taken from the half of the sum, gives for the resolution $3 + R\,(9 + \frac{1}{3}q - \frac{56}{3R})$ which shall be the greatest sought number, and $3 - R\,Q\,(9 + \frac{1}{3}q - \frac{56}{3R})$ which shall be the least. Now these two numbers resolves the Question indefinitely, in such sort, that if you take two for the value of the root, you will find that your two sought numbers shall be 4 and 2, and every other number (taken for the value of 1 R) above 2, will resolve the Question.

Appendix.

Appendix.

Queſtions in *Algebra*, moſt of which require the Rule of Three in their Operation.

Queſtion I.

A Merchant receives in exchange for 568 Crowns, four kinds of Money; of the firſt 7 make one Crown, of the ſecond 18, of the third 21, and of the fourth 28 make one Crown. Moreover, he received of each ſort of Money alike number. I demand how much he received of each kind of Money?

Put 1 R for the quantity of each kind of Money, and then conſtitute the Rule of Three after this manner:

						Crowns,
If	7	Pieces of	Crown	money	I an-	$\frac{1}{9}$ R
	18	money be	1 what	1 R	ſwer	$\frac{1}{18}$ R
	21	worth	is			$\frac{1}{21}$ R
	28					$\frac{1}{28}$ R

For if 7 pieces of money make 1 Crown, 1 R of money will make $\frac{1}{7}$ R of Crowns, and ſo of the reſt. Now their Fractions deſcribed in the fourth place, do make together $\frac{71}{252}$ R of Crowns, equal to the number of 568 Crowns. Divide therefore 568 by $\frac{71}{252}$ and you have for the value of 1 R 2106, and ſo many pieces of each kind of money he received: which thus I prove:

If

Crowns.

If { 7 / 18 / 21 / 28 } Pieces are worth { 1 Crown } what { Pieces 2016 / Answer { 288 / 112 / 96 / 72 } }

For 2016 pieces of the first kind of money do make 288 Crowns, and as many of the second kind make 112, as many of the third kind make 96, and of the fourth kind 72, which added together, do make 568 Crowns.

Question II.

A certain man hath two measures of Wine, the one worth 12 Crowns, the other 15. Now he desires of both these Wines to fill another equal measure, whose worth may be 13 Crowns. I demand what part of each of those Wines he must take to fill the other to be worth that price?

Put 1 R for the part of the measure of the worst Wine, and for the part of the measure of the best Wine 1 — 1 R, then work by the Rule of Three thus:

	Meas.	Cro.		Crowns
Worst Wine	1	12	1 R?	make 12 R
Best Wine	1	15	1 — 1 R?	make 15 — 15 R

For if one measure of the worst Wine be worth 12 Crowns, 1 R of one measure of the same Wine will be worth 12 R of Crowns, and if one measure of the best Wine be worth 15 Crowns 1 = 1 R of a measure of the same Wine will be worth 15 — 15 R. Therefore 1 R measure of the worst Wine, and 1 — 1 R measure of the best will be worth 15 — 3 R Crowns, which ought to be equal to 13 Crowns. Add therefore 3 R to each part of the equation, and the equation will be between 3 R + 13 and 15; take away therefore 13 from both sides, and the equation will be between

twen 3 R and 2. Divide therefore 2 by 3, and you ſhall have $\frac{2}{3}$ for the value of 1 R, and ſo much ought to be taken of the meaſure of the worſt Wine, and $\frac{1}{3}$ part of the meaſure of the beſt Wine, which thus I prove by the Rule of Three.

	Meaſ.	Cro.		Crowns	
Worſt Wine	1	12	$\frac{2}{3}$? worth	8	13 Cr.
	1	15	$\frac{1}{3}$?	5	

For $\frac{2}{3}$ of a meaſure of the worſt Wine is worth 8 Crowns, and $\frac{1}{3}$ of the meaſure of the beſt Wine is worth 5 Crowns, which added together make three.

Queſtion III.

I have a meaſure of Wine worth ten Crowns; How much Water muſt I mix with one meaſure, that a mixed (like) meaſure may be worth ſeven Crowns?

Put for the meaſure of Water 1 R, then frame the queſtion by the Rule of Three, thus:

Meaſ. Wine,	Meaſ. Water,	Crowns	Meaſ. mixt		Crowns
1 + 1 R		10	1?	worth	$\frac{10}{1 + 1\,R}$

For if a meaſure of Wine, together with 1 R of a meaſure of Water, be worth 10 Crowns, one meaſure of Wine and Water mingled together will be worth $\frac{10}{1 + 1R}$ and ſo the equation will be between $\frac{10}{1 + 1\,R}$ and 7, which by croſs multiplication is reduced to 10 and 7 + R, take away 7 from each part of the equation, and it will be between 3 and 7 R;

7 R; divide 3 by 7., and you have for 1R, $\frac{3}{7}$, and so much Water ought to be mingled with a measure of Wine, that a measure of the mixture may be worth seven Crowns, and is thus proved:

Measure of Wine and Water	Crowns	Measure	Crowns
$1\frac{3}{7}$	10	1 ? worth	7

For if 1 of Wine and $\frac{3}{7}$ of Water be worth 10 Crowns, 1 of that mixture is worth 7 Crowns.

Question IV.

There are in a certain Vessel 20 measures of Wine, of which each of them are worth 12 Crowns the measure: Now this Vessel is filled up with Water, and then one measure of this mixture is worth 10 Crowns, I demand the content of the Vessel.

Put for the measure $20 + 1$ R, then work by the Rule of Three thus:

Measures of Wine	Measures of Water	Cro.	Meas. Mixt	Crowns
$20 + 1$ R		240	1 ? worth	$\frac{240}{20 + 1 R}$

For if one measure of Wine be worth twelve Crowns, a measure of Wine $20 + 1$ R measure of Water together will be worth 240 Crowns: therefore one measure mixt will be worth

$\frac{240}{20 + 1 R}$ Crowns, so the Equation is found between $\frac{240}{20 + 1 R}$ and 10, which by cross multiplication

cation is reduced to 240 and 200 -|- 10 R; take away 200 from both parts, and there will remain an equation between 40 and 10 R: Divide 40 by 10, and you have 4 for the price of the root, and ſo many meaſures of Water were put into the Veſſel, and therefore the whole Veſſel contains 24 meaſures, thus proved: 24 meaſures Wine aud Water worth 240 Crowns, 1 meaſure worth 10 Crowns.

Queſtion V.

Two Letter-Carriers belonging to two Cities diſtant each from other 140 leagues ſet forth towards one another, at one and the ſame time; the one travels eight leagues a day, the other ſix. I demand on what day they ſhall meet together.

Put 1 R for the day, then work by the Rule of Three thus:

Dayes	Leagues	Dayes	Leagues
1	8	1 R?	8 R.
1	6	1 R?	6 R.

The firſt therefore in 1 R of days ſhall travel 8 R of leagues, and the later 6 R. and both of them together will have meaſured 14 R, of leagues, that is 140 leagues. There is therefore an equation between 14 R and 140. Divide then 140 by 14, and you ſhall have ten for the value of 1 R. So the tenth day finiſhed, they met together, which thus I prove.

Dayes	Leagues	Dayes	Leagues
1	8	10?	80
1	6	10?	60

For the firſt in 10 days went 80 leagues, and the later went 60 leagues, which both together make 140, the diſtance of the two Cities from one another.

Queſtion

Queſtion VI.

A certain Merchant bought a quantity of Wooll, and another quantity of Wax, which coſt him together 124 *Crowns. Now* 100 *pound weight of Wooll coſt him* 7 *Crowns, and* 100 *pound of Wax coſt him* 14 *Crowns; but the quantity of Wooll that he bought was double to the quantity of Wax. I demand how many pounds of each ſort the Merchant bought?*

Put for the Wax 1 R, and for the Wooll 2 R of pounds, then work by the Rule of Three thus:

Pounds	Crowns	Pounds	Crowns
100 Wooll	7	2 R Wooll?	$\frac{14}{100}$ R
100 Wax	14	1 R Wax?	$\frac{14}{100}$ R

Therefore there will be an equation between $\frac{28}{100}$ R of Crowns, and 124 Crowns. Divide therefore 124 by $\frac{28}{100}$, and you ſhall have for the value of 1 R 442$\frac{4}{7}$, and ſo many pounds of Wax he bought, and of Wooll 885 $\frac{1}{7}$, which is thus proved:

100 Pounds, Wooll	7 Crowns	885 Pounds $\frac{1}{7}$	62 Cr.
100 Pounds, Wax	14 Crow.	442 Pounds $\frac{4}{7}$	62 Cr.

And ſo for Wooll and Wax together he expended 124 Crowns.

Queſtion VII.

One buys a number of Ells of Velvet, which he ſelleth again; he buys 5 *Ells for* 7 *Crowns, and ſells* 7 *Ells for* 11 *Crowns, and gained on the Bargain* 100 *Crowns. I demand how many Ells of Velvet he bought and ſold in all?*

Put

Put for the quantity of Ells 1 R, then work by the Rule of Three thus:

Ells	Crowns	Ells	Crowns
5	7	1 R?	$\frac{7}{5}$ R
7	11	1 R?	$\frac{11}{7}$ R

You see if $\frac{7}{5}$ R of Crowns which he laid out be substracted from $\frac{11}{7}$ R of Crowns which he received, there will remain $\frac{6}{35}$ R of Crowns for the profit. Therefore the equation will be between $\frac{6}{35}$ R and 100 R, divide 100 by $\frac{6}{35}$, and you have 583 $\frac{1}{3}$ for the value of 1 R, and so many Ells he bought and sold. Thus proved:

Ells	Crowns	Ells	Crowns
5	1	583 $\frac{1}{3}$?	816 $\frac{2}{3}$
7	11	583 $\frac{1}{3}$?	916 $\frac{2}{3}$

By which you may perceive there is 100 Crowns gotten.

Question VIII.

A certain man buys a number of Ells of Velvet, paying 11 Crowns for 7 Ells: Now he sells the whole again after the rate of 5 Ells for 7 Crowns, and lost 100 Crowns by the bargain. I demand how many Ells he bought and sold in all.

Put for the number of Ells 1 R, and then work by the rule of Three thus:

Ells	Crowns	Ells	Crowns
7	11	1 R?	$\frac{11}{7}$ R.
5	7	1 R?	$\frac{7}{5}$ R.

Then

Then if $\frac{7}{5}$ R of crowns which he received, be subtracted from $\frac{11}{7}$ R of crowns which he expended, the loss will happen to be $\frac{4}{35}$ R of crowns. So the equation will be between $\frac{4}{35}$ R of crowns, and 100 crowns. Divide therefore 100 by $\frac{4}{35}$, and you shall have for the value of 1 R, $583\frac{1}{3}$ Ells, and so many Ells were bought and sold, which thus I prove:

Ells	Crowns	$583\frac{1}{3}$?	$916\frac{2}{3}$
7	11		
5	7	$583\frac{1}{3}$?	$816\frac{2}{3}$

Where you see he lost 100 crowns by the bargain.

Question IX.

A man buys 100 pounds of Wax for 17 crowns. I demand how many pounds he must sell for one crown, that so on 102 crowns he may gain 18 crowns?

Put 1 R for the number of pounds, then work by the Rule of Three thus framed:

Crowns	Pounds	Crowns	Pounds
17	100	102	600
5	1 R	132 + 18?	120 R

For 102 crowns do give 600 pounds, and if for one crown there be given 1 R of pounds, there will be 120 R of pounds given for 102 + 18, which said 120 are equal to 600 pounds, the quantity of the Wax sold. Therefore the equation shall be between 120 R of pounds and 600 pounds. Divide therefore 600 by 120, and so you have for the value of 1 R, 5 pounds, and so many pounds are sold for one crown; so as that in 600 pounds 18 crowns may be gained on 102, thus proved:

Pounds	Crowns	Pounds	Crowns
100	17	600?	102
5	1	600?	120

Where in the first example 600 pound made 102, in this it makes 120, that is 102 + 18.

Question X.

A Man buys 100 pounds of Wax for 17 crown s, disposing of which he loseth 18 crowns, on 102 crowns. I demand how many pounds he sold for one crown?

Put for the number of pounds 1 R, then work by the Rule of Three, thus constituted.

Crowns	Pounds	Crowns	Pounds
17	100	102?	600
1	1 R	102—18?	48 R.

So there will be equatiou between 84 R of pounds and 600 pounds. Divide therefore 600 by 84, and the value of 1 R will be 7 pounds and $\frac{1}{7}$, and so many pounds he sold for one crown, and lost 18 crowns on 102 by the bargain, which thus I prove:

Pounds	Crowns	Pounds	Crowns
100	17	600?	102
$7\frac{1}{7}$	1	600?	84

Where you see he laid out 102 crowns for 600 pounds, and instead thereof received but 84 crowns, that is 102—18 crowns.

Question. XI.

A certain man agrees with a servant for 12 months service, for ten crowns and a coat; but at the end of seven months, he gives him the coat and two crowns

crowns. I demand then at what rate he eſteemed the coat.

Put for the price of the coat 1 R of crowns, and ſay by the rule of Three. If 12 moneths require 1 R + 10 crowns, how much will one moneth require? and you ſhall find that it will require $\frac{1\,R + 10}{12}$ of crowns: Again ſay, If 7 moneths require 1 R + 2 crowns, how much will one moneth require, and you ſhall find it to be $\frac{1\,R + 2}{7}$ as here under appears.

Moneths	Crowns	Moneth	Crowns
12	1 R + 10	1?	$\frac{1\,R + 10}{12}$
7	1 R + 2	1?	$\frac{1\,R + 2}{7}$

Therefore there will be an equation between $\frac{1\,R + 10}{12}$ and $\frac{1\,R + 2}{7}$ ſeeing both are the reward of one moneth, which equation by croſs multiplication is reduced to 7 R + 70, and 12 R + 24, take away 24 from both parts, and the equation will be between 7 R + 46 and 12 R. Again, take away 7 from both parts, then it will be 46, equal to 5 R; divide 46 by 5, and the price of a root, and ſo of the coat is 9 $\frac{1}{5}$ crowns, as appears hereunder. The reward of 12 moneths is 19 $\frac{1}{5}$, and of 7, 11 $\frac{1}{5}$.

Moneths	Crowns	Moneths	Crowns
12	19 $\frac{1}{5}$	7?	11 $\frac{1}{5}$

Queſtion XII.

A certain Citizen agrees with a ſloathful ſervant for 30 days, that every day he wrought he would give him 7 Groats; but for every day that he idled, and wrought not, he was to allow his Maſter five Groats: when the 30 days were paſt, it happens, that the ſervant was to receive nothing from his Maſter, nor the Maſter from the Servant, I demand then, how many days he laboured, and how many he idled?

Put 1 R for the days of labour, and 30 — 1 R for the days of idleneſs, and then frame the Rule of Three thus:

Day	Groats	Days	Groats
Labour 1	7	1 R?	7 R
Idleneſs 1	5	30—1 R?	150—5 R

Now ſeeing that his work and play came to one reckoning, there will be an equation between 7 R and 150—5 R, add 5 R to each part, and the equation will be between 12 R and 150. Divide therefore 150 by 12, and you have for the value of 1 R, $12\frac{1}{2}$, and ſo many days he laboured, and 17 days $\frac{1}{2}$ he idled, which is thus proved:

Day	Groats	Days	Groats
Labour 1	5	$12\frac{1}{2}$	$87\frac{1}{2}$
Idleneſs 1	1	$17\frac{1}{2}$	$87\frac{1}{2}$

Where you ſee the reward is the ſame with the mulct.

Queſtion XIII.

One ſells 20 pound weight, part Saffron, and part Ginger, for 45 crowns; but he ſold 1 pound of Saffron for three crowns, and 1 pound of Ginger for $\frac{1}{2}$ a crown.

The

The question is, how many pounds of each sort he sold?

Put for the Saffron 1 R of pounds, and for the Ginger 20 — 1 R of pounds, then by Rule of Three work thus:

	Pound	Crowns	Pounds	Crowns
Saffron	1	3	1 R?	3 R
Ginger	1	½	20 — 1 R?	$\frac{20 - 1\text{ R}}{2}$

Therefore the equation is between the sum of 3 R of crowns, and $\frac{20 - 1\text{ R}}{2}$ of crowns added together, and 45 crowns, now that sum is 10 + ½ R of crowns (for $\frac{20 - 1\text{ R}}{2}$ is equal to 10 1 ½ R, to which if you add 3 R of crowns, it makes the sum 10 + ½ R) therefore the equation shall be between 10 + ½ R of crowns, and 45 crowns. Take away 10 from both parts, and it will be between ½ R and 35. Divide therefore 25 by ½, and you shall have 14 for the value of 1 R. And so many pounds of Saffron were sold, and 6 pounds of Ginger, which thus I prove:

	Pound	Crowns	Pounds	Crown
Saffron	1	3	14?	42
Ginger	1	½	6?	3

Where you see that the price of 14 pound Saffron, and 6 pound Ginger added make 45 crowns.

Question XIV.

A certain Tradesman hath 2 sorts of coyn, in number

560 pieces, worth 160 crowns; a certain part thereof is worth each piece $\frac{1}{3}$ of a crown, and each piece of the rest $\frac{1}{4}$ of a crown. I demand the number of the first and later sort of money?

Put 1 R for the first, and 560 — 1 R for the later, and then constitute the Rule of Three after this manner:

Money	Crown	Money	
1	$\frac{1}{3}$	1 R?	$\frac{1}{3}$ R
1	$\frac{1}{4}$	560 — 1 R?	$\frac{560 - 1 R}{4}$

The fourth number found is equal to 160 crowns, and the sum of their numbers makes $140 + \frac{1}{12}$ R (for $\frac{560 - 1 R}{4}$ is equal to $140 + \frac{1}{4}$ R, to which if you add $\frac{1}{3}$ R, makes the sum $140 + \frac{1}{12}$ R.) There is therefore an equation between $140 + \frac{1}{12}$ R, and 160 crowns. Take away 140 from both parts, and then the equation is between $\frac{1}{12}$ and 20. Divide therefore 20 by $\frac{1}{12}$, and you have 240 for the value of 1 R, and so much money there was of the first sort, of which each piece was worth $\frac{1}{3}$ of a crown of the latter kind 320, each worth $\frac{1}{4}$ of a crown, Thus proved:

Money		Money	Crowns
1	$\frac{1}{3}$	240?	80
1	$\frac{1}{4}$	320?	80

Where you see the numbers in the fourth place make 160 crowns.

Question XV.

In the Army of the Emperour, the number of the Infantry

Infantry were octuple to the number of the Cavalry, among them there is distributed 392000 *crowns, so as that every Foot Soldier had* 5 *crowns, and every Horseman* 16. *The question is, Of how many Horsemen the Army consisted, and of how many Footmen?*

Put 1 R for the Horsemen, and 8 R for the Foot, according to the condition of the question, and then constitute the Rule of Three thus:

	Crowns	Horsemen	Crowns	
Horse 1	16	1 R	16 R	} 56 R
Foot 1	5	Foot 8 R	40 R	}

Therefore 56 R of crowns shall be equal to 392000 crowns: wherefore divide 392000 by 56, and you shall have 1 R 7000 for the number of Horsemen; therefore the Foot shall be 56000, eight times as many, and so there will be distributed to the Horsemen 112000 crowns, and to the Foot 280000, which together make 392000 crowns.

Question XVI.

A man hath a certain sum of money in a purse, which a stander by judgeth to be 600 *crowns, whose errour he thus corrects. If to what I have in this purse, there be added* $\frac{1}{2}$, $\frac{1}{3}$ *and* $\frac{1}{4}$, *and from the sum there be substracted* $\frac{1}{12}$ *part of my money, then I should have* 600 *crowns. The question is, How many crowns he had in the purse?*

Put 1 R for the number of crowns. If the parts $\frac{1}{2}$ R, $\frac{1}{3}$ and $\frac{1}{4}$ R, which together make 1 $\frac{1}{12}$ R, be added, to 1 R, the whole makes 2 $\frac{1}{12}$ R; take away $\frac{1}{12}$ R, and there will remain 2 R equal to 600. Divide therefore 600 by 2, and you have 300 for the value of 1 R, and so much money was in the purse. For if you add the $\frac{1}{2}$, the $\frac{1}{3}$, and $\frac{1}{4}$, to wit 150, 100, and 75, it will make the sum of 625, and taking away $\frac{1}{12}$, to

wit 25, the reſt will be the number 600, which reſolves the queſtion.

Queſtion XVII.

A certain Traveller goes 9 miles a day, another Traveller after the tenth day paſt, begins his journey from the ſame place, and goes every day 14 miles. I demand in how many days the later will overtake the firſt.

Put 1 R for the number of days, therefore the firſt over and above 90 miles, which he hath gone in ten days, goeth in 1 R of days, beſides 9 R of miles, ſeeing that every day he goeth 9 miles. But the latter going 14 miles each day, goeth 1 R of days, 14 R of miles, and becauſe the firſt of neceſſity in 1 R of days goes as many miles together with 90, which he went in ten days as the latter went in 1 R of days; ſith that then they are to meet together, the equation ſhall be between 9 R + 90 and 14 R. Take away 90 from both parts, and it will be between 90 and 5 R. Divide therefore 90 by 5, and 1 R makes 18. Therefore in 18 days they ſhall come together. For the firſt in 18 days went 162 miles, which added to 90, he made the firſt 10 days, makes 252 miles, which the later went in 18 days.

Queſtion XVIII.

A Traveller goes nine miles a day, another Traveller after the end of ten days begins the ſame journey. I demand, how many miles a day the later ought to travel, that ſo in 18 days he may overtake the firſt.

Put 1 R for the miles. Therefore in 18 days he will have travelled 18 R of miles. Seeing therefore that the firſt travelling every day 9 miles, went in 18 days 162 miles, and adding thereto 90, which he went

went the first ten days before the second set forth, it is manifest, that he travelled 252 miles. Therefore the equation is between 18 R and 252. Divide 252 by 18, and you have 14 for the value of 1 R, and so many miles the later ought to travel, to overtake the first in 18 days.

Question XIX.

A certain man dying, made his Will and Testament, leaving 3000 *Crowns to be distributed between his Wife, Son, and two Daughters, on this condition, that the Portion of the Son might be double to that of the Mother, and the Portion of the Mother double also to the Portion of each of the Daughters. The question how much each one's Portion was?*

Put for the Portion of one of the Daughters 1 R, for the Mothers Portion 2 R, and for the Sons Portion 4 R. So there will be an equation between 8 R and 3000 Crowns.

The Portion of	one Daughter	1 R	that is	375 Cro.
	the other Daugh.	1 R		375
	the Mother	2 R		750
	the Son	4 R		1500

Divide 3000 by 8, so the value of 1 R will be 375, the Portion of one of the Daughters, and therefore the Mothers Portion will be 750, and the Sons 1000.

Question XX.

A certain man receiveth of a Merchant, a quantity of Saffron for 10 *Crowns; and again he receives of the same man* 24 *pounds more of Saffron, at length he returns to him* 30 *pounds thereof again, and the Merchant computing the*

the price of the Saffron, reſtoreth to him 14 Crowns. I demand the price of a pound of that Saffron.

Here you ſee 10 crowns, + 24 pounds to be the whole debt which the Buyer owed to the Merchant, and in like manner 30 pounds — 14 crowns. Therefore there will be an equation between 10 crowns + 24 pounds and 30 pounds — 14 crowns. Add 14 crowns on both parts, and the equation will be between 24 crowns + 24 pounds and 30 pounds, take away 24 pounds from both parts, and it will be between 24 and 6 pounds. Divide 24 by 6, and you have 4 for the root, and ſo many crowns one pound of Saffron is worth, which I prove thus, 2 ½ pounds are bought for 10 crowns, and ſo the Buyer received of the Merchant 26 ½, which were worth 106 crowns. If therefore to the Merchant there be reſtored 30 pounds, the Merchant oweth to the Buyer 3 ½, ſeeing he received only 26 ½ but 3 ½ pound are worth 14 crowns, which the Merchant rendered to the Buyer.

Queſtion XXI.

Two men enter into Fellowſhip in Trade, now the ſecond brings with him double the money that the firſt brings, and 5 Crowns over and above. They gain by their Traffick 960 Crowns, of which the firſt takes to himſelf 300 Crowns, and the ſecond 660. I demand how much each put in bank?

Put for the firſt 1 R, and for the ſecond 2 R + 5, the ſum of both together is 3 R + 5, which have gained 960 Crowns. Then work by the Rule of Three thus:

3 R + 5 — 960 — 1 R ? — $\frac{960\ R}{3\ R + 5}$

You ſhall find that the firſt which brought in 1 R, hath gained $\frac{960\ R}{3\ R + 5}$ which number is equal

qual to 300 Crowns which he received. This equation by croſs multiplying will be reduced to 960 R, equal to 900 R + 1500. Take away therefore 900 R from both parts, and there will remain 60 R equal to 1500, divide 1500 by 60, and you have 25 for one R, and ſo much the firſt put in bank, therefore the ſecond put in 55 Crowns, and is thus proved: For both put in 80 Crowns. 80. 960 25 ? 300 | 80. 960 55 ? 660.

Queſtion XXII.

Three Merchants gain together 700 Crowns, which thus they diſtribute amongſt themſelves (having regard to the ſum each one brought into Bank) ſo as that the portion of the ſecond ſurmounted the portion of the firſt by 12 Crowns, and the portion of the third ſurpaſſed that of the ſecond by 16 Crowns. I demand how much each mans portion was?

Put for the portion of the firſt man 1 R, and then ſhall the portion of the ſecond be 1 R + 12, and the portion of the third 1 R + 28. Theſe 3 portions together make 3 R + 40, equal to 700. Take away 40 from both parts of the equation, and the equation will be between 3 R and 660. Divide therefore 660 by 3, and you ſhall have 220 for the value of 1 R, and ſo much was the firſt mans portion, ſo the portion of the ſecond ſhall be 232, and of the third 248, which all together make 700.

Queſtion XXIII.

A Caterer buyes a number of Fowls, ſo as that if he had bought the $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ of the number, and over and above 6, he would then have 100 juſt. I demand the number of Fowls he bought?

Put 1 R for the number, whoſe $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ make $\frac{47}{60}$ R, which

which with 6, make $\frac{47}{60}$ R + 6 equal to 100. Take away 6 from both parts of the equation, and the remaining equation will be between $\frac{47}{60}$ R and 94. Divide therefore 94 by $\frac{47}{60}$, and the value of 1 R will be 120, to wit, the number of Fowls that were bought.

For its $\frac{1}{3}$ is 40, and its $\frac{1}{4}$ is 30, and its $\frac{1}{5}$ 24; which all together with 6 make the sum 100.

Some Examples in *ALGEBRA* concerning *Squares.*

To find two numbers in a given Exceſs, ſo as their Squares may have alſo a given Exceſs.

LEt there be ſought two numbers, whoſe Exceſs is 4, and the Exceſs of their Squares 144.

Put for the leſſer number 1 R, and therefore the greater number muſt be 1 R + 4, whoſe ſquares are 1 Q and 1 Q + 8 R + 16, their exceſs is 8 R + 16, which ought to be equal to 144. Take away 16 from both parts of the equation, and the equation will be between 8 R and 128. Divide therefore 128 by 8, and you ſhall have for the value of 1 R, 16 for the leſſer number, the greater therefore will be 20, that the exceſs may be 4. The ſquares of thoſe 2 numbers are 256, and 400, whoſe difference or exceſs is 144.

Two numbers being given, to find another, with which multiplying both the numbers, makes the greater number a ſquare, and its leſſer the ſide of that ſquare.

LEt the two given numbers be 200 and 5, and let the ſought number be put 1 R. Now 1 R multiplied in 200, produceth 200 R, and 1 R multiplied in 5, makes 5 R, which ought to be the ſide, and ſo multiplied

multiplied in it ſelf, ought to make a number equal to 200. But 5 R multiplied in it ſelf, makes 25 Q. the equation therefore is between 200 R and 25 Q. Divide therefore 200 by 25, and the value of 1 R is 8, which multiplied in 200, make the ſquare 1600, whoſe ſide is the number 40, which is alſo produced by the multiplication of 8 in 5.

Some Examples relating to *Cubes*.

To find a number, which multiplied in it ſelf, and the product multiplied by ſome given number, may produce a number in a given proportion to the Cube *of the found number.*

LEt the given number be 20, find another number, which being firſt multiplied in it ſelf, and then the product multiplied by the given number 20, may produce a number in a quintuple proportion to the Cube made of the found number.

Put for the ſought number 1 R, which multiplied in it ſelf make 1 Q, which alſo multiplied by 20, makes 20 Q, the Cube of 1 R is 1 C, to which 20 Q ought to have a quintuple proportion; ſo the equation is between 20 Q and 5 C: Divide 20 by 5, and you have for the value of 1 R, 4, the number ſought. This 4 multiplied in it ſelf makes 16, and 16 multiplied by 20, makes 320, which is quintuple to 64, the Cube of 4.

To divide a given number in two parts, ſo as that their Cubes may make a given ſum, which ſhall be greater than the quarter part of the Cube deſcribed of the given number.

LEt the given number be 10, to be divided into two parts, whoſe Cubes may make 370, which number is greater than the $\frac{1}{4}$ part of the Cube of 10. Put for the firſt number compounded of 1 R, and half of the given number, 1 R + 5, and for the ſecond 5 — 1 R. So theſe two numbers do make the given number. Their Cubes are C ✠ 15 Q + 75 R ✠ 125, and 125 — 75 R ✠ 15 Q — 1 C; their ſum is 30 Q ✠ 250: For ✠ 1 C, and — 1 C, as alſo + 75 R, and — 75 R, do mutually deſtroy one another; and of 15 Q and 15 Q, are made 30 Q: Alſo 125, and 125, make 250. Therefore the equation is between 30 Q + 250, and 370. Take away 250 from both parts, and the equation will be between 30 Q and 120. Divide 120 by 30, and you have 4 for the value of 1 Q, and the value of 1 R, 2. The firſt part put 1 R ✠ 5 ſhall be 7; and the ſecond put 5 — 1 R ſhall be 3. The Cubes of thoſe parts are 343 and 27; which together make 370.

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